1/19/11

- Finish review
- Start 5.6

Friday

5.6

Monday

5.7

THEOREM: THE CONSTANT RULE

Let k be a real number.

$$\int k dx = x + C$$

Example 1: Find the indefinite integral.

$$\boxed{\int -3dx = -3x + C}$$

THEOREM: THE POWER RULE

Let n be a rational number.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Example 2: Find the following indefinite integrals.

a. $\boxed{\int x^{-5} dx = \frac{x^{-4}}{-4} + C}$

b. $\int x^{1/2} dx$

$\int x^{1/2} dx = \frac{x^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{3} x^{3/2} + C}$

c. $\int x^{-2/3} dx = \frac{x^{1/3}}{1/3} + C$

$= \boxed{3x^{1/3} + C}$

THEOREM: THE CONSTANT MULTIPLE RULE

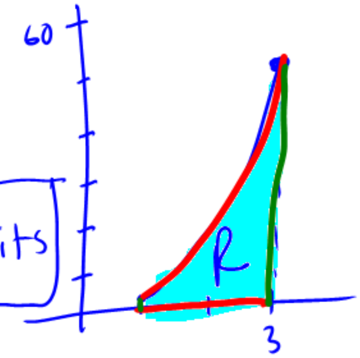
If f is an integrable function and c is a real number, then cf is also integrable and

$$\int cf(x)\,dx = c\int f(x)dx$$

Example 3: Find the area of the region bounded by $f(x) = 2x^3$, $x = 1$, $x = 3$, and $y = 0$.

$$A = \int_1^3 (2x^3 - 0)\,dx$$

$$A = \frac{\cancel{2}^{1} x^4}{2\cancel{4}}\Bigg|_1^3 = \frac{1}{2}(3^4 - 1^4) = \frac{1}{2}(81-1) = \boxed{40 \text{ sq. units}}$$



THEOREM: THE SUM AND DIFFERENCE RULES

The sum (or difference) of two integrable functions f and g is itself integrable. Moreover, the antiderivative of $f+g$ (or $f-g$) is the sum (or difference) of the antiderivatives of f and g.

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$$

Example 4: Find the indefinite integral.

a. $\int \left(\frac{\sqrt{x} - 5x^2}{\sqrt{x}} \right) dx = \int (1 - 5x^{3/2})\,dx$

$= x - 5\frac{x^{5/2}}{5/2} + C$

$= \boxed{x - 2x^{5/2} + C}$

b. $\int (x^3+1)^2\,dx = \int (x^6 + 2x^3 + 1)\,dx$

$= \frac{x^7}{7} + \frac{2x^4}{4} + x + C$

$= \boxed{\frac{x^7}{7} + \frac{x^4}{2} + x + C}$

THEOREM: ANTIDERIVATIVES OF THE TRIGONOMETRIC FUNCTIONS

$\int \sin x dx = -\cos x + C$	$\int \cos x dx = +\sin x + C$
$\int \csc x \cot x dx = -\csc x + C$	$\int \sec x \tan x dx = \sec x + C$
$\int \sec^2 x dx = \tan x + C$	$\int \csc^2 x dx = -\cot x + C$
$\int \tan x dx = -\ln\|\cos x\| + C$	$\int \cot x dx = \ln\|\sin x\| + C$
$\int \sec x dx = \ln\|\sec x + \tan x\| + C$	$\int \csc x dx = -\ln\|\csc x + \cot x\| + C$

$\sin^2 A = \frac{1-\cos 2A}{2}$

$\cos^2 A = \frac{1+\cos 2A}{2}$

Example 5: Integrate.

$u=2x$
$du=2dx$

a. $\int \sin^2 x dx = \frac{1}{2}\int \left(\frac{1-\cos 2x}{2}\right)dx$

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2} + C_1\right)$

$= \boxed{\frac{x}{2} - \frac{\sin 2x}{4} + C}$

$u=2x$

$\frac{du}{dx}=2 \quad dx=\frac{du}{2}$

$\int \cos 2x \, dx$

$= \int \cos u \frac{du}{2}$

$= \frac{1}{2}\sin u + C$

$= \frac{1}{2}\sin 2x + C$

b. $\int (-\csc\theta + \csc\theta\cot\theta)\,d\theta$

$= \boxed{\ln|\csc\theta + \cot\theta| - \csc\theta + C}$

c. $\int 3\tan x dx = \boxed{-3\ln|\cos x| + C}$

d. $\int \frac{1}{1+\cos\theta}d\theta = \int(\csc^2\theta - \csc\theta\cot\theta)d\theta = \boxed{-\cot\theta + \csc\theta + C}$

