

## CHAPTER PROBLEM

Does the MicroSort method of gender selection increase the likelihood that a baby will be a girl?

Gender-selection methods are somewhat controversial. Some people believe that use of such methods should be prohibited, regardless of the reason. Others believe that limited use should be allowed for medical reasons, such as to prevent gender-specific hereditary disorders. For example, some couples carry X-linked recessive genes, so that a male child has a 50% chance of inheriting a serious disorder and a female child has no chance of inheriting the disorder. These couples may want to use a gender-selection method to increase the likelihood of having a baby girl so that none of their children inherit the disorder.

Methods of gender-selection have been around for many years. In the 1980s, ProCare Industries sold a product called Gender Choice. The product cost only \$49.95, but the FDA told the company to stop distributing Gender Choice because there was no evidence to support the claim that it was 80% reliable.

The Genetics and IVF Institute developed a newer gender-selection method called MicroSort. The MicroSort XSORT method is designed to increase the likelihood of a baby girl, and the YSORT method is designed to increase

the likelihood of a boy. Here is a statement from the MicroSort web site: "The Genetics and IVF Institute is offering couples the ability to increase the chance of having a child of the desired gender to reduce the probability of X-linked diseases or for family balancing." Stated simply, for a cost exceeding \$3000, The Genetics and IVF Institute claims that it can increase the probability of having a baby of the gender that a couple prefers. As of this writing, the MicroSort method is undergoing clinical trials, but these results are available: Among 726 couples who used the XSORT method in trying to have a baby girl, 668 couples did have baby girls, for a success rate of 92.0%. Under normal circumstances with no special treatment, girls occur in 50% of the births. (Actually the current birth rate of girls is 48.79%, but we will use 50% to keep things simple.) These results provide us with an interesting question: Given that 668 out of 726 couples had girls, can we actually support the claim that the XSORT technique is effective in increasing the probability of a girl? Do we now have an effective method of gender selection?

## MATH 103 CHAPTER 8 HOMEWORK

8.2 1-15 odd, 17-24, 27, 29-35 odd, 37, 41, 43, 45

8.3 1-5, 7, 11, 13-16, 19, 24, 25, 26, 29, 32

8.4 1-5, 7, 9, 10, 11, 14, 16, 17, 19

8.5 1-5, 7, 9, 11, 13, 15, 16, 19, 23, 27

## 8.1 REVIEW AND PREVIEW

In Chapters 2 and 3 we used " \_\_\_\_\_  
 \_\_\_\_\_ " when we \_\_\_\_\_  
 data using tools such as the \_\_\_\_\_, and \_\_\_\_\_  
 \_\_\_\_\_. Methods of \_\_\_\_\_  
 statistics use \_\_\_\_\_ data to make an \_\_\_\_\_  
 or \_\_\_\_\_ about a \_\_\_\_\_.

The two main activities of \_\_\_\_\_  
 \_\_\_\_\_ are using sample data to (1) \_\_\_\_\_  
 a \_\_\_\_\_ and  
 (2) \_\_\_\_\_ a \_\_\_\_\_ or \_\_\_\_\_  
 about a \_\_\_\_\_. In  
 Chapter 7 we presented methods for \_\_\_\_\_ a  
 \_\_\_\_\_ with a

\_\_\_\_\_, and in this chapter we present the method of \_\_\_\_\_.

## DEFINITION

In statistics, a **hypothesis** is a \_\_\_\_\_ or \_\_\_\_\_ about a \_\_\_\_\_ of the \_\_\_\_\_.

A **hypothesis test (aka test of significance)** is a \_\_\_\_\_ for testing a \_\_\_\_\_ about a \_\_\_\_\_ of a \_\_\_\_\_.

The main objective of this chapter is to \_\_\_\_\_ the \_\_\_\_\_ to \_\_\_\_\_ tests for claims made about a population \_\_\_\_\_, a population \_\_\_\_\_, or a population \_\_\_\_\_.

## 8.2 BASICS OF HYPOTHESIS TESTING

Key Concept...

In this section we present individual \_\_\_\_\_ of a \_\_\_\_\_ . In Part 1 we discuss the basic \_\_\_\_\_ of \_\_\_\_\_

testing. Because these concepts are used in the following sections and chapters, we should \_\_\_\_\_ and

\_\_\_\_\_ the following:

$\pi$  How to \_\_\_\_\_ the \_\_\_\_\_  
 \_\_\_\_\_ and \_\_\_\_\_  
 \_\_\_\_\_ from a given \_\_\_\_\_,  
 and how to \_\_\_\_\_ both in \_\_\_\_\_  
 form

$\pi$  How to \_\_\_\_\_ the \_\_\_\_\_ of the  
 \_\_\_\_\_, given a  
 \_\_\_\_\_ and \_\_\_\_\_

