Differentials

u and v are functions of x.

$$\left(\frac{\partial}{\partial \cancel{x}}(cu)\right)\cancel{dx} = \left(c\,\frac{du}{\cancel{\partial x}}\right)\cancel{dx}$$

$$\partial cu = c\,\partial u$$

$$\frac{\partial}{\partial x}(uv) = \frac{(du)}{\partial x}v + u\left(\frac{dv}{\partial x}\right)$$

$$\partial(uv) = v\,du + u\,dv$$

ex: Use differential to approximate

$\sqrt{24.9}$

Let $f(x) = \sqrt{x}$

$f(x+\Delta x) \approx f(x) + f'(x)\,dx$

TRICK

$f(x+\Delta x)$

$\approx f(x) + dy$

$dy \approx f'(x)\,dx$

$\frac{dy}{dx} = f'(x)$

$\approx f(x) + f'(x)\,dx$

$x = 25$

$\Delta x = -0.1$

$\Delta x = dx$

$f'(x) = \frac{1}{2\sqrt{x}}$

↑ all the stuff we need yaay!

$f(x+\Delta x) \approx f(25) + f'(25)(-.1)$

$\approx \sqrt{25} + \frac{1}{2\sqrt{25}}(-.1)$

$= 5 + \frac{1}{2(5)}(-.1)$

$= 5 - \frac{1}{100}$

$= \frac{499}{100} = 4.99$

$$f(x+\Delta x) \approx f(x) + f'(x)\,dx \qquad dy$$

$$\Delta y = f(x+\Delta x) - f(x)$$

$$\frac{dy}{dx} = f'(x)$$

$$\Delta y \approx \partial y$$

$$\partial y \approx f(x+\Delta x) - f(x)$$

$$f'(x)\,dx + f(x) \approx f(x+\Delta x)$$

ex: Use differentials to approximate $\cos(0.1)$

$f(x) = \cos x$

$f'(x) = -\sin x$

$\Delta x = 0.1$

$x = 0$

$$f(x+\Delta x) \doteq f(x) + f'(x)\,dx$$

$$= \cos(0) - \sin(0)(.1)$$

$$= 1 - 0(.1)$$

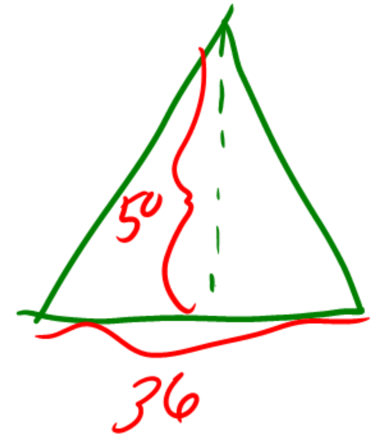
$$= \boxed{1}$$

ex: The measurements of the base and altitude of a triangle are found to be 36 and 50 centimeters, respectively. The possible error in each measurement is 0.25 cm. Use differentials to approximate the propagated error in computing the area.

measured lengths:

$h = 50\,cm$

$b = 36\,cm$



possible error:

$\pm .25\,cm = \partial b = \partial h$

$$\Delta A \approx \partial A$$

$$A = \frac{1}{2} b h$$

$$\partial A \doteq \frac{1}{2}\left(b\,\partial h + h\,\partial b\right)$$

$$\partial A = \frac{1}{2}\left((36)(\pm .25) + (50)(\pm .25)\right)$$

$$\partial A = \frac{1}{2}\left(9 + 12.5\right) \rightarrow \partial A = \pm 10.75\,cm$$

The propagated error in the area is $\pm 10.75\,cm$

Homework Questions

3.7 #17

$$\frac{\partial Q}{\partial x} = kx\left(Q_0 - x\right) = kQ_0x - kx^2$$

Q_0 : amount of original substance

x : " " catalyst

For what value of x will will the chem reaction be the greatest?

$$\frac{\partial^2 Q}{\partial x^2} = KQ_0 - 2Kx \xrightarrow[\text{to see if it's a max}]{\text{check}} \frac{\partial^3 Q}{\partial x^3} = -2k < 0$$

$$\Rightarrow \max$$

$$0 = KQ_0 - 2kx$$

$$2\cancel{k}x = \cancel{k}Q_0$$

$$x = \frac{1}{2}Q_0$$

Conclusion: When x is half the original substance the chemical reaction will be the greatest.

