2/14/11

- warm up by finish the last problem on the 7.3 worksheet
- lecture 7.4

2/16/11

- Prepare by completing 7.1 - 7.4 homework so you have questions ready

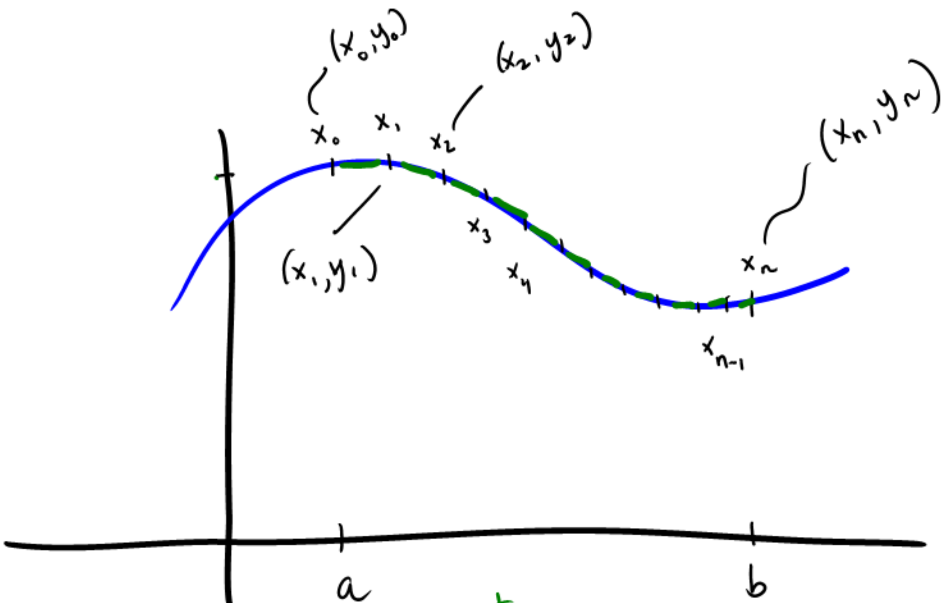
FRIDAY & next Monday

- DO NOT KILL BRAIN CELLS

AND

- PRACTICE CALCULUS!!!

Exam 2 / 7.1-7.4 is on 2/23/11



distance for 1 segment

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$x_i - x_{i-1} = \text{change in } x = \Delta x_i$$

$$y_i - y_{i-1} = \text{change in } y = \Delta y_i$$

arc length

$$s \approx \sum_{i=1}^{n} \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$

$$s = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2 \frac{(\Delta x_i)^2}{(\Delta x_i)^2}}$$

$$s = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + \frac{(\Delta y_i)^2}{(\Delta x_i)^2} \cdot (\Delta x_i)^2}$$

$$s = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 \left[1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2\right]}$$

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

MATH 251/GRACEY NAME____________________________________

WORKSHEET/CHAPTER 7.4~~/7.5~~

ARC LENGTH AND AREA OF A SURFACE OF REVOLUTION

Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a,b]$. The arc length of f between a and b is

$$s = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx,\ y \text{ is a function of } x,$$

If $x = g(y)$ on the interval $[c,d]$, then the arc length of g between c and d is

$$s = \int_c^d \sqrt{1+\left[g'(y)\right]^2}\,dy,\ x \text{ is a function of } y$$

---

1. Find the arc length of the graph of the function $y = \frac{3}{2}x^{2/3}$ over the interval $[1,4]$.

$y' = x^{-1/3}$

$(y')^2 = x^{-2/3}$

$1+(y')^2 = 1 + x^{-2/3}$

$= x^{-2/3}\left(x^{2/3}+1\right)$

$\sqrt{1+(y')^2} = \sqrt{\dfrac{x^{2/3}+1}{x^{2/3}}}$

$= \dfrac{\sqrt{x^{2/3}+1}}{x^{1/3}}$

$s = \displaystyle\int_1^4 \frac{\sqrt{x^{2/3}+1}}{x^{1/3}}\,dx$

$u = x^{2/3}+1$

$\dfrac{du}{dx} = \dfrac{2}{3}x^{-1/3}$

$dx = \dfrac{3}{2}x^{1/3}\,du$

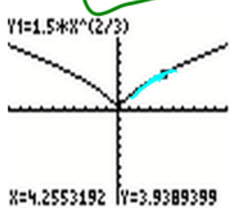
$s = \displaystyle\int \frac{u^{1/2}}{\cancel{x^{1/3}}}\left(\frac{3}{2}\cancel{x^{1/3}}\,du\right)$