$\pi$  How to \_\_\_\_\_ the \_\_\_\_\_  
 \_\_\_\_\_, given a \_\_\_\_\_  
 \_\_\_\_\_

$\pi$  How to \_\_\_\_\_ the \_\_\_\_\_, given  
 a \_\_\_\_\_ of the \_\_\_\_\_  
 \_\_\_\_\_

$\pi$  How to \_\_\_\_\_ the \_\_\_\_\_ about a

\_\_\_\_\_ in \_\_\_\_\_ and  
 \_\_\_\_\_ terms

In Part 2 we discuss the \_\_\_\_\_ of a \_\_\_\_\_  
 \_\_\_\_\_.

## PART 1: BASICS CONCEPTS OF HYPOTHESIS TESTING

The methods presented in this chapter are based on the \_\_\_\_\_  
 \_\_\_\_\_ for \_\_\_\_\_.

### RARE EVENT RULE FOR INFERENCE STATISTICS

If, under a given assumption, the \_\_\_\_\_ of a particular  
 observed is extremely \_\_\_\_\_, we conclude that the  
 \_\_\_\_\_ is probably not \_\_\_\_\_.

Following this rule, we \_\_\_\_\_ a \_\_\_\_\_ by  
 \_\_\_\_\_ sample data in an attempt to \_\_\_\_\_  
 between results that can \_\_\_\_\_ by  
 \_\_\_\_\_ and results that are \_\_\_\_\_  
 to \_\_\_\_\_ by \_\_\_\_\_. We can explain the  
 occurrence of \_\_\_\_\_ results by  
 saying that either a \_\_\_\_\_ has indeed occurred or

that the \_\_\_\_\_ is \_\_\_\_\_  
\_\_\_\_\_.

### WORKING WITH THE STATED CLAIM: NULL AND ALTERNATIVE HYPOTHESES

The **null hypothesis** denoted by \_\_\_\_\_ is a \_\_\_\_\_ that the value of a \_\_\_\_\_ is \_\_\_\_\_ to some \_\_\_\_\_ value. The term \_\_\_\_\_ is used to \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_.

The **alternative hypothesis** denoted by \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ is the \_\_\_\_\_ that the \_\_\_\_\_ has a value that somehow \_\_\_\_\_ from the \_\_\_\_\_.

For the methods of this chapter, the \_\_\_\_\_ form of the \_\_\_\_\_ must use one of these symbols: \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

### NOTE ABOUT FORMING YOUR OWN CLAIMS (HYPOTHESES)

If you are \_\_\_\_\_ a study and want to use a \_\_\_\_\_ to \_\_\_\_\_ your

\_\_\_\_\_, the \_\_\_\_\_ must be worded so that it becomes the \_\_\_\_\_. You can \_\_\_\_\_ a \_\_\_\_\_ that some parameter is \_\_\_\_\_ to some \_\_\_\_\_ value.

## IDENTIFYING \_\_\_\_\_ AND \_\_\_\_\_

### START

1

- Identify the specific \_\_\_\_\_ or \_\_\_\_\_ to be tested
- Express it in \_\_\_\_\_ form

2

- Give the symbolic form that must be \_\_\_\_\_ when the \_\_\_\_\_ is \_\_\_\_\_

3

- Using the two \_\_\_\_\_ expressions obtained so far, identify the \_\_\_\_\_ and the \_\_\_\_\_
- \_\_\_\_\_ is the symbolic expression that \_\_\_\_\_ contain \_\_\_\_\_
- \_\_\_\_\_ is the symbolic expression that the \_\_\_\_\_ the \_\_\_\_\_ value being \_\_\_\_\_

Example 1: Examine the given statement, then express the null hypothesis and the alternative hypothesis in symbolic form.

- a. The proportion of people aged 18-25 who currently use illicit drugs is equal to 0.20.
- b. The majority of college students have credit cards.
- c. The standard deviation of daily rainfall amounts in San Francisco is 0.66 cm.
- d. The mean weight of plastic discarded by households in one week is less than 1 kg.

## CONVERTING SAMPLE DATA TO A TEST STATISTIC

The \_\_\_\_\_ required for a \_\_\_\_\_ test typically involve \_\_\_\_\_ a \_\_\_\_\_ to a \_\_\_\_\_.

The **test statistic** is a \_\_\_\_\_ used in making a \_\_\_\_\_ about the \_\_\_\_\_. It is found by converting



the \_\_\_\_\_ (such as \_\_\_\_\_, \_\_\_\_\_, or \_\_\_\_\_) to a \_\_\_\_\_ (such as \_\_\_\_\_, \_\_\_\_\_, or \_\_\_\_\_) with the \_\_\_\_\_ that the \_\_\_\_\_ is \_\_\_\_\_.

