7/21/08

- Homework Questions 5.1–5.2
- Quiz mañana 5.1–5.3
- Lecture on 5.3

5.1 #26

expand $\ln(3e^2)$

$= \ln 3 + \ln e^2$

$= \ln 3 + 2\ln e$

$= \ln 3 + 2(1)$

$= \boxed{\ln 3 + 2}$

#48

derive $y = \underbrace{x}_{\text{factor}}\underbrace{\ln x}_{\text{factor}}$

PRODUCT!

$y' = 1\ln x + \cancel{x}\left(\frac{1}{\cancel{x}}\right)$

$\boxed{y' = \ln x + 1}$

#49

$y = \ln\left(x\sqrt{x^2-1}\right)$

The best strategy is to use properties of logs to expand BEFORE deriving

$y = \ln x + \frac{1}{2}\ln(x^2-1)$

$y' = \frac{1}{x} + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}x}{x^2-1}\right)$

$y' = \frac{1}{x} + \frac{x}{x^2-1}$

$y' = \frac{(1)(x^2-1) + x(x)}{x(x^2-1)}$

$y' = \frac{x^2-1+x^2}{x(x^2-1)}$

$$\boxed{y' = \frac{2x^2-1}{x(x^2-1)}}$$

5.2

#9

$$\int \frac{x^2-4}{x}\,dx$$

Strategy

I see a binomial being divided by a monomial, so I should divide!

$$\int \left(x - \frac{4}{x}\right)dx = \boxed{\frac{x^2}{2} - 4\ln|x| + C}$$

$$\int x\,dx - 4\int \frac{1}{x}\,dx$$

#25

$$\int \frac{1\,dx}{1+\sqrt{2x}}$$

Strategy

well the only thing I can see is to let u = the denominator & hope I can adjust it.

$u = 1 + \sqrt{2x} \longrightarrow \cancel{1 = u - \sqrt{2x}}$ or $\sqrt{2x} = u - 1$

$du = \frac{\cancel{2}\, dx}{\cancel{2}\sqrt{2x}} = \frac{1}{\sqrt{2x}} dx$

$dx = \sqrt{2x}\, du$

$dx = (u-1)\, du$

$\int \frac{(u-1)\,du}{u} = \int 1\,du - \int \frac{1}{u}\,du$

$= u - \ln|u| + c,$

$= 1 + \sqrt{2x} - \ln|1 + \sqrt{2x}| + c,$

$= \boxed{\sqrt{2x} - \ln(1 + \sqrt{2x}) + c}$

$u = 1 + (2x)^{1/2}$

$du = \left(0 + \frac{1}{\cancel{2}}(2x)^{-1/2} \cdot \cancel{2}\right) dx$

$du = \frac{1}{\sqrt{2x}} dx$

#19

$$\int \frac{(\ln x)^2}{x}\,dx$$

Strategy

well, $u = x$ doesn't help at all!

So let's try $u = \ln x$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$dx = x\,du$

$$\int \frac{u^2 \cancel{x}\,du}{\cancel{x}} = \int u^2\,du$$

$$= \frac{u^3}{3} + C$$

$$\boxed{= \frac{(\ln x)^3}{3} + C}$$

#20 $\displaystyle\int \frac{1}{x\ln(x^3)}\,dx$

Strategy

- use properties of logs
- the derivative of $\ln x$ is in there

$$= \int \frac{dx}{x[3\ln x]} = \frac{1}{3}\int \frac{dx}{x\ln x} = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C$$

$u = \ln x$

$du = \dfrac{dx}{x}$

$$= \frac{1}{3}\ln|\ln x| + C$$

$$= \boxed{\ln \sqrt[3]{|\ln x|} + C}$$

#32

$$\int \sec\frac{x}{2}\,dx = 2\int \sec u\,du$$

$$= 2\ln|\sec u + \tan u| + C$$

$u = \frac{x}{2}$

$du = \frac{dx}{2}$

$dx = 2\,du$

Strategy

- Freak out about the trig function and then get OVER IT!
- see if a u-sub for the quantity in the ∡ works.

$$= 2\ln\left|\sec\frac{x}{2} + \tan\frac{x}{2}\right| + C$$

$$= \boxed{\ln\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right)^2 + C}$$

5.2 Trig Integrals

$\int \cos u \, du = \sin u + C$

$\int \sin u \, du = -\cos u + C$

$\int \tan u \, du = -\ln|\cos u| + C$

$\int \cot u \, du = \ln|\sin u| + C$

$\int \sec u \, du = \ln|\sec u + \tan u| + C$

$\int \csc u \, du = -\ln|\csc u + \cot u| + C$

$\int \csc u \cot u \, du = -\csc u + C$

$\int \sec u \tan u \, du = \sec u + C$

$\int \sec^2 u \, du = \tan u + C$

$\int \csc^2 u \, du = -\cot u + C$

5.2 #40 Solve the differential equation

$$\frac{dr}{dt} = \frac{\sec^2 t}{\tan t + 1}; \quad (\pi, 4)$$

initial condition

Strategy

Try u = denominator

$$\int dr = \int \frac{\sec^2 t}{\tan t + 1} dt$$

$u = \tan t + 1$

$du = \sec^2 t \, dt$

$$r = \int \frac{du}{u}$$

$$r = \ln|u| + C$$

$$r = \ln|\tan t + 1| + C$$

$4 = \ln|\tan \pi + 1| + C$

$4 = \ln|1| + C$

$4 = 0 + C$

$C = 4$

$$\boxed{r = \ln|\tan t + 1| + 4}$$

5.3 Inverse Functions

If a function f is one-to-one:

1. Its inverse, denoted f^{-1}, is also a function
2. $(f \circ f^{-1})(x) = f[f^{-1}(x)] = x$
   and $(f^{-1} \circ f)(x) = f^{-1}[f(x)] = x$
3. The domain of f is the range of f^{-1} and the range of f is the domain of f^{-1}.