$\frac{1}{(1+\cos\theta)}\frac{(1-\cos\theta)}{(1-\cos\theta)}$

$= \frac{1-\cos\theta}{1-\cos^2\theta}$

$= \frac{1-\cos\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta} - \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}$

$= \csc^2\theta - \csc\theta\cot\theta$

THEOREM: ANTIDIFFERENTIATION OF A COMPOSITE FUNCTION

Let g be a function whose range is an interval I and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C$$

Letting $u = g(x)$ gives $du = g'(x)\,dx$ and

$$\int f(u)\,du = F(u) + C$$

Example 6: Find the following definite and indefinite integrals.

a. $\int \left(x\sqrt{1-x}\right)dx = \int x u^{1/2}(-du)$

$u = 1-x$, $x = 1-u$

$\frac{du}{dx} = -1$

$dx = -du$

$= -\int (1-u)u^{1/2}\,du$

$= -\int \left(u^{1/2} - u^{3/2}\right)du$

$= -\left(\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right) + C$

$= \boxed{-\frac{2}{3}(1-x)^{3/2} + \frac{2}{5}(1-x)^{5/2} + C}$

b. $-\frac{1}{4}\int -4x\left(5-2x^2\right)^5 dx = -\frac{1}{4}\frac{\left(5-2x^2\right)^6}{6} + C$

$g(x) = 5-2x^2$

$g'(x) = -4x$

$= \boxed{-\frac{1}{24}\left(5-2x^2\right)^6 + C}$

c. $\int \cos^2 3x dx$ $=\frac{1}{2}\int(1+\cos 6x)dx$

Recall...

$\cos^2 A = \frac{1+\cos 2A}{2}$

$\cos 6x$

$u=6x$

$\frac{du}{dx}=6$

$dx=\frac{du}{6}$

$=\frac{1}{2}\left(x+\int \cos u\left(\frac{du}{6}\right)\right)$

$=\frac{x}{2}+\frac{1}{12}\sin u + C$

$=\boxed{\frac{x}{2}+\frac{1}{12}\sin 6x + C}$

d. $\int\left(\frac{4+5x^{3/2}}{\sqrt{x}}\right)dx$ $=\int(4x^{-1/2}+5x)dx$

$=4\cdot 2x^{1/2}+\frac{5}{2}x^2+C$

$=\boxed{8x^{1/2}+\frac{5}{2}x^2+C}$

$(\cancel{5+x})(1+x)$

e. $\int_3^5 \frac{5+6x+x^2}{\cancel{5+x}}dx$ $=\int_3^5(1+x)dx$

$=\left(x+\frac{x^2}{2}\right|_3^5$

$=\left(5+\frac{25}{2}\right)-\left(3+\frac{9}{2}\right)$

$=2+8$

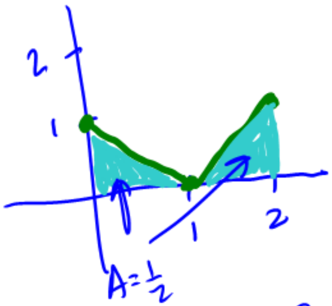
$=\boxed{10}$

f. $\int_0^2 |x-1|\,dx$ $= -\int_0^1 (x-1)\,dx + \int_1^2 (x-1)\,dx$

$= -\left(\frac{x^2}{2}-x\right)\Big|_0^1 + \left(\frac{x^2}{2}-x\right)\Big|_1^2$

$= -\left(-\frac{1}{2}\right) + \left[\left(\frac{4}{2}-2\right) - \left(\frac{1}{2}-1\right)\right]$

$= \frac{1}{2} + 0 - \left(-\frac{1}{2}\right)$ $= \boxed{1}$



$$|x-1| = \begin{cases} -(x-1), & x < 1 \\ (x-1), & x \geq 1 \end{cases}$$

g. $\int_{\pi/4}^{\pi/3} \tan^3 x \sec^2 x\,dx$

Theorem: LOG RULE FOR INTEGRATION

Let u be a differentiable function of x.

1. $\int \frac{1}{x}\,dx = \ln|x| + C$

2. $\int \frac{1}{u}\,du = \ln|u| + C$

Theorem: INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let u be a differentiable function of x.

