CHAPTER PROBLEM

How do we design airplanes, boats, cars, and homes for safety and comfort?

Ergonomics involves the study of people fitting into their environments. Ergonomics is used in a wide variety of applications such as these: Design a doorway so that most people can walk through it without bending or hitting their head; design a car so that the dashboard is within easy reach of most drivers; design a screw bottle top so that most people have sufficient grip strength to open it; design a manhole cover so that most workers can fit through it. Good ergonomic design results in an environment that is safe, functional, efficient, and comfortable. Bad ergonomic design can result in uncomfortable, unsafe, or possibly fatal conditions. For example, the following real situations illustrate the difficulty in determining safe loads in aircraft and boats.

- "We have an emergency for Air Midwest fifty-four eighty," said pilot Katie Leslie, just before her plane crashed in Charlotte, North Carolina. The crash of the Beech plane killed all of the 21 people on board. In the subsequent investigation, the weight of the passengers was suspected as a factor that contributed to the crash. This prompted the Federal Aviation Administration to order airlines to collect weight information from randomly selected flights, so that the old assumptions about passenger weights could be updated.
- Twenty passengers were killed when the Ethan Allen tour boat capsized on New York's Lake George. Based on an assumed mean weight of 140 lb, the boat was certified to carry 50 people. A subsequent investigation showed that most of the passengers weighed more than 200 lb, and the boat should have been certified for a much smaller number of passengers.
- A water taxi sank in Baltimore's inner Harbor. Among the 25 people on board, 5 died, and 16 were injured. An investigation revealed that the safe passenger load for the water taxi was 3500 lb. Assuming a mean passenger weight of 140 lb, the boat was allowed to carry 25 passengers, but the mean of 140 lb was determined 44 years ago when people were not as heavy as they are today. (The mean weight of the 25 passengers aboard the boat that sank was found to be 168 ln). The National Transportation and Safety Board suggested that the old estimated mean of 140 lb be updated to 174 lb, so the safe load of 3500 lb would now allow only 20 passengers instead of 25.

This chapter introduces the statistical tools that are basic to good ergonomic design. After completing this chapter, we will be able to solve problems in a wide variety of different disciplines, including ergonomics.

MATH 103 CHAPTER 6 HOMEWORK
6.2 1, 3, 4, 5-16, 17-35 odd, 37, 40, 41-44, 45-48, 51
6.3 1, 2, 5-12, 21-26, 28, 31
6.5 1, 2, 5-12, 21-26, 28, 31

6.1 REVIEW AND PREVIEW

In Chapter 2 we considered the ____________________________ of data, and in Chapter 3 we considered some important _______________________ of data sets, including measures of ____________________________ and ________________________. In Chapter 4 we discussed basic principles of ________________________, and in Chapter 5 we presented the concept of a ________________________ _____________________________ _____________________________. In this chapter we present _____________________________ _________________________________ _____________________________.

DEFINITION

If a continuous random variable has a distribution with a graph that is _______________________________ and ______________________________, and it can be described by the equation_________________________, we say that it has a **normal distribution**.

6.2 THE STANDARD NORMAL DISTRIBUTION

Key Concept…

In this section, we present the ______________________________

__________________________ ___________________________________, which

has these three properties:

1. Its _________________________ is __________________________________.
2. Its ________________________ is equal to ____________.
3. Its _____________________________ ___________________________ is equal to ____________.

In this section we develop the skill to find _________________ or

___________________________ or _________________________

____________________________ corresponding to various _________________

under the _____________________ of the ______________________________

_______________________________ distribution. In addition, we find

______________________ that correspond to ____________________ under the

_________________________.

UNIFORM DISTRIBUTIONS

The _________________________ ____________________________ allows us to see two

very important properties:

1. The ________________ under the ________________ of a _______________ distribution is equal to _____________.
2. There is a __________________________ between ________________ and ___________________________ (or ________________________ frequency), so some _________________________ can be found by __________________ the corresponding _________________.