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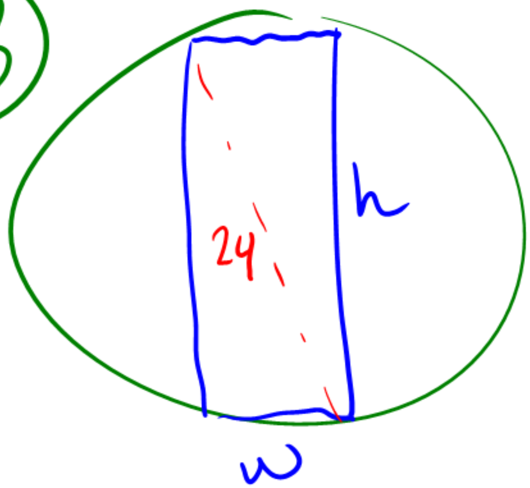


24 represents the diameter of the log. Don't need

$$S = kh^2w$$ ← primary equation

$$w^2 + h^2 = 24^2$$

secondary equation $h^2 = 576 - w^2$

$$S(w) = k(576 - w^2)w$$

$$S(w) = 576kw - kw^3$$

$$S'(w) = 576k - 3kw^2$$

$$S''(w) = -6kw$$

$$S''(13.85) < 0$$

$$\Rightarrow \text{max}$$

✓

$$0 = 576k - 3kw^2$$

$$3\cancel{k}w^2 = 576\cancel{k}$$

$$w^2 = 192$$

$$w = 8\sqrt{3} \text{ in}$$

$$w \approx 13.85 \text{ in}$$

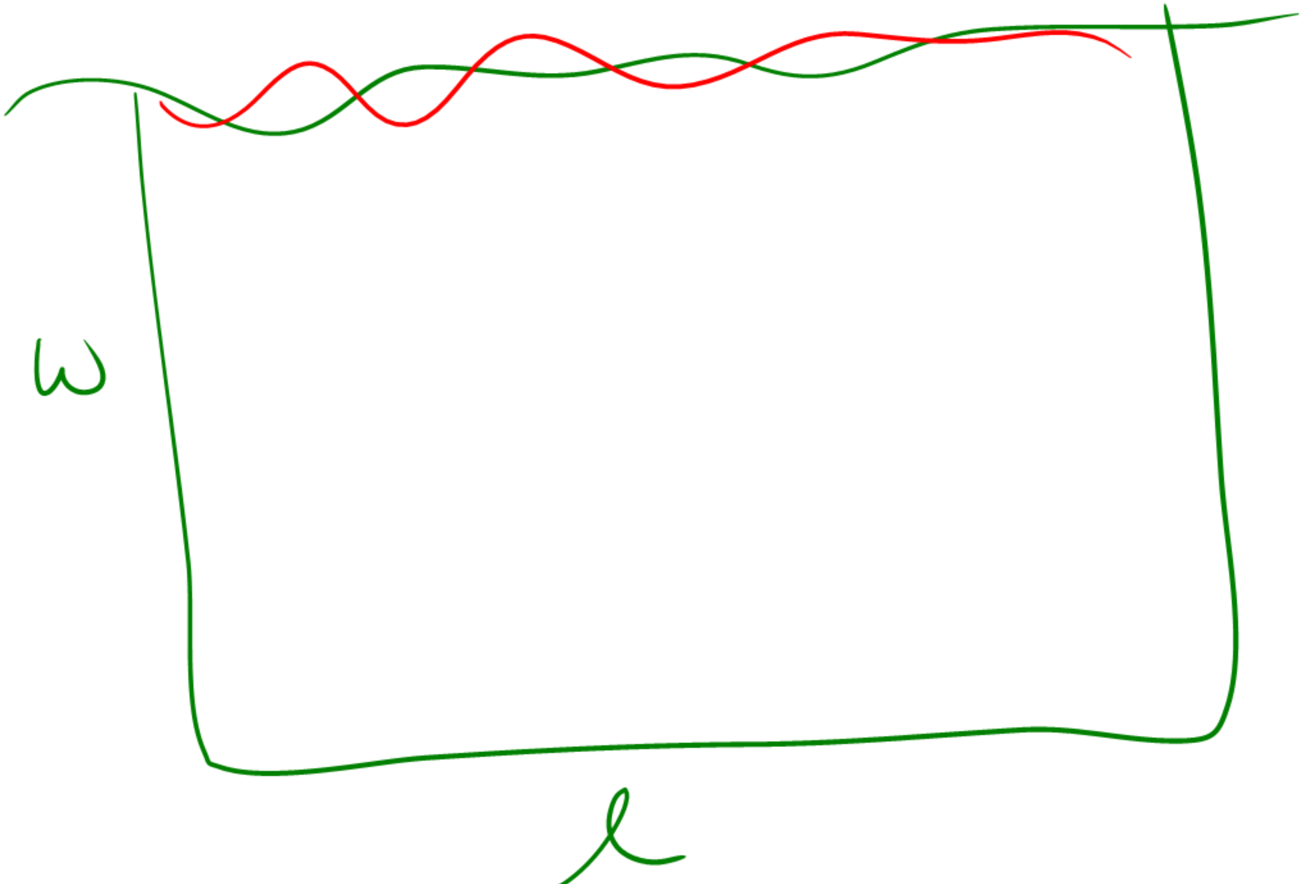
$$h^2 = 576 - w^2$$

$$h^2 = 576 - 192$$

$$h^2 = 384, \quad h = 8\sqrt{6} \text{ in}, \quad h \approx 19.6 \text{ in}$$

The dimensions of the strongest beam are $(8\sqrt{3} \times 8\sqrt{6})$ in

19 minimize fencing $\Rightarrow$ minimize perimeter



Primary equation: $P = 2w + l$

Secondary " : $A = 180000 m^2$

$A = lw$

$180000 = lw$

