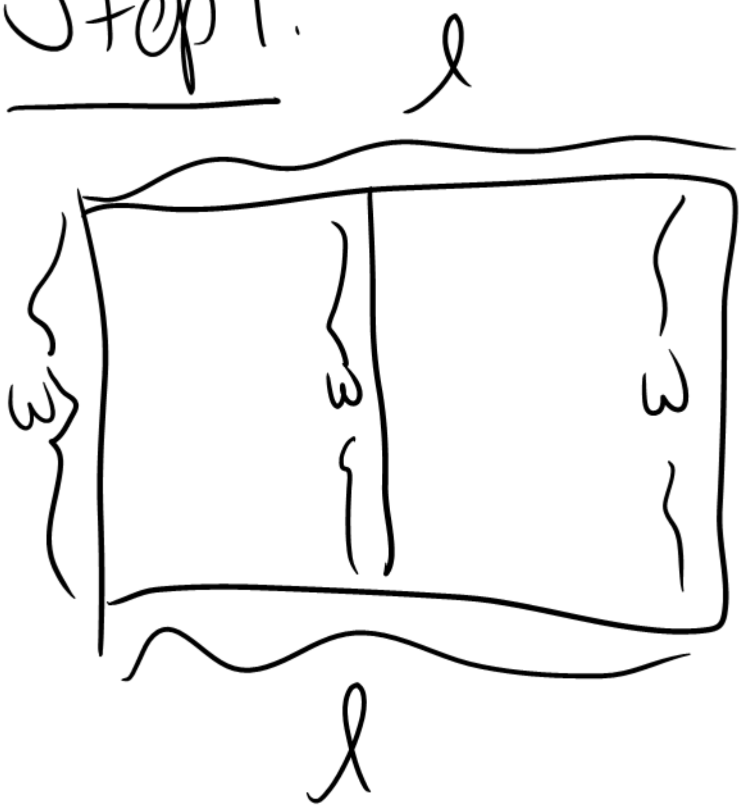
Step 1:



Step 2:

Primary equation: $A = lw$

Secondary equation: $P = 200\text{ ft}$

$P = 2l + 3w$

Step 3:

Step 3: $200 = 3w + 2l$

$$\frac{200 - 3w}{2} = \frac{2l}{2}$$

$\boxed{A = lw}$ $\quad l = 100 - \frac{3}{2}w$

$$A(w) = \left(100 - \frac{3}{2}w\right)(w)$$

$$A(w) = 100w - \frac{3}{2}w^2$$

$200 = 2l + 3w$

Step 4:

$(0, \square)$

Coming back

Step 5: $A(w) = 100w - \frac{3}{2}w^2$

$A'(w) = 100 - 3w$

$0 = 100 - 3w$

$3w = 100$

$w = \frac{100}{3}$ (this is my "c" for the area function)

We need to make sure $\frac{100}{3}$ yields a max.

$A''(w) = -3$

$A''\left(\frac{100}{3}\right) = -3 < 0$

By the 2nd deriv. test $w = \frac{100}{3}$ is a max

Conclusion

$$l = \frac{200-3w}{2} = \frac{200-\cancel{3}\left(\frac{100}{\cancel{3}}\right)}{2}$$

$$= 50$$

The area will be maximized when the width is $\frac{100}{3}$ ft and the length is 50 ft.

3.7: OPTIMIZATION PROBLEMS

1. Find two positive numbers which have a product of 192 and the sum is a minimum.

   a. Identify given quantities and quantities to be determined.

   Let x, y be positive numbers

   b. Write a primary equation for the quantity that is to be maximized or minimized.

   $S = x + y$

   c. Reduce the primary equation to one having a single independent variable. This may involve the use of secondary equations relating the independent variables of the primary equation.

   Secondary: $\frac{xy}{x} = \frac{192}{x} \rightarrow y = \frac{192}{x}$

   $S(x) = x + \frac{192}{x}$

   d. Determine the feasible domain of the primary equation.

   $x, y > 0$

   e. Determine the desired maximum or minimum value using the calculus techniques you've learned.

   $S(x) = x + \frac{192}{x}$

   $S'(x) = 1 - \frac{192}{x^2}$

   $S''(x) = \frac{384}{x^3}$

   $0 = 1 - 192/x^2$

   $\frac{192}{x^2} = 1$

   $\sqrt{x^2} = \sqrt{192}$

   $x = \cancel{\pm} 8\sqrt{3}$

   $2 \,|\, 192$, $2 \,|\, 96$, $2 \,|\, 48$, $2 \,|\, 24$, $2 \,|\, 12$, $2 \,|\, 6$, $3 \,|\, 3$, 1

2nd der. $S''(8\sqrt{3}) > 0 \Rightarrow \min$

$$xy = 192$$

$$8\sqrt{3}\,y = 192$$

$$y = \frac{24}{\sqrt{3}} = \frac{\cancel{24}^{\,8}\sqrt{3}}{\cancel{3}} = 8\sqrt{3}$$

When both x and y are $8\sqrt{3}$ the sum is minimized.

2. Find the point on the graph of $f(x)=\sqrt{x-8}$ that is closest to the point $(2,0)$.

a. Identify given quantities and quantities to be determined.

b. Write a primary equation for the quantity that is to be maximized or minimized.

c. Reduce the primary equation to one having a single independent variable. This may involve the use of secondary equations relating the independent variables of the primary equation.

d. Determine the feasible domain of the primary equation.

e. Determine the desired maximum or minimum value using the calculus techniques you've learned.

3. Determine the dimensions of a rectangular solid (with a square base) with maximum volume if its surface area is 337.5 square cm.

   a. Identify given quantities and quantities to be determined.

   b. Write a primary equation for the quantity that is to be maximized or minimized.

   c. Reduce the primary equation to one having a single independent variable. This may involve the use of secondary equations relating the independent variables of the primary equation.

   d. Determine the feasible domain of the primary equation.

   e. Determine the desired maximum or minimum value using the calculus techniques you've learned.

4. A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 12 cubic centimeters. Find the radius of the cylinder that produces the minimum surface area.

   a. Identify given quantities and quantities to be determined.

   b. Write a primary equation for the quantity that is to be maximized or minimized.

   c. Reduce the primary equation to one having a single independent variable. This may involve the use of secondary equations relating the independent variables of the primary equation.

   d. Determine the feasible domain of the primary equation.

   e. Determine the desired maximum or minimum value using the calculus techniques you've learned.