DEFINITION

A _________________________ __________________ ____________________ has a **uniform distribution** if its values are spread _______________________ over the ____________________ of ___________________________. The graph of a uniform distribution results in a _______________________________ shape.

Example 1: The Newport Power and Light Company provides electricity with voltage levels that are uniformly distributed between 123.0 volts and 125.0 volts. That is, any voltage amount between 123.0 volts and 125.0 volts is possible, and all of the possibilities are equally likely. If we randomly select one of the voltage levels and represent its value by the random variable *x*, then *x* has a distribution that can be graphed.

a. Sketch a graph of the uniform distribution of voltage levels.

b. Find the probability that the voltage level is greater than 124.0 volts.

c. Find the probability that the voltage level is less than 123.5 volts.

d. Find the probability that the voltage level is between 123.2 volts and 124.7 volts.

e. Find the probability that the voltage level is between 124.1 volts and 124.5 volts.

The graph of a probability distribution, such as part (a) in the previous example is called a ______________________ ________________________. A density curve must satisfy the following two requirements.

1. The total ____________________ under the _________________ must equal __________.

2. Every point on the _______________ must have a vertical

_______________ that is ___________ or ___________________.

STANDARD NORMAL DISTRIBUTION

DEFINITION

The **standard normal distribution** is a _________________________

_________________________ _________________________ with

_____________ and _____________. The total _____________ under its

_________________ _______________ is equal to _________.

FINDING PROBABILITIES WHEN GIVEN *z* SCORES

Using table ________, we can find _____________ or __________________

for many different _________________. Such areas can also be found using a

_____________________ ______________________. When using Table A-2,

it is essential to understand these points:

1. Table A-2 is designed only for the ____________________ _____________ distribution, which has a mean of ____________ and a standard deviation of _______.

2. Table A-2 is on ________ pages, with one page for _______________ ______________ and the other page for _______________ ____________.

3. Each value in the body of the table is a _____________________ ________ from the _______________ up to a ________________ _______________ above a specific ___________________.

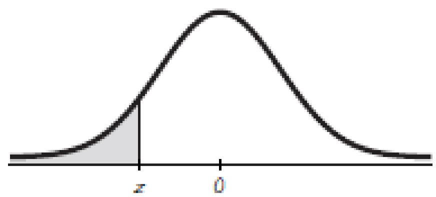
4. When working with a ___________________, avoid confusion between _________________ and ________________.

   z score: ______________________ along the _____________________ scale of the standard normal distribution; refer to the

   _____________________ column and _______________ row of Table A-2.

   Area: ___________________ under the ____________________; refer to the values in the _________________ of Table A-2.

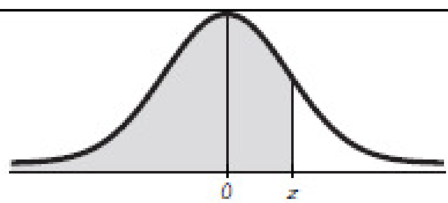
5. The part of the _______________ denoting ______________________ is found across the ___________ ___________ of Table A-2.



NEGATIVE *z* Scores

TABLE A-2 Standard Normal (*z*) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	[illegible]	[illegible]	[illegible]	.0023	.0023	.0022	.0021	[illegible]	.0020	.0019



POSITIVE *z* Scores

TABLE A-2 (continued) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.535
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.575
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.61
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.754
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.813
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.838
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.862
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.901
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.91
1.4	.9192	.9207	[illegible]	.9236	.9251	[illegible]	.9279	.9292	.9306	.93

NOTATION

$P(a < z < b)$

$P(z > a)$

$P(z < a)$

Example 2: Assume that thermometer readings are normally distributed with a mean of 0°C and a standard deviation of 1.00 °C. A thermometer is randomly selected and tested. In each case, draw a sketch and find the probability of each reading. The given values are in Celsius degrees.

a. Less than -2.75

b. Greater than 2.33

c. Between 1.00 and 3.00

e. Greater than 3.68

d. Between -2.87 and 1.34

USING THE TI-84

FINDING *z* SCORES WITH KNOWN AREAS

1. Draw a bell-shaped curve and ______________________ the ______________ under the ___________________ that ________________________ to the _________________ probability. If that region is not a ___________________ region from the __________________, work instead with a known region that

is a cumulative region from the _______________.

