7/17/08

5.1

$\ln \sqrt[3]{12} = \ln 12^{1/3}$

$\ln 2 \doteq .6931$

$\ln 3 \doteq 1.0986$

$= \frac{1}{3} \ln 12$

$= \frac{1}{3}\left[\ln 2^2 + \ln 3\right]$

$= \frac{1}{3}\left[2\ln 2 + \ln 3\right]$

$= \frac{1}{3}(2(.6931) + 1.0986)$

$= \frac{1}{3}(1.3862 + 1.0986)$

$\frac{2.4848}{3}$

$= 0.8283$

Question

$$\left[\frac{1}{3}(\ln 2 + \ln 3)\right](\ln 2)$$

$$(\ln 6^{1/3})(\ln 2) \approx 1.6$$

$$\frac{\ln 2}{3}$$

not equivalent

$$\frac{1}{3}(\ln 2 + \ln 3 + \ln 2)$$

① $f(x) = \ln\left(\frac{x}{x-1}\right) = \ln x - \ln(x-1)$

$$f'(x) = \frac{1}{x} - \frac{1}{x-1}$$

$$= \frac{(x-1)(1) - (1)(x)}{x(x-1)}$$

$$= \frac{x - 1 - x}{x(x-1)}$$

$$= \boxed{\frac{-1}{x(x-1)}}$$

② $y = \dfrac{x^2}{\sqrt{3x^2+5}}$

* Using logs to differentiate a simpler expression.

$\ln y = \ln\left(\dfrac{x^2}{(3x^2+5)^{1/2}}\right)$

$\ln y = 2\ln x - \frac{1}{2}\ln(3x^2+5)$

$$\frac{y'}{y} = \frac{2}{x} - \frac{1}{2}\left(\frac{6x}{3x^2+5}\right)$$

$$y' = y\left(\frac{2}{x} - \frac{3x}{3x^2+5}\right)$$

$$y' = \frac{x^2}{\sqrt{3x^2+5}}\left(\frac{2}{x} - \frac{3x}{3x^2+5}\right)$$

(3) $y = \ln[\ln 5x^2]$

$y' = \dfrac{\left(\frac{2}{x}\right)}{\ln(5x^2)}$

$$\boxed{y' = \frac{2}{x \ln(5x^2)}}$$

$\ln(5x^2)$

$u = 5x^2$

$u' = \dfrac{10x}{5x^2} = \dfrac{2}{x}$

$u_2 = \ln 5x^2$

$u_2' = \dfrac{\frac{d}{dx} \ln 5x^2}{\ln 5x^2}$

5.2 Warm-up

① $\int \frac{x^2-1}{x-1}\,dx = \int \frac{(x+1)(x-1)}{x-1}\,dx$

$= \boxed{x^2/2 + x + C}$

Find $y'$

② $y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{y'}{y} = \frac{1}{x} x + 1 \ln x$

$y' = y(1 + \ln x)$

$\boxed{y' = x^x(1 + \ln x)}$

(3) $y = x^5$

$y' = 5x^4$

$\boxed{y' = 5x^4}$

experimenting

$y = x^5$

$\ln y = 5 \ln x$

$\frac{y'}{y} = \frac{5}{x}$

$y' = \frac{5y}{x}$

$y' = \frac{5x^5}{x}$

$y' = 5x^4$

5.2

$$\int \frac{x^2 - 5x - 1}{x - 1} dx$$

Need to rewrite the rational expression as a polynomial plus a remainder

$$x - 4 - \frac{5}{x-1}$$

$$x-1 \overline{)\, x^2 - 5x - 1}$$

$$-(x^2 - x)$$

$$-4x - 1$$

$$-(-4x + 4)$$

$$= \int \left(x - 4 - \frac{5}{x-1}\right) dx$$

$$= \boxed{\frac{x^2}{2} - 4x - 5 \ln|x-1| + C}$$