7/14/08

4.2

68 $f(x) = \dfrac{8}{x^2+1}$, $[2, 6]$

$A = \cancel{\lim_{n\to\infty}} \sum_{i=1}^{n} f(c_i)\,dx$

use the values of n given

$dx = \dfrac{b-a}{n} = \dfrac{6-2}{n} = \dfrac{4}{n}$

right endpoint

$a + i\,dx = 2 + \dfrac{4i}{n}$

$f\left(2+\dfrac{4i}{n}\right) = \dfrac{8}{\left(2+\frac{4i}{n}\right)^2+1}$

$$\frac{8}{\left(2+\frac{4i}{n}\right)^2+1} = \frac{8}{\left(4+\frac{16i}{n}+\frac{16i^2}{n^2}\right)+1}$$

$$= \frac{8}{5+\frac{16i}{n}+\frac{16i^2}{n^2}}$$

Seq: $\left(\frac{8}{5+\frac{16i}{n}+\frac{16i^2}{n^2}}\right)\left(\frac{4}{n}\right)$ plug in 4 for n

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2nd 0 (catalog)

sum(seq(8/(5 + $\frac{16x}{4}$ + $\frac{16x^2}{4^2}$)*4/4, x, 1, 4, 1))

4.2 27

$f(x) = \sqrt{x}$

upper sum: $dx = \frac{1}{4}$

right endpoints

$\frac{1}{4}\left(f(1/4) + f(1/2) + f(3/4) + f(1)\right)$

$\frac{1}{4}\left(\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} + 1\right) \approx .76828$

lower sum: $\frac{1}{4}\left(f(0) + f\left(\frac{1}{4}\right) + f\left(\frac{1}{2}\right) + f\left(\frac{3}{4}\right)\right)$

4.5 Integration by Substitution

$$\int_0^3 3x^{4/5}\,dx = \cancel{3}\left(\frac{5x^{9/5}}{\cancel{9}_{3}}\right)\Bigg|_0^3 = \frac{5}{3}\left(3^{9/5} - 0\right)$$

$$3^{9/5} = 3^{5/5 + 4/5} = \underline{3^1} \cdot 3^{4/5}$$

$$= \frac{5}{\cancel{3}}\left(\cancel{3}\sqrt[5]{3^4}\right)$$

$$= \boxed{5\sqrt[5]{81}}$$

What if we have...

$$\int (2x+1)^3\,dx$$

$$\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$$

What is our $g(x)$?

$g(x) = 2x + 1$

$g'(x) = 2$

$\rightarrow u = 2x + 1$

$\left(\frac{du}{\cancel{dx}}\right)\cancel{dx} = (2)dx$

$\frac{du}{2} = \frac{2dx}{2}$

$$\int (2x+1)^3 dx$$

$$\int u^3 \frac{du}{2}$$

$$= \frac{1}{2}\int u^3 du$$

$$= \frac{1}{2}\frac{u^4}{4} + C$$

$$= u^4/8 + C$$

$$\Rightarrow \boxed{= \frac{(2x+1)^4}{8} + C}$$

Change of variables

Pattern recognition

$$\frac{2}{2}\int (2x+1)^3\,dx = \frac{1}{2}\int (2x+1)^3 \cdot \underbrace{2}_{g'(x)}\,dx$$

$$= \frac{1}{2}F(2x+1) + C$$

$$= \frac{\frac{1}{2}(2x+1)^4}{4} + C$$

$$= \frac{(2x+1)^4}{8} + C$$

$$\underline{\int \cos^2 x\,dx} \rightarrow \frac{1}{2}\int (1+\cos 2x)\,dx$$

recall that

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \frac{1}{2}\left[\int 1\,dx + \int \cos 2x\,dx\right]$$

$$= \frac{1}{2}\left[x + \frac{1}{2}\int \cos 2x \cdot 2\,dx\right]$$

$$= \frac{1}{2}\left[x + \frac{1}{2}(\sin 2x)\right] + C$$

$$= \boxed{\frac{x}{2} + \frac{\sin 2x}{4} + C}$$

$$\int \cos^2 x \boxed{-\sin x}\, dx$$

$g(x) = \cos x$

what is $g'(x)$?

$$\int \cos^2 x(-\sin x)\,dx$$

$u = \cos x$

$du = -\sin x\, dx$

$$\int u^2\, du = \frac{u^3}{3} + C$$

$$= \boxed{\frac{\cos^3 x}{3} + C}$$

$$\int \frac{x\,dx}{\sqrt{x^2+1}} = \int \frac{\cancel{x}\,du}{(u^{1/2})(2\cancel{x})}$$

$u = x^2 + 1$

$$\frac{du}{\cancel{\partial x}} = \frac{2x\,dx}{\cancel{\partial x}}$$

$$= \frac{1}{2}\int u^{-1/2}\,du$$

$$= \frac{1}{\cancel{2}}\left(\cancel{2}u^{1/2}\right) + C = \boxed{\sqrt{x^2+1} + C}$$