\_\_\_\_\_ . In this chapter we use the following \_\_\_\_\_ statistics:

Test statistic for proportion:

Test statistic for mean:

Test statistic for standard deviation

Example 2: Find the value of the test statistic. The claim is that less than  $\frac{1}{2}$  of adults in the United States have carbon monoxide detectors. A KRC Research survey of 1005 adults resulted in 462 who have carbon monoxide detectors.

### TOOLS FOR ASSESSING THE TEST STATISTIC: CRITICAL REGION, SIGNIFICANCE LEVEL, CRITICAL VALUE, AND P-VALUE

The \_\_\_\_\_ alone usually \_\_\_\_\_  
 \_\_\_\_\_ give us enough information to make a decision about the \_\_\_\_\_  
 being \_\_\_\_\_. The following tools can be used to \_\_\_\_\_  
 and \_\_\_\_\_ the \_\_\_\_\_.

$\pi$  The **critical region (aka rejection region)** is the \_\_\_\_\_ of  
 all \_\_\_\_\_ of the \_\_\_\_\_  
 that cause us to \_\_\_\_\_ the \_\_\_\_\_  
 \_\_\_\_\_

$\pi$  The **significance level (denoted by \_\_\_\_\_)** is the \_\_\_\_\_  
 that the \_\_\_\_\_ will fall in  
 the \_\_\_\_\_ when the  
 \_\_\_\_\_ is actually \_\_\_\_\_.

If the \_\_\_\_\_ falls in the \_\_\_\_\_, we \_\_\_\_\_ the \_\_\_\_\_, so \_\_\_\_\_ is the \_\_\_\_\_ of making the \_\_\_\_\_ of \_\_\_\_\_ the \_\_\_\_\_ when it is \_\_\_\_\_.

$\pi$  A **critical value** is any value that \_\_\_\_\_ the \_\_\_\_\_ from the \_\_\_\_\_ of the \_\_\_\_\_ that \_\_\_\_\_ lead to \_\_\_\_\_ of the \_\_\_\_\_. The \_\_\_\_\_ depend on the nature of the \_\_\_\_\_, the \_\_\_\_\_ that applies, and the \_\_\_\_\_ of \_\_\_\_\_.

$\pi$  The **P-value (aka p-value or probability value)** is the \_\_\_\_\_ of getting a \_\_\_\_\_ of the \_\_\_\_\_ that is \_\_\_\_\_

\_\_\_\_\_ as the one  
representing the \_\_\_\_\_, assuming that  
the \_\_\_\_\_ is \_\_\_\_\_.  
 $P$ -values can be found \_\_\_\_\_ finding the \_\_\_\_\_  
\_\_\_\_\_ the \_\_\_\_\_.

The procedure can be summarized as follows:

Critical region in the left tail:

Critical region in the right tail:

Critical region in two tails:

The \_\_\_\_\_ is \_\_\_\_\_ if  
 the \_\_\_\_\_ is very \_\_\_\_\_, such as  
 \_\_\_\_\_ or \_\_\_\_\_.

## DECISIONS AND CONCLUSIONS

***P*-value method:** Using the \_\_\_\_\_:  
 If *P*-value \_\_\_\_\_,  
 If *P*-value \_\_\_\_\_ to \_\_\_\_\_

**Traditional method:** If the \_\_\_\_\_ falls  
 \_\_\_\_\_ the \_\_\_\_\_  
 \_\_\_\_\_, \_\_\_\_\_. If the  
 \_\_\_\_\_  
 \_\_\_\_\_ fall \_\_\_\_\_ the  
 \_\_\_\_\_, \_\_\_\_\_ to  
 \_\_\_\_\_.

**Another option:** Instead of using a \_\_\_\_\_  
 such as \_\_\_\_\_, simply identify the  
 \_\_\_\_\_ and leave the \_\_\_\_\_  
 to the \_\_\_\_\_.

**Confidence intervals:** A \_\_\_\_\_  
 of a \_\_\_\_\_ contains  
 the \_\_\_\_\_ values of that  
 \_\_\_\_\_. If a \_\_\_\_\_  
 \_\_\_\_\_ does \_\_\_\_\_  
 \_\_\_\_\_ a \_\_\_\_\_ value of a  
 \_\_\_\_\_,  
 \_\_\_\_\_ that \_\_\_\_\_.

Example 3: Use the given information to find  $P$ -value.

a. The test statistic in a right-tailed test is  $z = 2.50$

c. With  $H_1: p \neq \frac{3}{4}$ , the test statistic is  $z = 0.35$

b. The test statistic in a two-tailed test is  $z = -0.55$

d. With  $H_1: p < 0.777$ , the test statistic is  $z = -2.95$

Example 4: State the final conclusion in simple non-technical terms. Be sure to address the original claim.

a. Original claim: The percentage of on-time U.S. airline flights is less than 75%. Initial conclusion: Reject the null hypothesis.

b. Original claim: The percentage of Americans who believe in heaven is equal to 90%. Initial conclusion: Reject the null hypothesis.

