MATH 104/GRACEY NAME_____

WORKSHEET/5.4-5.5

SUM AND DIFFERENCE IDENTITIES

Sum Identity for Cosine

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

Difference Identity for Cosine

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$

Sum Identity for Sine

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

Difference Identity for Sine

$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$

Sum Identity for Tangent

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

Difference Identity for Tangent

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

DOUBLE-ANGLE IDENTITIES

$$\cos 2A=\cos^2 A-\sin^2 A$$

$$\cos 2A=\cos-2\sin^2 A$$

$$\cos 2A=2\cos^2 A-1$$

$$\sin 2A=2\sin A\cos A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

- PRODUCT-TO-SUM AND SUM-TO-PRODUCT IDENTITIES

$$\cos A\cos B=\frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right] \quad \sin A\sin B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$

$$\sin A\cos B=\frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right] \quad \cos A\sin B=\frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$$

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \quad \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \quad \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

1. Find the mistake(s).

a.

$$\begin{aligned}(1-\sin\theta)^2 &= 1-\sin^2\theta\\ &= \cos^2\theta\end{aligned}$$

b.

$$\cos\left(\pi+\frac{\pi}{2}\right)=\cos(\pi)+\cos\left(\frac{\pi}{2}\right)$$
$$=-1+0$$
$$=-1$$

2. If $\cos s=-\frac{1}{5}$, $\sin s=\frac{3}{5}$, s and t in quadrant II, find

a. $\sin(s+t)$

b. $\cos(s+t)$

c. the quadrant of $s+t$

3. Verify that each equation is an identity.

a. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

b. $\csc\theta\cos^2\theta+\sin\theta=\csc\theta$

c. $\dfrac{\cos x+\sin x}{\cos x-\sin x}-\dfrac{\cos x-\sin x}{\cos x+\sin x}=2\tan 2x$

d. $\sin 2\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}$

4. If $\cos 2x = \frac{2}{3}$ and $\frac{\pi}{2} < x < \pi$, find

a. $\cos x$

b. $\sin x$

5. Write the following functions as a trigonometric function of *x*.

a. $\cos 4x$

b. $\sin 3x$

6. Write as a sum or difference of trigonometric functions

a. $8\sin 7x \sin 9x$

b. $2\cos 85^\circ \sin 140^\circ$

7. Write as a product of trigonometric functions

a. $\cos 5x + \cos 8x$

b. $\sin 102^\circ - \sin 95^\circ$