7/15/08

Warm-up

1. $\int \frac{5x\,dx}{(x^2-3)^5} = 5\int \frac{\cancel{x}}{u^5}\frac{du}{2\cancel{x}} = \frac{5}{2}\int u^{-5}du$

$= \frac{5}{2}\left(\frac{u^{-4}}{-4}\right) + C$

$= -\frac{5}{8u^4} + C = \boxed{\frac{-5}{8(x^2-3)^4} + C}$

$u = x^2-3$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

2. $\int x\sqrt{1-x}\,dx = -\int x u^{1/2}du = \int (1-u)u^{1/2}du$

$u = 1-x \Rightarrow x = 1-u$

$du = -dx$

$= -\int u^{1/2}du + \int u^{3/2}du = -\frac{u^{3/2}}{3/2} + \frac{u^{5/2}}{5/2} + C$

$$= -\frac{2}{3}u^{3/2} + \frac{2}{5}u^{5/2} + C = \boxed{-\frac{2}{3}(1-x)^{3/2} + \frac{2}{5}(1-x)^{5/2} + C}$$

4.4 HW questions 25, 29, 87

25 $\int_0^3 |x^2-4|\,dx = \int_0^2 -(x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$

$x^2-4=0$

$x=\pm 2$

$$= -\int_0^2 (x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$$

$$= \left(\frac{x^3}{3}-4x\right)\Big|_0^2 + \left(x^3/3 - 4x\right)\Big|_2^3$$

$$= -\left[\left(\frac{8}{3}-8\right)-(0-0)\right] + \left[\left(\frac{27}{3}-12\right)-\left(\frac{8}{3}-8\right)\right]$$

$$= -\frac{8}{3}+8+(-3)-\frac{8}{3}+8$$

$$= 13 - \frac{16}{3}$$

$$= \frac{39}{3} - \frac{16}{3}$$

$$= \boxed{\frac{23}{3}}$$

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29. $\int_{-\pi/6}^{\pi/6} \sec^2 x \, dx = \tan x \Big|_{-\pi/6}^{\pi/6} = \tan \pi/6 - \tan(-\pi/6)$

$$= \frac{\sqrt{3}}{3} - \left(-\frac{\sqrt{3}}{3}\right)$$

$$= \boxed{\frac{2\sqrt{3}}{3}}$$

87 $F(x) = \int_{x}^{x+2} (4t+1)\,dt$

$= 4(x+2)+1 - [4(x)+1]$

$= 4x+8+1-4x-1 = 8$

$\int_{x}^{a} (4t+1)\,dt + \int_{x}^{x+2} (4t+1)\,dt$

$-\int_{a}^{x} (4t+1)\,dt + \int_{a}^{x+2} (4t+1)\,dt$

4.5 51, 4, 23, 67

4 $\int \sec 2x \tan 2x \, dx$

u	du
$2x$	$2dx$

$u = 2x$

$du = 2dx$

$dx = \frac{du}{2}$

$= \frac{1}{2} \int \sec u \tan u \cdot 2dx$

$= \frac{1}{2} \int \sec u \tan u \, du$

$= \frac{1}{2} \sec u + C = \boxed{\frac{1}{2} \sec 2x + C}$

23 $\int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{\cancel{x}}{u^{1/2}} \frac{du}{-2\cancel{x}} = -\frac{1}{2} \int u^{-1/2} du$

$u = 1 - x^2$

$du = -2x \, dx \Rightarrow dx = \frac{du}{-2x}$

$= -\frac{1}{\cancel{2}} \left[\cancel{2} u^{1/2} \right] + C$

$$= \boxed{-\sqrt{1-x^2}+C}$$

pattern recognition

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int x\,(\overbrace{1-x^2}^{g(x)})^{-1/2}\,dx\,(-2)$$

need $-2x\,dx$ for $g'(x)$

$$= -\frac{1}{2}\left[2(1-x^2)^{1/2}\right]+C$$

$$= -(1-x^2)^{1/2}+C$$

51 $\int \tan^4 x \sec^2 x\,dx = \int (u)^4 \cancel{\sec^2 x}\,\frac{du}{\cancel{\sec^2 x}} = \frac{u^5}{5}+C$

$$= \boxed{\frac{\tan^5 x}{5}+C}$$

$u = \tan x$

$du = \sec^2 x\,dx \rightarrow dx = \frac{du}{\sec^2 x}$

(47) $\int \frac{x^2-1}{\sqrt{2x-1}}\,dx = \int \frac{x^2}{\sqrt{2x-1}}\,dx + \int \frac{-1}{\sqrt{2x-1}}\,dx$

