7/30/08

6.1

29 Need to isolate y

$$4y^2 - x^2 = c$$

$$4y^2 = x^2 + c$$

$$y^2 = \frac{x^2 + C}{4}$$

$$y = \pm\sqrt{\frac{x^2 + C}{4}}$$

$$y = \pm \frac{\sqrt{x^2 + c}}{2}$$

$$y_1 = \frac{\sqrt{x^2 + 0}}{2} = \frac{\sqrt{x^2}}{2}$$

$$y_2 = -\frac{\sqrt{x^2}}{2}$$

$c = 0$

then you do the same for 1, -1, 4, -4

6.3 Example 1

$$\cancel{dx}\left(\frac{dy}{\cancel{dx}}\right)=\left(\frac{8x}{y}\right)dx$$

$$y\left(dy\right)=\left(\frac{8x\,dx}{\cancel{y}}\right)\cancel{y}$$

$$\int y\,dy=\int 8x\,dx$$

$$\cancel{2}\left(\frac{y^2}{\cancel{2}}\right)=\left(4x^2+C\right)2$$

$$y^2=8x^2+2C$$

$$\boxed{y^2=8x^2+C}$$

Example 2

$$f(x,y)=\frac{xy}{\sqrt{x^2+y^2}}$$

$$f(tx,ty)=\frac{(tx)(ty)}{\sqrt{(tx)^2+(ty)^2}}$$

$$=\frac{t^2xy}{\sqrt{t^2x^2+t^2y^2}}$$

$$=\frac{t^2xy}{\sqrt{t^2(x^2+y^2)}}$$

$$=\frac{t^{\cancel{2}}xy}{\cancel{t}\sqrt{x^2+y^2}}$$

$$=t\left(\frac{xy}{\sqrt{x^2+y^2}}\right)=t\,f(x,y)$$

This is a homogeneous differential equation of order 1.

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Sidenote: Is $\sqrt{x^2+y^2} = x+y$

well, I forgot so I'll test $\sqrt{4}$.

I know $\sqrt{4} = 2$ It is also $\sqrt{2+2}$,

Is $\sqrt{2} + \sqrt{2} \stackrel{?}{=} 4$

approx: $1.41 + 1.41 \stackrel{?}{=} 4$

$2.82 \stackrel{?}{=} 4$

So I guess

$$\sqrt{x^2+y^2} = \sqrt{x^2+y^2}$$

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Consider $y = vx$, $v$ is a diff. function of $x$

$$\left(\frac{dy}{d\cancel{x}}\right) = \left(v(1) + \frac{dv}{dx}x\right)dx$$

$$\boxed{dy = v\,dx + x\,dv}$$

6.3 #38

$$y' = \frac{x^2+y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$

$M(x,y)dx + N(x,y)dy = 0$

use the substitution

$y = vx$

$dy = v\,dx + x\,dv$

$$2xy\,dy = (x^2+y^2)\,dx$$

$$2x(vx)(v\,dx + x\,dv) = [x^2+(vx)^2]\,dx$$

$$\frac{2x^2 v(v\,dx+x\,dv)}{x^2} = \frac{\cancel{x}^2(1+v^2)\,dx}{\cancel{x^2}}$$

$$2v(v\,dx+x\,dv) = (1+v^2)\,dx$$

$$2v^2\,dx + 2xv\,dv = (1+v^2)\,dx$$

$$(2v^2-(1+v^2))\,dx = -2xv\,dv$$

$$\frac{1}{(-2x)}\frac{(\cancel{v^2-1})\,dx}{\cancel{v^2-1}} = \frac{-\cancel{2}xv\,dv}{v^2-1}\frac{1}{(-\cancel{2}x)}$$

(cross multiply)

Is this separable right now? NO!

So we should check to see if $M(x,y)$ and $N(x,y)$ are homogen. diff equations of the same order.

$$M(x,y) = x^2+y^2$$

$$M(tx,ty) = (tx)^2+(ty)^2$$

$$= t^2x^2+t^2y^2$$

$$= t^2(x^2+y^2)$$

$$= t^2 M(x,y)$$

$$N(x,y) = 2xy$$

$$N(tx,ty) = 2xt\,yt$$

$$\int -\frac{dx}{dx} \quad \frac{1}{2}\int \frac{2v\,dv}{v^2-1}$$

$u = v^2 - 1$
$du = 2v\,dx$

$= t^2(2xy)$

$= t^2 N(x,y)$

So we can use $y = vx$ to sub. and make this diff. equation separable

$$-\frac{1}{2}\int \frac{1}{x}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$$

$$-\frac{1}{2}\ln|x| = \frac{1}{2}\ln|v^2-1| + C$$

$$\boxed{-\frac{1}{2}\ln|x| = \frac{1}{2}\ln\left|\left(\frac{y}{x}\right)^2 - 1\right| + C}$$

