4.2 Area

① $$\int \frac{dx}{\sin^2 x} = \int \csc^2 x\,dx = \boxed{-\cot x + C}$$

② $$\int \frac{\sqrt[3]{t} - 5\sqrt[4]{t^3} + 8\sqrt[5]{t^4}}{t^2}\,dt$$

$$= \int \frac{t^{1/3} - 5t^{3/4} + 8t^{4/5}}{t^2}\,dt$$

$$= \int \left(t^{1/3 - 6/3} - 5t^{3/4 - 8/4} + 8t^{4/5 - 10/5}\right) dt$$

$$= \int \left(t^{-5/3} - 5t^{-5/4} + 8t^{-6/5} \right) dt$$

$$= \frac{t^{-2/3}}{-2/3} - \frac{5t^{-1/4}}{-1/4} + \frac{8t^{-1/5}}{-1/5} + C$$

$$= \boxed{-\frac{3}{2}t^{-2/3} + 20t^{-1/4} - 40t^{-1/5} + C}$$

$$\sum_{i=2}^{6} i^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = \boxed{90}$$

$$\sum_{i=1}^{n} k a_i = k \sum_{i=1}^{n} a_i$$

$$a_i = i^3$$

$$\sum_{i=1}^{3} 5i^3 = 5(1^3) + 5(2^3) + 5(3^3)$$

$$= 5\left[1^3 + 2^3 + 3^3\right]$$

$$= 5 \sum_{i=1}^{3} i^3 \;//$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

ex: Find the area (using the limit process) bounded by $y = x^3$, x-axis, $x = 0$ and $x = 1$.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

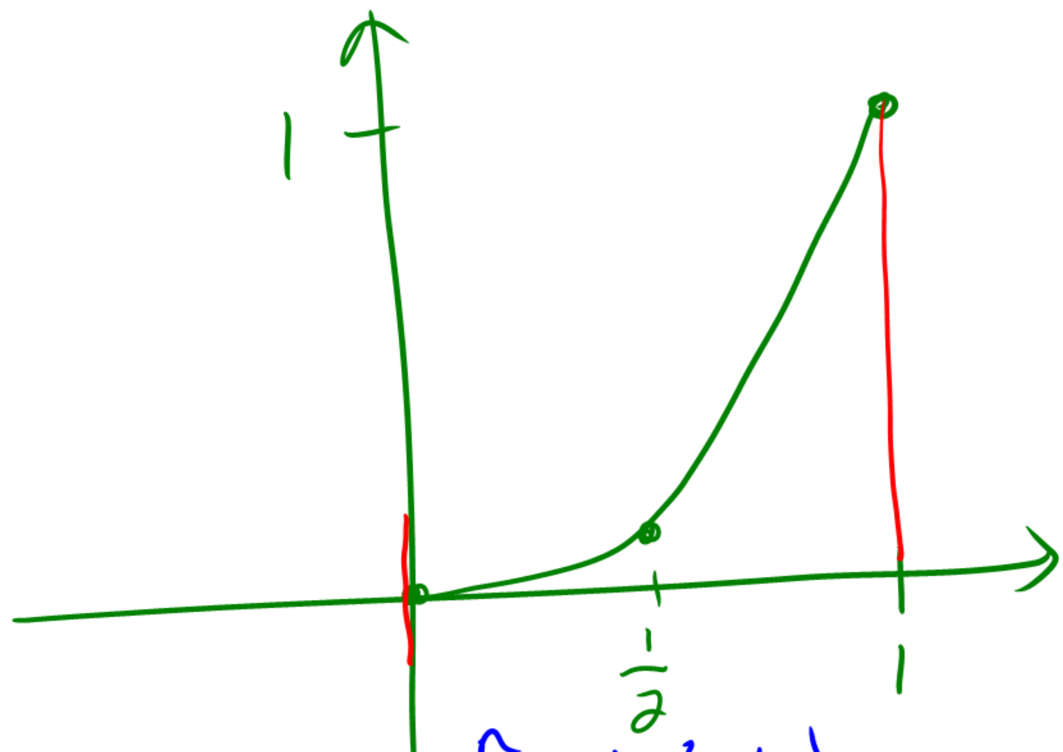
$$\Delta x_i = dx, \quad dx = \frac{b-a}{n}$$

$$dx = \frac{1-0}{n} = \frac{1}{n}$$

$$c_i = a + i\Delta x$$

$$c_i = 0 + i\left(\frac{1}{n}\right) = \frac{i}{n}$$

$$f(c_i) = (c_i)^3 = \left(\frac{i}{n}\right)^3$$



$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3$$

$$= \lim_{n\to\infty} \frac{1}{n^{4}} \left[\frac{\cancel{n^2} (n+1)^2}{4} \right]$$

$$= \lim_{n\to\infty} \left(\frac{n^2 + 2n + 1}{4n^2} \right)$$

$$= \frac{1}{4} + 0 + 0$$

$$= \boxed{\frac{1}{4}}$$

$$(n+1)^2 = n^2 + 2n + 1$$

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$y = x^2 - x^3$, x-axis, $x = -1$, $x = 1$