## ERRORS IN HYPOTHESIS TESTS

When testing a null hypothesis, we arrive at a \_\_\_\_\_ of \_\_\_\_\_ it or \_\_\_\_\_ to \_\_\_\_\_ it.

Such conclusions are sometimes \_\_\_\_\_ and sometimes \_\_\_\_\_ (even if we do everything \_\_\_\_\_).

$\pi$  **Type I error:** The \_\_\_\_\_ of \_\_\_\_\_ the \_\_\_\_\_ when it is actually \_\_\_\_\_. The symbol \_\_\_\_\_ is used to represent the \_\_\_\_\_ of a \_\_\_\_\_

error.

$\pi$  **Type II error:** The \_\_\_\_\_ of \_\_\_\_\_ to \_\_\_\_\_ the \_\_\_\_\_ when it is actually \_\_\_\_\_. The symbol \_\_\_\_\_ is used to represent the \_\_\_\_\_ of a \_\_\_\_\_ error.

## NOTATION

$\alpha$  (alpha) = \_\_\_\_\_ of a \_\_\_\_\_ (the \_\_\_\_\_ of \_\_\_\_\_ the \_\_\_\_\_ when it is \_\_\_\_\_)

$\beta$  (beta) = \_\_\_\_\_ of a \_\_\_\_\_ (the \_\_\_\_\_ of \_\_\_\_\_ to \_\_\_\_\_ the \_\_\_\_\_ when it is \_\_\_\_\_)

## CONTROLLING TYPE I AND TYPE II ERRORS

One step in our standard procedure for testing \_\_\_\_\_ involves the \_\_\_\_\_ of the \_\_\_\_\_ level \_\_\_\_\_, which is the \_\_\_\_\_ of a \_\_\_\_\_ error. The values of \_\_\_\_\_, \_\_\_\_\_, and the sample size \_\_\_\_\_ are all \_\_\_\_\_, so when you



choose or \_\_\_\_\_ any \_\_\_\_\_ of them, the \_\_\_\_\_ is automatically \_\_\_\_\_. One common practice is to select the \_\_\_\_\_ level \_\_\_\_\_, then select a \_\_\_\_\_ size that is \_\_\_\_\_, so the value of \_\_\_\_\_ is \_\_\_\_\_.

Generally, try to use the \_\_\_\_\_ that you can tolerate, but for \_\_\_\_\_ errors with more serious consequences, select \_\_\_\_\_ values of \_\_\_\_\_. Then choose a \_\_\_\_\_ as \_\_\_\_\_ as is \_\_\_\_\_, based on considerations of \_\_\_\_\_, \_\_\_\_\_, and other relevant factors. Another common practice is to select \_\_\_\_\_ and \_\_\_\_\_, so the required sample size \_\_\_\_\_ is automatically determined.

---

**TRUE STATE OF NATURE**

THE NULL  
HYPOTHESIS  
IS TRUE

THE NULL  
HYPOTHESIS  
IS FALSE

<b>DECISION</b>	We decide to reject $H_0$	TYPE I ERROR	CORRECT DECISION
	We fail to reject $H_0$	CORRECT DECISION	TYPE II ERROR

---

Example 5: Identify the type I error and the type II error that correspond to the given hypothesis.

- a. The percentage of Americans who believe that life exists only on earth is equal to 20%.
- b. The percentage of households with at least two cell phones is less than 60%.

## COMPREHENSIVE HYPOTHESIS TEST

In this section we describe the \_\_\_\_\_  
used in a \_\_\_\_\_ test, but the following sections will  
combine those components in \_\_\_\_\_.  
We can \_\_\_\_\_ claims about \_\_\_\_\_ by  
using the \_\_\_\_\_ method, the \_\_\_\_\_ method, or we  
can use a \_\_\_\_\_.

## CONFIDENCE INTERVAL METHOD

For \_\_\_\_\_ hypothesis tests \_\_\_\_\_ a \_\_\_\_\_  
interval with a \_\_\_\_\_ of \_\_\_\_\_; but  
for a \_\_\_\_\_ hypothesis test with \_\_\_\_\_

\_\_\_\_\_, construct a \_\_\_\_\_ of \_\_\_\_\_.

A \_\_\_\_\_ of a \_\_\_\_\_ contains the \_\_\_\_\_ values of that parameter. We should therefore \_\_\_\_\_ a \_\_\_\_\_ that the population parameter has a \_\_\_\_\_ that is \_\_\_\_\_ included in the \_\_\_\_\_.