$l = 180000/w$

$$P(w) = 2w + 180000w^{-1}$$

$$P'(w) = 2 - \frac{180000}{w^2}$$

$$P''(w) = \frac{360000}{w^3} \rightarrow P''(300) > 0 \checkmark$$

$$\Rightarrow \text{min}$$

$$0 = 2 - \frac{180000}{w^2}$$

$$\frac{180000}{w^2} = 2$$

$$2w^2 = 180000$$

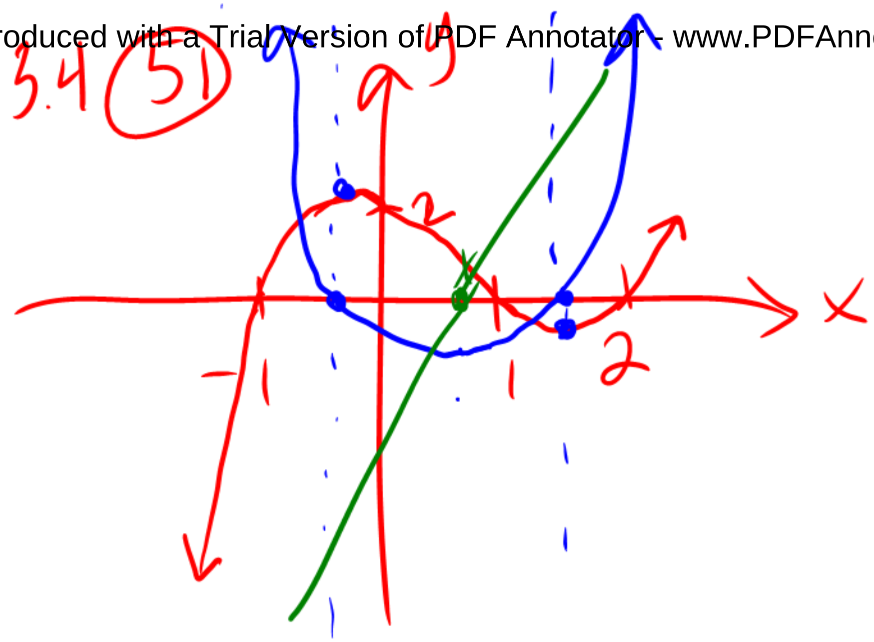
$$w^2 = 90000$$

$$w = 300\text{m}$$

$$l = \frac{180000}{300} = 600$$

The dimensions to minimize the fencing are 300m for the 2 widths and 600m for the 1 length

3.4 (51)



$f \uparrow$ on $(-\infty, -\frac{1}{2}) \cup (\frac{3}{2}, \infty)$

$f \downarrow$ on $(-\frac{1}{2}, \frac{3}{2})$

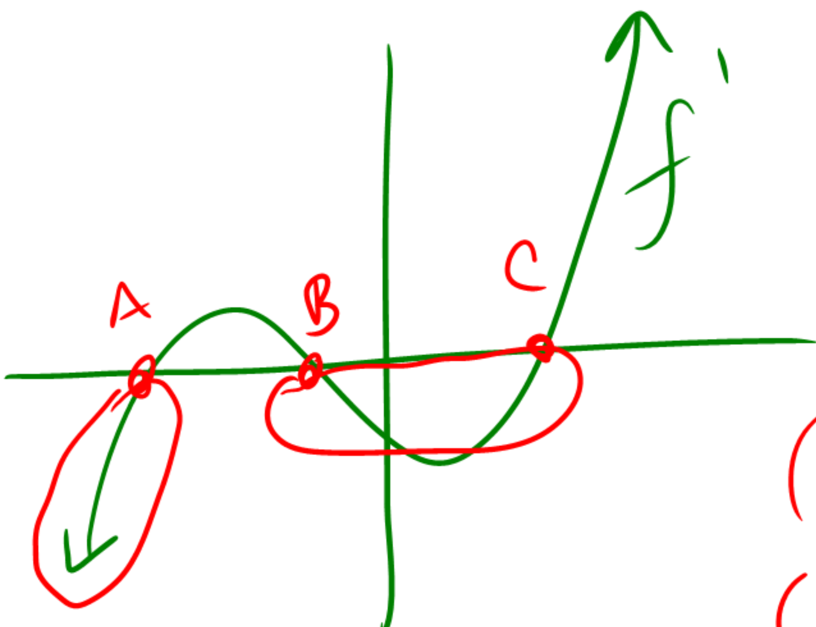
f has a rel max at $-\frac{1}{2}$

f " " " min at $\frac{3}{2}$

f' will have positive outputs

f' " " negative "

f' will cross the x-axis



$(-\infty, A) \cup (B, C)$ f is $\downarrow$

$(A, B) \cup (C, \infty)$ f is $\uparrow$

A is rel min, B rel max, C rel min