THEOREM: THE CONSTANT RULE

Let k be a real number.

$$\int k\,dx = x + C$$

Example 1: Find the indefinite integral.

$\int -3\,dx = -3x + C$

THEOREM: THE POWER RULE

Let n be a rational number.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Example 2: Find the following indefinite integrals.

a. $\int x^{-5} dx = \frac{x^{-4}}{-4} + C$

b. $\int x^{1/2} dx = \frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C$

c. $\int x^{-2/3} dx = \frac{x^{1/3}}{1/3} + C = 3x^{1/3} + C$

THEOREM: THE CONSTANT MULTIPLE RULE

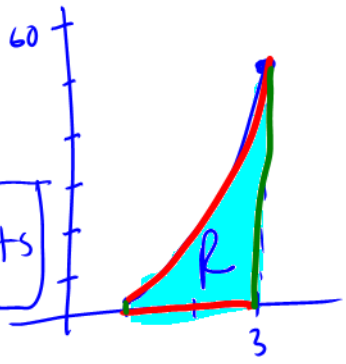
If f is an integrable function and c is a real number, then cf is also integrable and

$$\int cf(x)\,dx = c\int f(x)\,dx$$

Example 3: Find the area of the region bounded by $f(x)=2x^3$, $x=1$, $x=3$, and $y=0$.

$A=\int_1^3 (2x^3-0)\,dx$

$A=\left.\frac{2x^4}{2\cdot 4}\right|_1^3 = \frac{1}{2}(3^4-1^4)=\frac{1}{2}(81-1)=$ 40 sq. units



THEOREM: THE SUM AND DIFFERENCE RULES

The sum (or difference) of two integrable functions f and g is itself integrable. Moreover, the antiderivative of $f+g$ (or $f-g$) is the sum (or difference) of the antiderivatives of f and g.

$$\int[f(x)+g(x)]dx=\int f(x)\,dx+\int g(x)\,dx$$

$$\int[f(x)-g(x)]dx=\int f(x)\,dx-\int g(x)\,dx$$

Example 4: Find the indefinite integral.

a. $\int\left(\frac{\sqrt{x}-5x^2}{\sqrt{x}}\right)dx = \int(1-5x^{3/2})\,dx$

$= x-5\frac{x^{5/2}}{5/2}+C$

$= x-2x^{5/2}+C$

b. $\int(x^3+1)^2\,dx = \int(x^6+2x^3+1)\,dx$

$= \frac{x^7}{7}+\frac{2x^4}{4}+x+C$

$= \frac{x^7}{7}+\frac{x^4}{2}+x+C$

THEOREM: ANTIDERIVATIVES OF THE TRIGONOMETRIC FUNCTIONS

$\int \sin x dx = \cos x + C$	$\int \cos x dx = -\sin x + C$
$\int \csc x \cot x dx = -\csc x + C$	$\int \sec x \tan x dx = \sec x + C$
$\int \sec^2 x dx = \tan x + C$	$\int \csc^2 x dx = -\cot x + C$
$\int \tan x dx = -\ln\lvert\cos x\rvert + C$	$\int \cot x dx = \ln\lvert\sin x\rvert + C$
$\int \sec x dx = \ln\lvert\sec x + \tan x\rvert + C$	$\int \csc x dx = -\ln\lvert\csc x + \cot x\rvert + C$

$\sin^2 A = \frac{1-\cos 2A}{2}$

$\cos^2 A = \frac{1+\cos 2A}{2}$

Example 5: Integrate.

a. $\int \sin^2 x dx = \frac{1}{2}\int (1-\cos 2x)dx$ ($u=2x$, $du=2dx$)

$= \frac{1}{2}\left(x - \frac{\sin 2x}{2} + C_1\right)$

$= \frac{x}{2} - \frac{\sin 2x}{4} + C$

($u=2x$, $\frac{du}{dx}=2$, $dx=\frac{du}{2}$)

$\int \cos 2x dx$

$= \int \cos u \frac{du}{2}$

$= \frac{1}{2}\sin u + C$

$= \frac{1}{2}\sin 2x + C$

b. $\int (-\csc\theta + \csc\theta\cot\theta)d\theta$

c. $\int 3\tan x dx$

d. $\int \frac{1}{1+\cos\theta} d\theta$

THEOREM: ANTIDIFFERENTIATION OF A COMPOSITE FUNCTION

Let g be a function whose range is an interval I and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f\left(g\left(x\right)\right)g'\left(x\right)dx = F\left(g\left(x\right)\right)+C$$

Letting $u = g\left(x\right)$ gives $du = g'\left(x\right)dx$ and

$$\int f\left(u\right)du = F\left(u\right)+C$$

Example 6: Find the following definite and indefinite integrals.

a. $\int\left(x\sqrt{1-x}\right)dx$

b. $\int x\left(5-2x^2\right)^5\,dx$

c. $\int \cos^2 3x dx$

d. $\int \left(\frac{4+5x^{3/2}}{\sqrt{x}} \right) dx$

e. $\int_3^5 \frac{5+6x+x^2}{5+x} dx$

f. $\int_0^2 |x-1|\,dx$

g. $\int_{\pi/4}^{\pi/3} \tan^3 x \sec^2 x\,dx$

Theorem: LOG RULE FOR INTEGRATION

Let u be a differentiable function of x.

1. $\int \frac{1}{x}dx = \ln|x| + C$

2. $\int \frac{1}{u}du = \ln|u| + C$

Theorem: INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let u be a differentiable function of x.

1. $\int e^x dx = e^x + C$
2. $\int e^u du = e^u + C$
3. $\int a^x dx = \left(\frac{1}{\ln a}\right) a^x + C$, a is a positive real number, $a \neq 1$

Example 7: Find the following definite and indefinite integrals.

a. $\int \frac{5t^2 - t - 1}{2 - t} dx$

c. $\int \left(x + \frac{1}{x}\right)^2 dx$

b. $\int \frac{5}{\left(\sqrt{x} \ln x\right)^2} dx$

d. $\int \frac{1}{x^{2/3}\left(1 + x^{1/3}\right)} dx$

e. $\int_1^2 \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

f. $\int_0^{2e} \frac{x}{1-x} dx$

g. $\int_{\pi/3}^{\pi/2} \left(\sec^2 x\right) dx$

h. $\int_1^{\pi} \left(3 - \frac{1}{2x} + \tan 2x\right) dx$

i. $\int_{-\pi/2}^{\pi/2} \sin x \cos^2 x dx$

j. $\int 2^{-x} dx$