## PART 2: BEYOND THE BASICS OF HYPOTHESIS TESTING: THE POWER OF A TEST

We use \_\_\_\_\_ to denote the \_\_\_\_\_ of \_\_\_\_\_ to \_\_\_\_\_ a \_\_\_\_\_, so

$P(\text{type II error}) = \beta$ . It follows that \_\_\_\_\_ is the \_\_\_\_\_ of \_\_\_\_\_ a \_\_\_\_\_, and

statisticians refer to this probability as the \_\_\_\_\_ of a \_\_\_\_\_, and they often use it to \_\_\_\_\_ the \_\_\_\_\_

of a hypothesis test in allowing us to recognize that a \_\_\_\_\_ is \_\_\_\_\_.

**DEFINITION**

The **power** of a \_\_\_\_\_ test is the \_\_\_\_\_ of \_\_\_\_\_ a \_\_\_\_\_ hypothesis. The \_\_\_\_\_ of the \_\_\_\_\_ is \_\_\_\_\_ by using a particular \_\_\_\_\_ and a \_\_\_\_\_ value of the \_\_\_\_\_ that is an \_\_\_\_\_ to the value assumed \_\_\_\_\_ in the \_\_\_\_\_.

**POWER AND THE DESIGN OF EXPERIMENTS**

Just as \_\_\_\_\_ is a common choice for a \_\_\_\_\_ level, a power of at least \_\_\_\_\_ is a common requirement for \_\_\_\_\_ that a \_\_\_\_\_ test is \_\_\_\_\_. When \_\_\_\_\_ an \_\_\_\_\_, a goal of having a \_\_\_\_\_ value of \_\_\_\_\_ can often be used in \_\_\_\_\_ the \_\_\_\_\_.

Example 6: Chantix tablets are used as an aid to help people stop smoking. In a clinical trial, 129 subjects were treated with Chantix twice a day for 12 weeks, and 16 subjects experienced abdominal pain. If someone claims that more than 8% of Chantix users experience abdominal pain, that claim is supported with a hypothesis test conducted with a 0.05 significance level. Using 0.18 as an alternative value of  $p$ , the power of the test is 0.96. Interpret this value of the power of the test.

### 8.3 TESTING A CLAIM ABOUT A PROPORTION

Key Concept...

In section 8.2 we presented the individual \_\_\_\_\_ of

a \_\_\_\_\_. In this

section we present \_\_\_\_\_ for

\_\_\_\_\_ a \_\_\_\_\_ (or

\_\_\_\_\_ ) made about a \_\_\_\_\_.

We illustrate \_\_\_\_\_ testing with the \_\_\_\_\_

method, the \_\_\_\_\_ method, and the use of

\_\_\_\_\_. In

addition to testing \_\_\_\_\_ about population proportions,  
 we can use the \_\_\_\_\_ procedure for testing claims about  
 \_\_\_\_\_ or the \_\_\_\_\_  
 of \_\_\_\_\_.

Two common methods for testing a claim about a \_\_\_\_\_  
 proportion are (1) to use a \_\_\_\_\_  
 \_\_\_\_\_ as an \_\_\_\_\_ to the  
 \_\_\_\_\_ distribution, and (2) to use an  
 \_\_\_\_\_ method based on the \_\_\_\_\_  
 \_\_\_\_\_.

## PART 1: BASIC METHODS OF TESTING CLAIMS ABOUT A POPULATION PROPORTION $p$

### REQUIREMENTS

#### OBJECTIVE

#### NOTATION

 $n =$ 
 $p =$ 
 $\hat{p} = \text{---}$ 
 $q =$

**REQUIREMENTS**

1. The \_\_\_\_\_ observations are a \_\_\_\_\_ sample.
2. The \_\_\_\_\_ for a \_\_\_\_\_ are satisfied.
3. The conditions \_\_\_\_\_ and \_\_\_\_\_ are \_\_\_\_\_ satisfied so the \_\_\_\_\_ of \_\_\_\_\_ proportions can be \_\_\_\_\_ by a \_\_\_\_\_ with \_\_\_\_\_ and \_\_\_\_\_. Note that \_\_\_\_\_ is the \_\_\_\_\_ used in the \_\_\_\_\_.

**TEST STATISTIC FOR TESTING A CLAIM ABOUT A PROPORTION**

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)}}$$
 $P$  - values:

Critical values:

**FINDING THE NUMBER OF SUCCESSES  $x$** 

Computer software and \_\_\_\_\_ designed for \_\_\_\_\_ tests of \_\_\_\_\_ usually require \_\_\_\_\_ consisting of the \_\_\_\_\_ and the number of \_\_\_\_\_, but the \_\_\_\_\_ is often given instead of \_\_\_\_\_.

