1/24/11

- Warm-up
  - → take out a blank piece of paper and write out the unit circle
- Finish 5.6

$$\frac{d}{dx}(\arcsin u) = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}(\arccos u) = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}(\operatorname{arcsec} u) = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}(\arctan u) = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}(\operatorname{arccot} u) = -\frac{u'}{1+u^2}$$

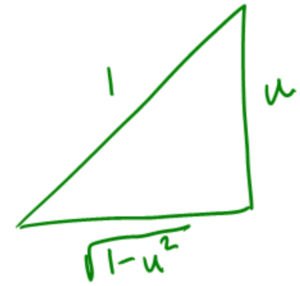
Wednesday

5.7

$u = f(x)$

$\sin y = \frac{u}{1}$

$$\sin y = \sin(\arcsin u)$$



$$\frac{d}{dx}\sin y = \frac{d}{dx}(u)$$

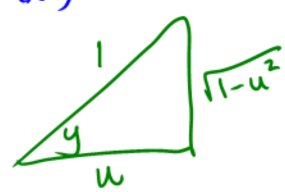
$$(\cos y)\frac{dy}{dx} = \frac{u'}{\cos y}$$

$$\frac{dy}{dx} = \frac{u'}{\left(\frac{\sqrt{1-u^2}}{1}\right)}$$

$$\frac{dy}{dx} = \frac{u'}{\sqrt{1-u^2}}$$

$u = f(x)$

$$\cos y = \cos(\arccos u)$$



$$\frac{d}{dx}\cos y = \frac{d}{dx}u$$

$$(-\sin y)\left(\frac{dy}{dx}\right) = u'$$

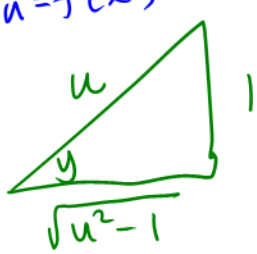
$$\frac{dy}{dx} = -\frac{u'}{\sin y}$$

$$\frac{dy}{dx} = -\frac{u'}{\frac{\sqrt{1-u^2}}{1}}$$

$$\frac{dy}{dx} = -\frac{u'}{\sqrt{1-u^2}}$$

$u = f(x)$

$$\csc y = \csc(\operatorname{arccsc} u)$$



$$\frac{d}{dx}\csc y = \frac{d}{dx}u$$

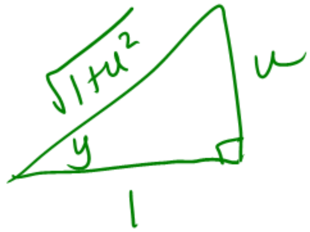
$$(-\csc y \cot y)\left(\frac{dy}{dx}\right) = u'$$

$$\frac{dy}{dx} = -\frac{u'}{\csc y \cot y}$$

$$\frac{dy}{dx} = \frac{-u'}{\left(\frac{u}{1}\right)\left(\frac{\sqrt{u^2-1}}{1}\right)}$$

$$\boxed{\frac{dy}{dx} = -\frac{u'}{|u|\sqrt{u^2-1}}}$$

$$u = f(x)$$



$$\tan y = \tan(\arctan u)$$

$$\frac{d}{dx}(\tan y) = \frac{d}{dx}(u)$$

$$(\sec^2 y)\frac{dy}{dx} = u'$$

$$\frac{dy}{dx} = \frac{u'}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{u'}{\left(\frac{\sqrt{1+u^2}}{1}\right)^2}$$

$$\frac{dy}{dx} = \frac{u'}{1+u^2}$$

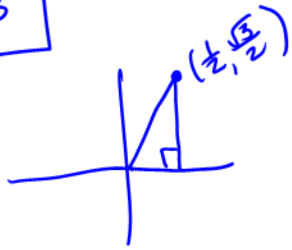
c. $\operatorname{arcsec}\left(\frac{2\sqrt{3}}{3}\right) = \boxed{\frac{\pi}{6}}$

$\frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$

e. $\arccos\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

d. $\arctan\left(\sqrt{3}\right) = \boxed{\frac{\pi}{3}}$

$\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}}$



f. $\operatorname{arccsc}\left(-\sqrt{2}\right) = \boxed{-\frac{\pi}{4}}$

$\csc\theta = \frac{1}{\sin\theta}$

and $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Example 2: Solve the equation for *x*.

$\tan\left(\arctan(2x-5)\right) = \tan(-1)$

$2x - 5 = \tan(-1)$

$$\boxed{\begin{aligned} x &= \frac{5 + \tan(-1)}{2} \\ x &\approx 1.72 \end{aligned}}$$

exact

approximate

Example 3: Write the expression in algebraic form. (HINT: Sketch a right triangle)

a. $\sec(\arctan 4x)$

Let $\theta = \arctan 4x$

$\tan\theta = \frac{4x}{1}$

So...

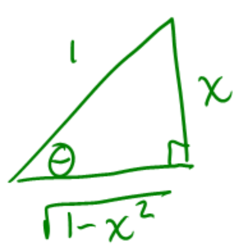
$\sec\theta = \frac{\sqrt{16x^2+1}}{1}$

$\boxed{\sec\theta = \sqrt{16x^2+1}}$

b. $\cos(\arcsin x)$

Let $\theta = \arcsin x$

$\sin\theta = \frac{x}{1}$



So...