1. $\int e^x dx = e^x + C$
2. $\int e^u du = e^u + C$
3. $\int a^x dx = \left(\dfrac{1}{\ln a}\right)a^x + C$, a is a positive real number, $a \neq 1$

Example 7: Find the following definite and indefinite integrals.

a. $\int \dfrac{5t^2 - t - 1}{2-t}dx = \int\left(-5t - 9 + \dfrac{17}{2-t}\right)dt$

$$\begin{array}{r} -5t - 9 + \frac{17}{-t+2} \\ (-t+2)\overline{\big)\,5t^2 - t - 1} \\ -(5t^2 - 10t) \\ \hline 9t - 1 \\ -(9t - 18) \\ \hline 17 \end{array}$$

$= -\frac{5}{2}t^2 - 9t$

$+17\int \frac{1}{u}(-du)$

$= \boxed{-\frac{5}{2}t^2 - 9t - 17\ln|2-t| + C}$

$u = 2-t$

$\frac{du}{dt} = -1 \rightarrow dt = -du$

b. $\int \dfrac{5}{\left(\sqrt{x}\ln x\right)^2}dx = 5\int \dfrac{1}{x(\ln x)^2}dx$

try $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$= 5\int \frac{1}{\cancel{x}}u^{-2}(\cancel{x}du)$

$= 5\,\frac{u^{-1}}{-1} + C$

$= \boxed{-\frac{5}{\ln x} + C}$

c. $\int\left(x + \dfrac{1}{x}\right)^2 dx = \int\left(x^2 + 2 + \frac{1}{x^2}\right)dx$

$= \boxed{\frac{x^3}{3} + 2x - \frac{1}{x} + C}$

d. $\int \dfrac{1}{x^{2/3}\left(1 + x^{1/3}\right)}dx = \int \frac{1}{\cancel{x^{2/3}}\cdot u}(3\cancel{x^{2/3}}du)$

try $u = 1 + x^{1/3}$

$\frac{du}{dx} = \frac{1}{3}x^{-2/3}$

$dx = 3x^{2/3}du$

$= 3\ln|u| + C$

$= 3\ln\left|1 + x^{1/3}\right| + C$

$= \boxed{\ln\left|\left(1 + x^{1/3}\right)^3\right| + C}$

e. $\int_1^2 \frac{e^x+e^{-x}}{e^x-e^{-x}}dx = \int_{e-e^{-1}}^{e^2-e^{-2}} \frac{\cancel{e^x+e^{-x}}}{u}\left(\frac{du}{\cancel{e^x+e^{-x}}}\right)$

$u=e^x-e^{-x}$

upper: e^2-e^{-2}

lower: $e-e^{-1}$

$\frac{du}{dx}=e^x+e^{-x}$

$dx=\frac{du}{e^x+e^{-x}}$

$= \ln|u| \Big|_{e-e^{-1}}^{e^2-e^{-2}}$

$= \ln|e^2-e^{-2}| - \ln|e-e^{-1}|$

$= \ln\left|\frac{e^4-1}{e^2}\right| - \ln\left|\frac{e^2-1}{e}\right|$

$= \ln\left|\frac{\frac{e^4-1}{\cancel{e^2}\,e}}{\frac{e^2-1}{\cancel{e}}}\right|$

$= \ln\left|\frac{e^4-1}{e(e^2-1)}\right|$

$= \ln\left|\frac{\cancel{(e^2-1)}(e^2+1)}{e\cancel{(e^2-1)}}\right|$

$= \boxed{\ln\left(\frac{e^2+1}{e}\right)}$

h. $\int_1^{\pi}\left(3-\frac{1}{2x}+\tan 2x\right)dx$

f. $\int_0^{2e}\frac{x}{1-x}dx$

i. $\int_{-\pi/2}^{\pi/2}\sin x\cos^2 x\,dx$

g. $\int_{\pi/3}^{\pi/2}(\sec^2 x)dx$

j. $\int 2^{-x}dx = \int e^{x\ln\frac{1}{2}}\,dx$

$2^{-x}=\left(\frac{1}{2}\right)^x$

$= e^{\ln\left(\frac{1}{2}\right)^x}$

$= e^{x\ln\left(\frac{1}{2}\right)}$

$= \int e^u\left(\frac{du}{\ln\frac{1}{2}}\right)$

$= \frac{1}{\ln\frac{1}{2}}e^u + C$

$= \boxed{\frac{\left(\frac{1}{2}\right)^x}{\ln\left(\frac{1}{2}\right)} + C}$

$u = x\ln\frac{1}{2}$

$\frac{du}{dx} = \ln\frac{1}{2} \rightarrow dx = \frac{du}{\ln\frac{1}{2}}$