Example 1: Identify the indicated values. Use the normal distribution as an approximation to the binomial distribution. In a survey, 1864 out of 2246 randomly selected adults in the United States said that texting while driving should be illegal (based on data from Zogby International). Consider a hypothesis test that uses a 0.05 significance level to test the claim that more than 80% of adults believe that texting while driving should be illegal.

- a. What is the test statistic?
- b. What is the critical value?
- c. What is the  $P$ -value?
- d. What is the conclusion?



Example 2: The company Drug Test Success provides a "1-Panel-THC" test for marijuana usage. Among 300 tested subjects, results from 27 subjects were wrong (either a false positive or a false negative). Use a 0.05 significance level to test the claim that less than 10% of the test results are wrong. Does the test appear to be good for most purposes?

- a. Identify the null hypothesis
- b. Identify the alternative hypothesis
- c. Identify the test statistic
- d. Identify the  $P$ -value or critical value(s)
- e. What is your conclusion about the null hypothesis?
- f. What is your final conclusion?

Example 3: In recent years, the town of Newport experienced an arrest rate of 25% for robberies (based on FBI data). The new sheriff compiles records showing that among 30 recent robberies, the arrest rate is 30%, so she claims that her arrest rate is greater than the 25% rate in the past. Is there sufficient evidence to support her claim that the arrest rate is greater than 25%?

- a. Identify the null hypothesis
- b. Identify the alternative hypothesis
- c. Identify the test statistic
- d. Identify the  $P$ -value or critical value(s)
- e. What is your conclusion about the null hypothesis?
- f. What is your final conclusion?

## PART 2: EXACT METHOD FOR TESTING CLAIMS ABOUT A POPULATION PROPORTION $p$

Instead of using the \_\_\_\_\_ distribution as an \_\_\_\_\_ to the \_\_\_\_\_, we can get \_\_\_\_\_ results by using the \_\_\_\_\_ itself. This exact approach \_\_\_\_\_ require that \_\_\_\_\_ and \_\_\_\_\_, so we have a method that applies when the requirement is not satisfied.

Left-tailed test: The  $P$ -value is the \_\_\_\_\_ of getting \_\_\_\_\_ or \_\_\_\_\_ among \_\_\_\_\_ trials.

Right-tailed test: The  $P$ -value is the \_\_\_\_\_ of getting \_\_\_\_\_ or \_\_\_\_\_ among \_\_\_\_\_ trials.

### 8.4 TESTING A CLAIM ABOUT A MEAN: SIGMA KNOWN

Key Concept...

In this section, we discuss \_\_\_\_\_ methods for \_\_\_\_\_ made about a \_\_\_\_\_, assuming the \_\_\_\_\_

\_\_\_\_\_ is a \_\_\_\_\_ value. Here we use the  
 \_\_\_\_\_ with the \_\_\_\_\_  
 components of \_\_\_\_\_ that were introduced  
 in Section 8.2.

## TESTING CLAIMS ABOUT A POPULATION MEAN (WITH $\sigma$ KNOWN)

### OBJECTIVE

### NOTATION

$n =$

$\mu_{\bar{x}} =$

$\bar{x} =$

$\sigma =$

### REQUIREMENTS

- The \_\_\_\_\_ is a \_\_\_\_\_  
 (\_\_\_\_\_).
- The \_\_\_\_\_ of the \_\_\_\_\_  
 \_\_\_\_\_ is \_\_\_\_\_.
- The \_\_\_\_\_ is \_\_\_\_\_  
 and/or \_\_\_\_\_.

**TEST STATISTIC FOR TESTING A CLAIM ABOUT A MEAN (WITH  $\sigma$  KNOWN)** $z =$  \_\_\_\_\_ $P$  – values:

Critical values:

Example 1: When a fair die is rolled many times, the outcomes of 1, 2, 3, 4, 5, and 6 are equally likely, so the mean of the outcomes should be 3.5. The author drilled holes into a die and loaded it by inserting lead weights, then rolled it 40 times to obtain a mean of 2.9375. Assume that the standard deviation of the outcomes is 1.7078, which is the standard deviation for a fair die. Use a 0.05 significance level to test the claim that outcomes from the loaded die have a mean different from the value of 3.5 expected with a fair die.

- a. Identify the null hypothesis
- b. Identify the alternative hypothesis
- c. Identify the test statistic

d. Identify the  $P$ -value or critical value(s)

e. What is your conclusion about the null hypothesis?

f. What is your final conclusion?

Example 2: Listed below are recorded speeds (in mi/h) of randomly selected cars traveling on a section of Highway 405 in Los Angeles (based on data from Sigalert). That part of the highway has a posted speed limit of 65 mi/h. Assume that the standard deviation of speeds is 5.7 mi/h and use a 0.01 significance level to test the claim that the sample data is from a population with a mean greater than 65 mi/h.