$\cos\theta = \frac{\sqrt{1-x^2}}{1}$

$\boxed{\cos\theta = \sqrt{1-x^2}}$

Example 4: Differentiate with respect to *x*.

a. $\frac{\partial}{\partial x}(y) = \frac{\partial}{\partial x}(\arcsin x)$

$u = x$

$u' = 1$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$$

d. $y = \operatorname{arc}\csc x$

$$\boxed{\frac{dy}{dx} = -\frac{1}{|x|\sqrt{x^2-1}}}$$

b. $\frac{\partial}{\partial x} y = \frac{\partial}{\partial x} \arccos x)$

$$\boxed{\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}}$$

e. $y = \operatorname{arc}\sec x$

$$\boxed{\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}}$$

c. $y = \arctan x$

$$\boxed{\frac{dy}{dx} = \frac{1}{1+x^2}}$$

f. $y = \operatorname{arc}\cot x$

$$\boxed{\frac{dy}{dx} = -\frac{1}{1+x^2}}$$

What have we found out?!

DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$

2. $\frac{d}{dx}[\arccos u] = -\frac{u'}{\sqrt{1-u^2}}$

3. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$

4. $\frac{d}{dx}[\operatorname{arc}\cot u] = -\frac{u'}{1+u^2}$

5. $\frac{d}{dx}[\operatorname{arc}\sec u] = \frac{u'}{|u|\sqrt{u^2-1}}$

6. $\frac{d}{dx}[\operatorname{arc}\csc u] = -\frac{u'}{|u|\sqrt{u^2-1}}$

Example 5: Find the derivative of the function. Simplify your result to a single ration expression with positive exponents.

a. $\frac{\partial}{\partial t} f(t) = \frac{\partial}{\partial t}(\arcsin t^3)$

$u = t^3$

$u' = 3t^2$

$$f'(t) = \frac{3t^2}{\sqrt{1-(t^3)^2}}$$

$$\boxed{f'(t) = \frac{3t^2}{\sqrt{1-t^6}}}$$

b. $\frac{\partial}{\partial x} g(x) = \frac{\partial}{\partial x}\arcsin x + \frac{\partial}{\partial x}\arccos x$

$$g'(x) = \frac{1}{\sqrt{1-x^2}} + \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$g'(x) = 0$$

c. $\frac{\partial}{\partial x} y = \frac{\partial}{\partial x}(x \arctan 2x) - \frac{\partial}{\partial x}\left(\frac{1}{4}\ln\left(1+4x^2\right)\right)$

$$y' = 1\arctan 2x + x\left(\frac{2}{1+4x^2}\right) - \frac{1}{\cancel{4}}\left(\frac{\cancel{8}^{\,2}x}{1+4x^2}\right)$$

$$y' = \frac{(1+4x^2)\arctan 2x + \cancel{2x} - \cancel{2x}}{1+4x^2}$$ zero out

$$\boxed{y' = \arctan 2x}$$

d. $\frac{\partial}{\partial x} y = \frac{\partial}{\partial x}\left[25\arcsin\frac{x}{5} - x\sqrt{25-x^2}\right]$

$$\frac{dy}{dx} = \cancel{25}^{\,5}\left(\frac{\frac{1}{\cancel{5}_1}}{\sqrt{1-\left(\frac{x}{5}\right)^2}}\right) - \left(1\sqrt{25-x^2} + x\left(\frac{-\cancel{2}^{\,1}x}{\cancel{2}_1\sqrt{25-x^2}}\right)\right)$$

$$\frac{dy}{dx} = \frac{5}{\sqrt{\frac{25-x^2}{25}}} - \sqrt{25-x^2} - \frac{x^2}{\sqrt{25-x^2}}$$

$$\frac{dy}{dx} = \frac{25 - \left[\sqrt{25-x^2}\left(\sqrt{25-x^2}\right)\right] + x^2}{\sqrt{25-x^2}}$$

$$\frac{dy}{dx} = \frac{25-(25-x^2)+x^2}{\sqrt{25-x^2}}$$

$$\boxed{\frac{dy}{dx} = \frac{2x^2}{\sqrt{25-x^2}}}$$

Example 6: Find an equation of the tangent line to the graph of the function $y=\frac{1}{2}\arccos x$ at the point $\left(-\frac{\sqrt{2}}{2},\frac{3\pi}{8}\right)$.

$\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$

Step 1: Find dy/dx when $x=-\frac{\sqrt{2}}{2}$

$$\frac{dy}{dx}=\frac{1}{2}\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

at $x=-\frac{\sqrt{2}}{2}$

$$\frac{dy}{dx}=-\frac{1}{2\sqrt{1-\frac{1}{2}}}$$

$$\frac{dy}{dx}=-\frac{1}{\frac{2}{\sqrt{2}}}$$

$$\frac{dy}{dx}=-\frac{\sqrt{2}}{2}$$

Step 2: Find equation of line passing through $\left(-\frac{\sqrt{2}}{2},\frac{3\pi}{8}\right)$

$$\boxed{y-\frac{3\pi}{8}=-\frac{\sqrt{2}}{2}\left(x+\frac{\sqrt{2}}{2}\right)}$$