ch. 2 review

#36 position function $s(t) = -16t^2 + v_0 t + s_0$

$s_0 = 14{,}400$ ft

$v_0 = 0$

$0 = -16t^2 + 14400$

$16t^2 = 14400$

$t^2 = 900$

$t = 30$

$s'(t) = v(t)$

$v(t) = -32t + v_0$

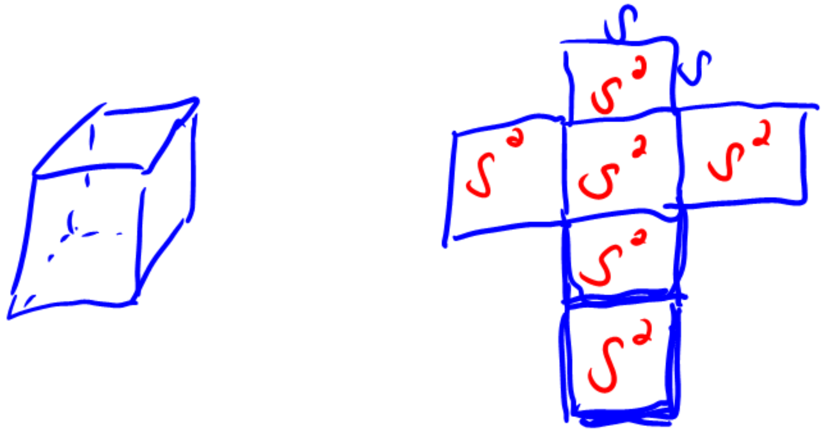
in ft/sec

→ 600 mph

$\frac{\cancel{600}\,mi}{1\,\cancel{hr}} \times \frac{1\,\cancel{hr}}{\cancel{3600}\,sec} = \frac{1}{6}\,m/sec$

at $t = 30$ sec, we have $\frac{1}{6}(30) = \boxed{5 \text{ miles}}/sec$

#110 The edges of a cube are expanding at a rate of 5 cm/sec. How fast is the surface area changing when each edge is 4.5 cm?



A = surface area

$\frac{ds}{dt} = 5\,cm/sec$

want $\frac{dA}{dt}$

when $s = 4.5\,cm$

$$\frac{d}{dt}A = \frac{d}{dt}6s^2$$

$$\frac{dA}{dt} = 12s\frac{ds}{dt}$$

$$\frac{dA}{dt} = 12(4.5)(5) = \boxed{270\ cm^2/sec}$$

#74 differentiate

$$\frac{d}{dx}y = \frac{d}{dx}\left(\frac{\sec^7 x}{7} - \frac{\sec^5 x}{5}\right)$$

$$y' = \frac{\cancel{7}(\sec x)^6}{\cancel{7}} \cdot \sec x \tan x - \frac{\cancel{5}(\sec x)^4}{\cancel{5}} \sec x \tan x$$

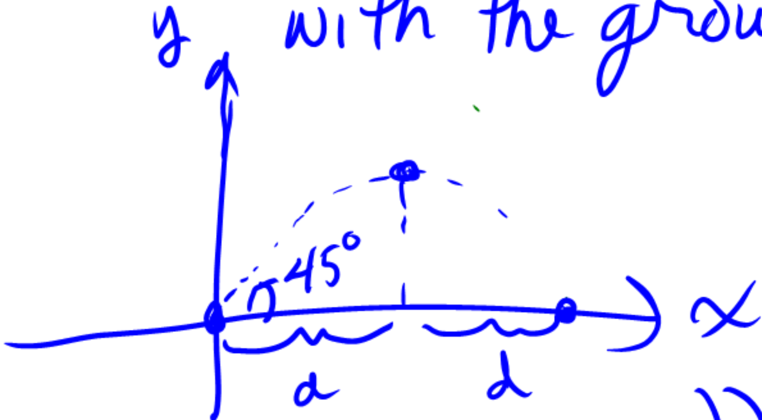
$$y' = \sec^5 x \tan x \left(\sec^2 x - 1\right)$$

$$y' = \sec^5 x \tan x \tan^2 x = \boxed{\sec^5 x \tan^3 x}$$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