68 68 72 73 65 74 73 72 68 65 65 73 66 71 68 74 66 71 65 73  
59 75 70 56 66 75 68 75 62 72 60 73 61 75 58 74 60 73 58 75

a. Identify the null hypothesis

b. Identify the alternative hypothesis

c. Identify the test statistic

d. Identify the  $P$ -value or critical value(s)

e. What is your conclusion about the null hypothesis?

f. What is your final conclusion?

## 8.5 TESTING A CLAIM ABOUT A MEAN: SIGMA NOT KNOWN

Key Concept...

In Section 8.4 we discussed methods for testing a \_\_\_\_\_

about a \_\_\_\_\_, but that section

is based on the \_\_\_\_\_ assumption that the value of

the \_\_\_\_\_

is \_\_\_\_\_. In this section, we present methods for testing a claim about a \_\_\_\_\_, but we \_\_\_\_\_ require that \_\_\_\_\_ is known. The methods of this section are referred to as a \_\_\_\_\_ because they use the \_\_\_\_\_ that was introduced in Section 7.4.

### TESTING CLAIMS ABOUT A POPULATION MEAN (WITH $\sigma$ NOT KNOWN)

#### OBJECTIVE

#### NOTATION

$n =$

$\mu_{\bar{x}} =$

$\bar{x} =$

$s =$

#### REQUIREMENTS

1. The \_\_\_\_\_ is a \_\_\_\_\_ (\_\_\_\_\_).
2. The \_\_\_\_\_ of the \_\_\_\_\_



\_\_\_\_\_ is \_\_\_\_\_.

3. The \_\_\_\_\_ is \_\_\_\_\_ and/or \_\_\_\_\_.

### TEST STATISTIC FOR TESTING A CLAIM ABOUT A MEAN (WITH $\sigma$ KNOWN)

$t =$  \_\_\_\_\_

$P$  – values:

Critical values:

### IMPORTANT PROPERTIES OF THE STUDENT $t$ DISTRIBUTION

1. The \_\_\_\_\_ is \_\_\_\_\_ for \_\_\_\_\_ sample sizes.

2. The Student \_\_\_\_\_ distribution has the same general \_\_\_\_\_ as the \_\_\_\_\_

distribution; its \_\_\_\_\_ shape reflects the \_\_\_\_\_

\_\_\_\_\_ that is expected when \_\_\_\_\_ is used to estimate \_\_\_\_\_.

3. The \_\_\_\_\_ has a mean of \_\_\_\_\_.

4. The \_\_\_\_\_ of the \_\_\_\_\_ with the \_\_\_\_\_ and is \_\_\_\_\_ than \_\_\_\_\_.

5. As the \_\_\_\_\_ gets \_\_\_\_\_, the \_\_\_\_\_ gets \_\_\_\_\_ to the \_\_\_\_\_.

### CHOOSING THE CORRECT METHOD

When \_\_\_\_\_ a \_\_\_\_\_ about a \_\_\_\_\_, first be sure that the sample data have been collected with an appropriate \_\_\_\_\_ method. If we have a \_\_\_\_\_, a \_\_\_\_\_ test of a \_\_\_\_\_ about \_\_\_\_\_ might use the \_\_\_\_\_, the \_\_\_\_\_ distribution, or it might require \_\_\_\_\_ methods or \_\_\_\_\_ resampling techniques.

To test a \_\_\_\_\_ about a \_\_\_\_\_, use the \_\_\_\_\_ when the sample is a \_\_\_\_\_.

\_\_\_\_\_, \_\_\_\_\_ is \_\_\_\_\_, and  
\_\_\_\_\_ or \_\_\_\_\_ of these conditions is  
satisfied:

The \_\_\_\_\_ is \_\_\_\_\_  
distributed or \_\_\_\_\_.

Example 1: Determine whether the hypothesis test involves a sampling distribution of means that is a normal distribution, Student  $t$  distribution, or neither.

- a. Claim about FICO credit scores of adults:  $\mu = 678$ ,  $n = 12$ ,  $\bar{x} = 719$ ,  $s = 92$ .  
The sample data appear to come from a population with a distribution that is not normal and  $\sigma$  is not known.

- b. Claim about daily rainfall amounts in Boston:  
 $\mu < 0.20$  in.,  $n = 52$ ,  $\bar{x} = 0.10$  in.,  $s = 0.26$  in. The sample data appear to come from a population with a distribution that is very far from normal, and  $\sigma$  is known.

