7/24/08

① Find the derivative

a) $y = \ln x^{x} = x \ln x$

$y' = 1 \ln x + \cancel{x}\left(\frac{1}{\cancel{x}}\right)$

$\boxed{y' = \ln x + 1}$

b) $\ln y = \ln \frac{\sqrt{x^2+4}}{2x^2} \rightarrow \ln y = \frac{1}{2}\ln(x^2+4) - \ln 2 - 2\ln x$

$\frac{y'}{y} = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{x^2+4}\right) - 0 - \frac{2}{x}$

$y' = y\left[\left(\frac{x}{x^2+4}\right) - \left(\frac{2}{x}\right)\right]$

$\boxed{y' = \frac{\sqrt{x^2+4}}{2x^2}\left[\frac{x}{x^2+4} - \frac{2}{x}\right]}$

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c) $$y = \frac{e^{x} - e^{-x}}{2} = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$y' = \frac{1}{2}\left(e^{x} - e^{-x}(-1)\right) = \boxed{\frac{1}{2}\left(e^{x} + e^{-x}\right)}$$

d) $$y = x^{2}e^{-x}$$

$$y' = 2xe^{-x} + x^{2}e^{-x}(-1)$$

$$\boxed{y' = xe^{-x}[2-x]}$$

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e) $f(x) = e^{\sqrt{x}}$

$f'(x) = e^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)$

$$\boxed{f'(x) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}}$$

f) Find $\frac{dy}{dx}$

$$\frac{d}{dx}\left[\boxed{e^{xy}} + x^2 - y^2\right] = (10)\frac{d}{dx}$$

(5)

$$e^{xy}\left[1y + xy'\right] + 2x - 2(y)^1 y' = 0$$

$$ye^{xy} + xy'e^{xy} + 2x - 2yy' = 0$$

$$y'\left(xe^{xy} - 2y\right) = -\left(ye^{xy} + 2x\right)$$

$$y' = \frac{-\left(ye^{xy} + 2x\right)}{xe^{xy} - 2y}$$

$$y' = \frac{ye^{xy} + 2x}{2y - xe^{xy}}$$

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② Integrate

a) $\int \frac{(\ln x)^3}{x}\,dx = \int \frac{(u)^3}{\cancel{x}}\cancel{x}\,du = \frac{u^4}{4} + C$

$u = \ln x$

$du = \frac{1}{x}\,dx \rightarrow dx = x\,du$

$\boxed{= \frac{(\ln x)^4}{4} + C}$

b) $\int \frac{x^3 - 6x - 20}{x+5}\,dx = \int x^2\,dx - 5\int x\,dx + \int 19\,dx - 115\int \frac{1}{x+5}\,dx$

$\boxed{= \frac{x^3}{3} - \frac{5x^2}{2} + 19x - 115\ln|x+5| + C}$

$$
\begin{array}{r}
x^2 - 5x + 19 - \frac{115}{x+5} \\
x+5 \,\overline{)\, x^3 + 0x^2 - 6x - 20} \\
-(x^3 + 5x^2) \\
\hline
-5x^2 - 6x \\
-(-5x^2 - 25x) \\
\hline
19x - 20 \\
-(19x + 95) \\
\hline
-115
\end{array}
$$

⑥

c) $\int \frac{e^{2x}}{1+e^{2x}}\,dx = \int \frac{\cancel{e^{2x}}}{u}\,\frac{du}{2\cancel{e^{2x}}}$

$u = 1+e^{2x}$

$du = 2e^{2x}\,dx$

$dx = \frac{du}{2e^{2x}}$

$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln\left(1+e^{2x}\right) + C$

$= \boxed{\ln\sqrt{1+e^{2x}} + C}$

d) $\int \frac{e^{2x} + 2e^{x} + 1}{e^{x}}\,dx$

$= \int e^{x}\,dx + \int 2\,dx + \int e^{-x}\,dx$

$u = -x$

$du = -dx$

$= \boxed{e^{x} + 2x - e^{-x} + C}$

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e) $\int \ln(e^{2x-1})\,dx = \int (2x-1)\,dx$

$= \boxed{x^2 - x + C}$

f) $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{\sin^2 x - \cos^2 x}{\cos x}\,dx = 2\int_0^{\pi/4} (\sec x - 2\cos x)\,dx$

$= 2\left[\ln|\sec x + \tan x| - 2\sin x\right]_0^{\pi/4}$

$= 2\left[\left(\ln|2+1| - 2\left(\frac{\sqrt{2}}{2}\right)\right) - \left(\ln|1+0| - 2(0)\right)\right]$

$= \boxed{2\left[\ln 3 - \sqrt{2}\right]}$

$\frac{\sin^2 x - \cos^2 x}{\cos x} = \frac{1-\cos^2 x - \cos^2 x}{\cos x}$

$= \sec x - 2\cos x$

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③ Derive...

a) $y = 4^x$

$y' = 4^x (1) \ln 4$

$\boxed{y' = (\ln 4) 4^x}$

b) $g(\alpha) = 5^{-\alpha/2} \sin 2\alpha$

$g'(\alpha) = \left[5^{-\alpha/2}\left(-\frac{1}{2}\right)\ln 5\right]\sin 2\alpha + 5^{-\alpha/2}(2\cos 2\alpha)$

$= \boxed{5^{-\alpha/2}\left(2\cos 2\alpha - \frac{\ln 5}{2}\sin 2\alpha\right)}$

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c) $y = \log_5 \sqrt{x^2-1} = \frac{1}{2}\log_5(x^2-1)$

$y' = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{\ln 5(x^2-1)}\right) = \boxed{\frac{x}{\ln 5(x^2-1)}}$

d) $f(t) = t^{3/2}\log_2\sqrt{t+1} = t^{3/2}\left[\frac{1}{2}\log_2(t+1)\right]$

$f'(t) = \frac{3}{2}t^{1/2}\left[\frac{1}{2}\log_2(t+1)\right] + t^{3/2}\left[\frac{1}{2}\frac{1}{\ln 2(t+1)}\right]$

$\boxed{f'(t) = \frac{t^{1/2}}{2}\left[\frac{3}{2}\log_2(t+1) + \frac{t}{\ln 2(t+1)}\right]}$

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④ Integrate

a) $\int (3-x)\left(7^{(3-x)^2}\right)dx$

$u=(3-x)^2$

$du = 2(3-x)(-1)\,dx$

$du = -2(3-x)\,dx$

$dx = \dfrac{du}{-2(3-x)}$

$= \int \cancel{(3-x)}\,7^u \dfrac{du}{-2\cancel{(3-x)}}$

$= -\frac{1}{2}\int 7^u\,du$

$= -\frac{1}{2}\left[\frac{1}{\ln 7}7^u\right] + C$

$\boxed{= -\dfrac{7^{(3-x)^2}}{2\ln 7} + C}$

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b) $\displaystyle\int_{-2}^{2} 4^{x/2}\,dx = 2\int_{-1}^{1} 4^{u}\,du = 2\,\frac{4^{u}}{\ln 4}\Bigg|_{-1}^{1}$

$u = \frac{x}{2}$ upper limit: $u = \frac{(2)}{2} = 1$

$du = \frac{1}{2}dx$ lower limit: $u = \frac{(-2)}{2} = -1$

$dx = 2du$

$= \frac{2}{\ln 4}\left(4^{1} - 4^{-1}\right)$

$= \frac{\cancel{2}}{\ln 4}\left(\frac{15}{\cancel{4}_{2}}\right)$

$= \frac{15}{2\ln 4}$ or

power rule $\longrightarrow \frac{15}{\ln 16}$

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