#38 The path of a projectile thrown at an angle of 45° with the ground is $y = x - \frac{32}{V_0^2}(x^2)$



$$0 = x - \frac{32}{V_0^2}(x^2)$$

Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$$-\frac{b}{2a} = \frac{-1}{-2\left(\frac{32}{V_0^2}\right)} = \frac{V_0^2}{64}$$

this is the x-coord where the max height

$$0 = x\left(1 - \frac{32}{V_0^2}x\right)$$

$x = 0$ or $1 - \frac{32}{V_0^2}x = 0$

this is the starting point

$$1 = \frac{32}{V_0^2}x$$

$$x = \frac{V_0^2}{32}$$

this is the x-coord where it hits the ground

b) $y = x - \frac{32}{v_0^2}x^2$

$y' = 1 - \frac{64}{v_0^2}x \longrightarrow$ at $x = \overset{\text{input of}}{\text{max height}} = \frac{v_0^2}{64}$

$\boxed{\frac{64}{v_0^2} = 1}$

$y' = 1 - \frac{\cancel{64}}{\cancel{v_0^2}}\left(\frac{\cancel{v_0^2}}{\cancel{64}}\right) = 1 - 1 = \boxed{0}$

c) double the initial velocity

$2v_0$

max height: $f\left(-\frac{b}{2a}\right) = f\left(\frac{v_0^2}{64}\right)$

$f(x) = x - \frac{32}{v_0^2}x^2$

$f\left(\frac{v_0^2}{64}\right) = \frac{v_0^2}{64} - \frac{\cancel{32}}{\cancel{v_0^2}}\left(\frac{v_0^{\cancel{4}\,2}}{\cancel{64}_2 \cdot 64}\right) = \frac{2v_0^2 - v_0^2}{128} = \frac{v_0^2}{128}$

what changes when we replace $2v_0$

$(2v_0)^2 = 4v_0^2$ so $\frac{v_0^2}{128}$ would be $4\left(\frac{v_0^2}{128}\right)$ when we double the initial velocity. Since the max of the function is the y-coord. of the vertex this will increase the range by a factor of 4 as well.

d) set $v_0 = 70$: $y = \frac{v_0^2}{128}$ is the max

$y = \frac{70^2}{128} \doteq 38.3$ ft is the max height

R: $[0, 38.3\text{ft}]$

#90 $y = 2\csc^3\sqrt{x}$; $(1, 2\csc^3 1)$

$$y' = \cancel{6}^{3}(\csc\sqrt{x})^2(-\csc\sqrt{x}\cot\sqrt{x})\frac{1}{\cancel{2}\sqrt{x}}$$

$$y' = \frac{-3\csc^3\sqrt{x}\cot\sqrt{x}}{\sqrt{x}}$$ → slope of the graph

we want the slope of the tangent line at $x = 1$

$$y' = \frac{-3\csc^3\sqrt{1}\cot\sqrt{1}}{\sqrt{1}} = -3\csc^3(1)\cot(1)$$

$$y - 2\csc^3(1) = -3\csc^3(1)\cot(1)(x-1)$$

$$\boxed{y - 3.35 = -3.23(x-1)}$$

$$2*\left(\frac{1}{(\sin(1))^3}\right) \approx 3.35$$

$$-3*\left(\frac{1}{(\sin 1)^3}\right)*\left(\frac{1}{\tan 1}\right) \approx -3.23$$

#68 differentiate

$$g(x) = x^5, \quad g'(x) = 5x^4$$

$$h(x) = x^2 + \frac{1}{x}$$

$$h'(x) = 2x + \left(-\frac{1}{x^2}\right)$$

$$f(x) = \left(x^2 + \frac{1}{x}\right)^5$$

$$f'(x) = 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)$$

$$\boxed{f'(x) = 5\left(2x - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x}\right)^4}$$

#54 $g(x) = 3x\sin x + x^2\cos x$

$$g'(x) = \left[3\sin x + 3x\cos x\right] + \left[2x\cos x + x^2(-\sin x)\right]$$

$$g'(x) = 3\sin x + 3x\cos x + 2x\cos x - x^2\sin x$$

$$\boxed{g'(x) = \sin x\,(3 - x^2) + 5x\cos x}$$

#76 $f(x) = \dfrac{3x}{\sqrt{x^2+1}} = 3x\,(x^2+1)^{-1/2}$

$$f'(x) = 3(x^2+1)^{-1/2} + 3x\left[-\tfrac{1}{\cancel{2}}(x^2+1)^{-3/2}\cdot\cancel{2}x\right]$$

$$= 3(x^2+1)^{-3/2}\left[(x^2+1)^{-1/2-(-3/2)} + x^2\right]$$

$$= 3(x^2+1)^{-3/2}\left[(x^2+1)^{1} + x^2\right] = \frac{3(2x^2+1)}{(x^2+1)^{3/2}}$$

Consider...

$$x^2 + 5x$$

$$x(x^{2-1} + 5) = x(x+5)$$