$s = \dfrac{3}{2}\displaystyle\int u^{1/2}\,du$

$s = \dfrac{\cancel{3}}{\cancel{2}}\,\dfrac{u^{3/2}}{\cancel{3}/\cancel{2}}$

$s = \left(x^{2/3}+1\right)^{3/2}\Big|_1^4$

$\boxed{s = \left(4^{2/3}+1\right)^{3/2} - 2^{3/2}}$



Is $\sqrt{4+9} \stackrel{?}{=} \sqrt{4} + \sqrt{9}$

$(4+9)^{1/2} \stackrel{?}{=} 4^{1/2} + 9^{1/2}$

$13^{1/2} \stackrel{?}{=} 2+3$

$3.6 \stackrel{?}{=} 5$ NOT

correct

NOT UNLESS YOU'VE BEEN DRINKING

so let's go with $\sqrt{a+b} = \sqrt{a+b}$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$= \frac{99}{48} = \boxed{\frac{33}{16} \text{ units}}$

2. Find the arc length of the graph of the function $y=\frac{x^3}{6}+\frac{1}{2x}$ over the interval $\left[\frac{1}{2},2\right]$.

$(y')^2 = \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2$

$(y')^2 = \frac{x^4}{4} - \cancel{2}\left(\frac{x^2}{\cancel{2}}\right)\left(\frac{1}{2\cancel{x^2}}\right) + \frac{1}{4x^4}$

$(y')^2 = \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}$

$\sqrt{1+(y')^2} = \sqrt{\frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4}}$

$= \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}$

$= \frac{1}{2}\left(x^2 + x^{-2}\right)$

$S = \frac{1}{2}\int_{1/2}^{2}\left(x^2+x^{-2}\right)dx = \frac{1}{2}\left(\frac{x^3}{3} - \frac{1}{x}\right|_{1/2}^{2}$

$= \frac{1}{2}\left[\left(\frac{8}{3} - \frac{1}{2}\right) - \left(\frac{1}{24} - 2\right)\right]$

Let $y=f(x)$ have a continuous derivative on the interval $[a,b]$. The area S of the surface of revolution formed by revolving the graph of about a horizontal or vertical axis is

$$S = 2\pi\int_a^b r(x)\sqrt{1+\left[f'(x)\right]^2}\,dx,$$

y is a function of x, where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x=g(y)$ on the interval $[c,d]$, then the surface area is

$$S = 2\pi\int_c^d r(y)\sqrt{1+\left[g'(y)\right]^2}\,dy,$$

x is a function of y, where $r(y)$ is the distance between the graph of g and the axis of revolution.

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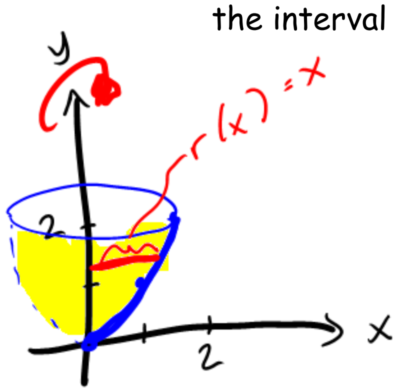
3. Find the **area of the surface** formed by revolving the graph of $f(x)=x^2$ on the interval $\left[0,\sqrt{2}\right]$ about the y-axis.



$\frac{dy}{dx} = 2x$

$S = 2\pi\int_0^{\sqrt{2}}(x)\sqrt{1+(2x)^2}\,dx$

$S = 2\pi\frac{1}{8}\int_0^{\sqrt{2}} 8x\left(1+4x^2\right)^{1/2}dx$

$S = \frac{\pi}{4}\cdot\frac{2}{3}\left[\left(1+4x^2\right)^{3/2}\right]_0^{\sqrt{2}}$

$S = \frac{\pi}{6}\left(9^{3/2}+1^{3/2}\right)$

$S = \frac{\pi}{6}(27+1)$

$S = \frac{28\pi}{6}$

$\boxed{S = \frac{14\pi}{3}\text{ units}^2}$