$y = vx$
$v = \frac{y}{x}$

Example: $y' = \frac{2x+3y}{x}$

$$\frac{dy}{dx} = \frac{2x+3y}{x}$$

$\rightarrow \underline{x}\,dy = (2x+3\underline{y})\,dx$

$y = vx$
$dy = v\,dx + x\,dv$

$$x(v\,dx + x\,dv) = (2x + 3(vx))\,dx$$

$$xv\,dx + x^2\,dv = 2x\,dx + 3xv\,dx$$

$$x^2\,dv = 2x\,dx + 3xv\,dx - xv\,dx$$

$$x^2\,dv = 2x\,dx + 2xv\,dx$$

$$\frac{\cancel{x^2}\,dv}{\cancel{x^2}(1+v)} = \frac{2\cancel{x}\,dx\,\cancel{(1+v)}}{x^{\cancel{2}}\,\cancel{(1+v)}}$$

$$\int \frac{dv}{1+v} = \int \frac{2\,dx}{x}$$

$$\ln|1+v| = 2\ln|x| + \ln C$$

$$\ln|1+v| = \ln(Cx^2)$$

$$1+v = Cx^2$$

$$1 + \frac{y}{x} = Cx^2$$

$$\frac{x+y}{x} = Cx^2$$

$$\boxed{x+y = Cx^3}$$

# 7.1 Area Between Curves

$$A = \int_a^b \left(f(x) - g(x)\right) dx \quad \text{or}$$

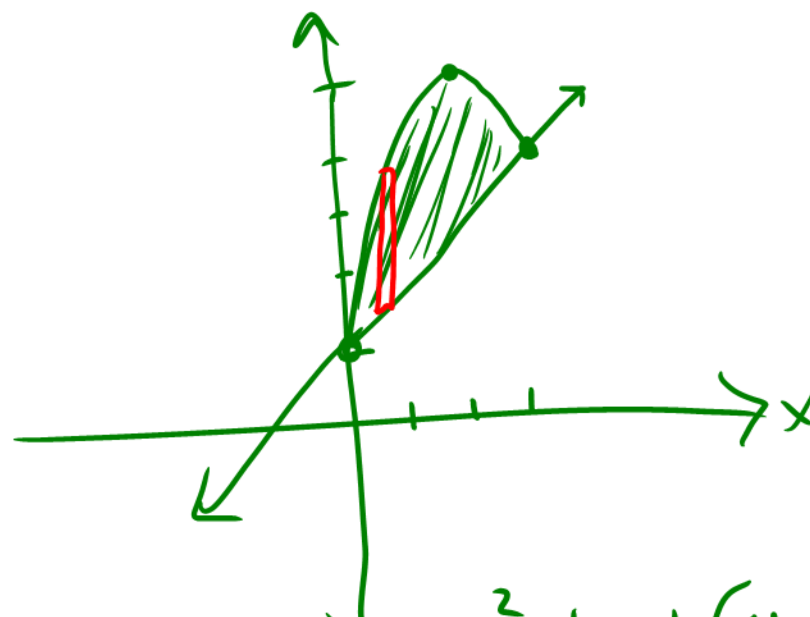
$$A = \int_c^d \left(f(y) - g(y)\right) dy$$

Example: 7.1 #20

$f(x) = -x^2 + 4x + 1$

$g(x) = x + 1$

Find limits of integration by setting $f(x) = g(x)$




$$f(x) - g(x) = -x^2 + 4x + 1 - (x+1)$$
$$= -x^2 + 3x$$

$x+1=-x^2+4x+1$

$0=x^2-3x$

$0=x(x-3)$

$x=0$ or $x=3$

$V:\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$-\frac{b}{2a}=-\frac{4}{2\cdot(-1)}=2$

$f(2)=5$

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$$A=\int_0^3(-x^2+3x)\,dx=\left(-\frac{x^3}{3}+\frac{3x^2}{2}\right)\Big|_0^3=\left(-9+\frac{27}{2}\right)-(0)$$

$$=\frac{-18+27}{2}$$

$$=\frac{9}{2}\text{ sq. units}$$