* ④ If the graph of f contains the point (a,b), the graph of f^{-1} MUST contain the point (b,a).

Finding an inverse function

- A function will have an inverse if and only if it is 1-1.
- If f is strictly monotonic on its entire domain $\Rightarrow$ it is 1-1 $\Rightarrow$ it has an inverse

$\hookrightarrow$ monotonic means if $x_1 < x_2 < \cdots < x_n$ then

$$f(x_1) < f(x_2) < \cdots < f(x_n)$$ or

if $x_1 < x_2 < \cdots < x_n$ then

$$f(x_1) > f(x_2) > \cdots > f(x_n).$$

Steps to find an inverse

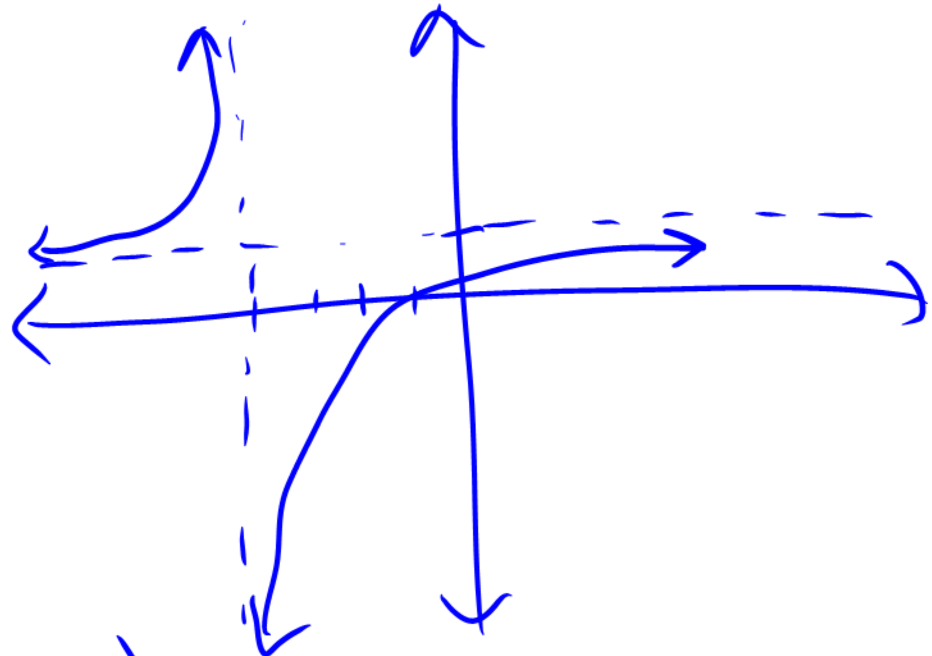
① Write in equation notation [change $f(x)$ to y]

② Switch x and y and then isolate y

③ Change y to $f^{-1}(x)$

example:

$$f(x) = \frac{x}{x+4}$$



f is 1-1 → passes the HLT (see graph)

① $y = \dfrac{x}{x+4}$

② $x = \dfrac{y}{y+4}$

$x(y+4) = y$ → $xy + 4x = y$

$xy - y = -4x$

$y(x-1) = -4x$

$y = \dfrac{-4x}{x-1}$

$y = \dfrac{4x}{1-x}$

3. $f^{-1}(x) = \dfrac{4x}{1-x}$

Continuity and Differentiability of Inverse Functions

Let f be a function whose domain is an interval I. If f has an inverse function then the following stuff is true:

1. If f is continuous on its domain, then f^{-1} is also continuous on its domain.
2. If f is $\uparrow$ on its domain so is f^{-1}.

③ If f is $\downarrow$ on its domain so is f^{-1}!

④ If f is differentiable at c and $f'(c) \neq 0$ then f^{-1} is differentiable at $f(c)$.

The derivative of an inverse function

Let f be a differentiable function on an interval I. If f has an inverse function, f^{-1}, then f^{-1} is differentiable at any x for which $f'[f^{-1}(x)] \neq 0$.

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'\left[f^{-1}(x)\right]}$$

Example 5.3 # 76

$f(x) = \sqrt{x-4}$, $a = 2$

we want $(f^{-1})'(a)$

Remember that if (x,y) is on the graph of f

(y,x) " " " " f^{-1}

Step 1: find the input value which yield the output given.

$(2)^2 = (\sqrt{x-4})^2$

$4 = x-4$

$8 = x$

$(8,2)$ is on the graph of f so $(2,8)$ is on the graph of f^{-1}. So

$$f^{-1}(2) = 8$$

<u>Step 2</u>: Find $f'(x)$ and then evaluate at $f^{-1}(a)$

$$f'(x) = -2\sqrt{x-4}$$

$$f'(8) = 2\sqrt{(8)-4}$$

$$f'(8) = \frac{1}{2\sqrt{4}}$$

$$= \frac{1}{4}$$

$$f'[f^{-1}(2)] = \frac{1}{4}$$

$$(f^{-1})'(2) = \frac{1}{\left(\frac{1}{4}\right)} = \boxed{4}$$