**FINDING  $P$ -VALUES WITH THE STUDENT  $t$  DISTRIBUTION**

1. Use software or a \_\_\_\_\_.

2. If \_\_\_\_\_ is not available, use Table A-3 to identify a \_\_\_\_\_ of \_\_\_\_\_ as follows: Use the number of \_\_\_\_\_ of \_\_\_\_\_ to \_\_\_\_\_ the \_\_\_\_\_ row of Table A-3, then determine where the \_\_\_\_\_ lies \_\_\_\_\_ to the \_\_\_\_\_ in that \_\_\_\_\_. Based on a comparison of the \_\_\_\_\_ and the \_\_\_\_\_ in the row of Table A-3, \_\_\_\_\_ a \_\_\_\_\_ of \_\_\_\_\_ by referring to the \_\_\_\_\_ given at the \_\_\_\_\_ of Table A-3.

Example 2: Either use technology to find the  $P$ -value or use Table A-3 to find a range of values for the  $P$ -value.

a. Movie Viewer Ratings: Two-tailed test with  $n = 15$ , and test statistic  $t = 1.495$ .

b. Body Temperatures: Test a claim about the mean body temperature of healthy adults. Left-tailed test with  $n = 11$  and test statistic  $t = -3.518$ .

Example 3: Assume that a SRS has been selected from a normally distributed population and test the given claim. A SRS of 40 recorded speeds (in mi/h) is observed from cars traveling on a section of Highway 405 in Los Angeles. The sample has a mean of 68.4 mi/h and a standard deviation of 5.7 mi/h (based on data from Sigalert). Use a 0.05 significance level to test the claim that the mean speed of all cars is greater than the posted speed limit of 65 mi/h.

- a. Identify the null hypothesis
- b. Identify the alternative hypothesis
- c. Identify the test statistic
- d. Identify the  $P$ -value or critical value(s)
- e. What is your conclusion about the null hypothesis?

f. What is your final conclusion?

Example 2: Assume that a SRS has been selected from a normally distributed population and test the given claim. The trend of thinner Miss America winners has generated charges that the contest encourages unhealthy diet habits among young women. Listed below are body mass indexes (BMI) of recent Miss America winners. Use a 0.01 significance level to test the claim that recent Miss America winners are from a population with a mean BMI less than 20.16, which was the BMI for winners from the 1920s and 1930s.

19.5 20.3 19.6 20.2 17.8 17.9 19.1 18.8 17.6 16.8

a. Identify the null hypothesis

b. Identify the alternative hypothesis





	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	Area in Two Tails				
	0.01	0.02	0.05	0.10	0.20
1	63.657	31.821	12.706	6.314	3.078
2	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.476
6	3.707	3.143	2.447	1.943	1.440
7	3.499	2.998	2.365	1.895	1.415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372
11	3.106	2.718	2.201	1.796	1.363
12	3.055	2.681	2.179	1.782	1.356
13	3.012	2.650	2.160	1.771	1.350
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325
21	2.831	2.518	2.080	1.721	1.323
22	2.819	2.508	2.074	1.717	1.321
23	2.807	2.500	2.069	1.714	1.319
24	2.797	2.492	2.064	1.711	1.318
25	2.787	2.485	2.060	1.708	1.316
26	2.779	2.479	2.056	1.706	1.315
27	2.771	2.473	2.052	1.703	1.314
28	2.763	2.467	2.048	1.701	1.313
29	2.756	2.462	2.045	1.699	1.311
30	2.750	2.457	2.042	1.697	1.310
31	2.744	2.453	2.040	1.696	1.309
32	2.738	2.449	2.037	1.694	1.309
34	2.728	2.441	2.032	1.691	1.307
36	2.719	2.434	2.028	1.688	1.306
38	2.712	2.429	2.024	1.686	1.304
40	2.704	2.423	2.021	1.684	1.303
45	2.690	2.412	2.014	1.679	1.301
50	2.678	2.403	2.009	1.676	1.299
55	2.668	2.396	2.004	1.673	1.297
60	2.660	2.390	2.000	1.671	1.296
65	2.654	2.385	1.997	1.669	1.295
70	2.648	2.381	1.994	1.667	1.294
75	2.643	2.377	1.992	1.665	1.293
80	2.639	2.374	1.990	1.664	1.292
90	2.632	2.368	1.987	1.662	1.291
100	2.626	2.364	1.984	1.660	1.290
200	2.601	2.345	1.972	1.653	1.286
300	2.592	2.339	1.968	1.650	1.284
400	2.588	2.336	1.966	1.649	1.284
500	2.586	2.334	1.965	1.648	1.283
750	2.582	2.331	1.963	1.647	1.283
1000	2.581	2.330	1.962	1.646	1.282
2000	2.578	2.328	1.961	1.645	1.282
∞	2.576	2.327	1.961	1.645	1.282