$a = -1$

$b = 1$

$dx = \frac{b-a}{n}$

$dx = \frac{1-(-1)}{n} = \frac{2}{n}$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$c_i = a + i\,dx$

$c_i = -1 + i\left(\frac{2}{n}\right) = -1 + \frac{2i}{n}$

$f(c_i) = \left[\left(-1+\frac{2i}{n}\right)^2 - \left(-1+\frac{2i}{n}\right)^3\right]$

$\left(-1+\frac{2i}{n}\right)^2$

$= (-1)^2 + 2(-1)\left(\frac{2i}{n}\right) + \left(\frac{2i}{n}\right)^2$

$= 1 - \frac{4i}{n} + \frac{4i^2}{n^2}$

$$\left(-1+\frac{2i}{n}\right)^3$$

$$=(-1)^3+3(-1)^2\left(\frac{2i}{n}\right)^1+3(-1)^1\left(\frac{2i}{n}\right)^2+\left(\frac{2i}{n}\right)^3$$

$$=\cancel{-1}+\cancel{\frac{6i}{n}}-\frac{12i^2}{n^2}-\frac{8i^3}{n^3}$$

$$2-\frac{10i}{n}+\frac{16i^2}{n^2}-\frac{8i^3}{n^3}$$

$$A=\lim_{n\to\infty}\sum_{i=1}^{n}\left(2-\frac{10i}{n}+\frac{16i^2}{n^2}-\frac{8i^3}{n^3}\right)\left(\frac{2}{n}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[\sum_{i=1}^{n}2+\frac{16}{n^2}\sum_{i=1}^{n}i^2-\frac{8}{n^3}\sum_{i=1}^{n}i^3-\frac{10}{n}\sum_{i=1}^{n}i\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n + \frac{\cancel{16}^{\,8}}{n^2}\left(\frac{n\overbrace{(n+1)(2n+1)}^{(2n^2+3n+1)}}{\cancel{6}\,3}\right) - \frac{\cancel{8}^{\,2}}{n^3}\left(\frac{n^2\overbrace{(n+1)^2}^{(n^2+2n+1)}}{\cancel{4}}\right) - \frac{\cancel{10}^{\,5}}{\cancel{n}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right)\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n + \frac{16n^{\cancel{3}\,2} + \cancel{24n^{\cancel{2}\,1}} + \cancel{8n}}{3n^{\cancel{2}}} - \frac{2n^{\cancel{4}\,2} + 4n^{\cancel{3}} + \cancel{2n^2}}{n^{\cancel{3}}} - 5n - 5\right]$$

$$= \lim_{n\to\infty} \left[4 + \frac{32}{3} + \frac{(24n-8)(2)}{n^2} - 4 + \frac{(4n+2)2}{n^2} - 10 - \frac{10}{n}\right]$$

$$= \cancel{4} + \frac{32}{3} + 0 - \cancel{4} + 0 - 10 - 0$$

$$= \boxed{\frac{2}{3}}$$

10.3

$$\int_1^3 3x^2\,dx$$

$$\Delta x = \frac{3-1}{n} = \frac{2}{n}$$

$$c_i = 1 + \frac{2i}{n}$$

$$f(c_i) = 3\left(1 + \frac{2i}{n}\right)^2$$

$$= 3\left(\frac{n+2i}{n}\right)^2$$

$$= 3\left(\frac{n^2 + 4ni + 4i^2}{n^2}\right)$$

$$A = \lim_{n\to\infty} \frac{6}{n^3}\left[\sum_{i=1}^{n} n^2 + 4n\sum_{i=1}^{n} i + 4\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \frac{6}{n^3}\left[n^3 + \frac{\overset{2}{\cancel{4}}n\left(\overset{n^2+n}{n(n+1)}\right)}{\cancel{2}} + \frac{\overset{2}{\cancel{4}}\left(\overset{2n^2+3n+1}{n(n+1)(2n+1)}\right)}{\underset{3}{\cancel{6}}}\right]$$

$$= \lim_{n\to\infty}\left(\frac{6\cancel{n^3}}{\cancel{n^3}} + \frac{6(2n^2+2n)}{n^2} + \frac{\overset{2}{\cancel{6}}}{n^2}\left(\frac{4n^2+6n+2}{\cancel{3}}\right)\right)$$

$$= \lim_{n\to\infty}\left(6+12+\frac{12}{n}+8+\frac{12}{n}+\frac{4}{n^2}\right)$$

$$= 6+12+0+8+0+0$$

$$= \boxed{26}$$