$u = 2x-1$ | $x = \frac{u+1}{2}$

$du = 2dx$ | $x^2 = \frac{u^2+2u+1}{4}$

$dx = \frac{du}{2}$

$-\int (u)^{-1/2}\,\frac{du}{2} = -\frac{1}{\cancel{2}}\left[\cancel{2}u^{1/2}\right] = -\sqrt{2x-1}$

$\int \frac{x^2}{(u)^{1/2}}\,\frac{du}{2} = \frac{1}{2}\int \frac{u^2+2u+1}{4u^{1/2}}\,du$

$$= \frac{1}{8}\int \left(u^{3/2} + 2u^{1/2} + u^{-1/2}\right) du$$

$$= \frac{1}{8}\left[\frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2} + 2u^{1/2}\right]$$

1st int we eval.

$$= \frac{1}{8}\left(\frac{2}{5}(2x-1)^{5/2} + \frac{4}{3}(2x-1)^{3/2}\right) + \frac{1}{4}(2x-1)^{1/2} - (2x-1)^{1/2} + C$$

$$= \boxed{\frac{1}{8}\left(\frac{2}{5}(2x-1)^{5/2} + \frac{4}{3}(2x-1)^{3/2}\right) - \frac{3}{4}(2x-1)^{1/2} + C}$$

4.6 Trapezoidal and Simpson's Rules

12 $\displaystyle\int_0^2 \frac{dx}{\sqrt{1+x^3}}$; use $n=4$

$f(x) = \dfrac{1}{\sqrt{1+x^3}}$, $n=4$, $a=0$, $b=2$

$a = x_0$ $\quad b = x_n$

x_0, x_1, x_2, x_3, x_4

$\downarrow$ 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2

Trapezoidal Rule

$$\int \frac{1}{\sqrt{1+x^3}}\,dx \approx \frac{2-0}{2(4)}\left(f(0)+2f\left(\tfrac{1}{2}\right)+2f(1)+2f\left(\tfrac{3}{2}\right)+f(2)\right)$$

$$\approx \frac{1}{4}\left(1+2(.942)+2(.707)+2(.478)+.333\right)$$

$$\approx \boxed{1.397}$$

Trapezoidal and Simpson's Rules

4.6

12 $\displaystyle\int_0^2 \frac{dx}{\sqrt{1+x^3}}$; use $n=4$

$f(x) = \dfrac{1}{\sqrt{1+x^3}}$, $n=4$, $a=0$, $b=2$

$a = x_0$ $\quad b = x_n$

x_0, x_1, x_2, x_3, x_4

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$

Simpson's Rule

$$\int \frac{1}{\sqrt{1+x^3}}\,dx \approx \frac{2-0}{3(4)}\left(f(0)+4f\left(\tfrac{1}{2}\right)+2f(1)+4f\left(\tfrac{3}{2}\right)+f(2)\right)$$

$$\approx \frac{1}{6}\left(1+4(.942)+2(.707)+4(.478)+.333\right)$$

$$\approx \boxed{1.404}$$

✓ on calculator:

under math key find fnint$((1+x^3)^{\wedge -.5}, x, 0, 2)$

$\boxed{1.402}$ Simpson's off by .002

Trap. off by .005

Practice

Whoops!! I need the damn power reducing formula

① $\int \sin^2 4x\,dx$ ~~$= \int \sin^2 u \frac{du}{4} = \frac{1}{4}\int \sin^2 u\,du$~~

~~$u = 4x$~~

~~$du = 4dx$~~

Recall $\sin^2 x = \frac{1-\cos 2x}{2}$

our "x" is 4x so 2·4x is 8x

$$\int \sin^2 4x\,dx = \int \frac{1-\cos 8x}{2}\,dx = \frac{1}{2}\left[\int 1\,dx - \frac{1}{8}\int \cos u\,du\right]$$

$u = 8x$

$du = 8\,dx \Rightarrow dx = \frac{du}{8}$

$$= \frac{x}{2} - \frac{1}{16}\sin u + C$$

$$\int \frac{1-\cos 8x}{2}\,dx = \frac{1}{2}\int(1-\cos 8x)\,dx = \boxed{\frac{x}{2} - \frac{1}{16}\sin 8x + C}$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos 8x\,dx$$