2. Using the ________________________ ___________________ from the ____________, locate the ___________________ probability in the ________________ of Table A-2 and identify the ______________________ ________________.

NOTATION

The expression z_{α} denotes the *z* score with an area of _________ to its ____________.

Example 3: Find the value of $z_{.075}$.

Example 4: Assume that thermometer readings are normally distributed with a mean of 0°C and a standard deviation of 1.00 °C. A thermometer is randomly selected and tested. In each case, draw a sketch and find the probability of each reading. The given values are in Celsius degrees.

a. Find the 1st percentile.

b. If 0.5% of the thermometers are rejected because they have readings that are too low and another 0.5% are rejected because they have readings that are too high, find the two readings that are cutoff values separating the rejected thermometers from the others.

6.3 APPLICATIONS OF NORMAL DISTRIBUTIONS

Key Concept…

In this section we introduce ______________ and _________________

applications involving ______________________ normal distributions by

extending the procedures presented in Section 6-2. We use a simple

____________________ that allows us to ________________________ any

_________________________ distribution so that the methods of the

preceding section can be used with normal distributions having a

_____________ that is ___________ _____ and a ____________________

__________________________ that is not ______.

TO STANDARDIZE VALUES USE THE FOLLOWING FORMULA:

STEPS FOR FINDING AREAS WITH A NONSTANDARD NORMAL DISTRIBUTION:

1. Sketch a __________________ curve, label the ___________________ and the specific _______________, then _________________ the region representing the desired _______________________.
2. For each relevant value x that is a _____________________ for the shaded region, convert the relevant value to a standard ______________.
3. Refer to table ___________ or use a ____________________ to find the ________________ of the shaded region.

Example 1: Assume that adults have IQ scores that are normally distributed with a mean of 100 and a standard deviation of 15.

a. Find the probability that a randomly selected adult has an IQ that is less than 115.

b. Find the probability that a randomly selected adult has an IQ greater than 131.5 (the requirement for the Mensa organization).

c. Find the probability that a randomly selected adult has an IQ between 90 and 110 (referred to as the normal range).

d. Find the probability that a randomly selected adult has an IQ between 110 and 120 (referred to as bright normal).

e. Find P_{30}, which is the IQ score separating the bottom 30% from the top 70%.

f. Find the first quartile Q_1, which is the IQ score separating the bottom 25% from the top 75%.

g. Find the third quartile Q_3, which is the IQ score separating the top 25% from the others.

h. Find the IQ score separating the top 37% from the others.

FINDING VALUES FROM KNOWN AREAS

1. Don't confuse __________________ and ______________. Remember,

   ______________ are __________________ along the ________________

   scale, but ___________________ are ___________________ under the

   ______________.

2. Choose the correct ________________ of the __________________. A value separating the top 10% from the others will be located on the

   ______________ side of the graph, but a value separating the bottom 10%

   will be located on the __________________ side of the graph.

3. A ________________ must be ____________________ whenever it is

   located in the ______________ half of the __________________ distribution.

4. Areas (or ____________________) are ___________________ or

   ________ values, but they are never __________________.

Always use graphs to ___________________________!!!

STEPS FOR FINDING VALUES USING TABLE A-2:

1. Sketch a ___________________ distribution curve, enter the given

   ________________________ or _________________________ in the

   appropriate _________________ of the ____________________, and

   identify the _________________ being sought.

2. Use Table A-2 to find the _______________ corresponding to the

   ______________________ __________________ area bounded by

   ____. Refer to the ______________ of Table A-2 to find the

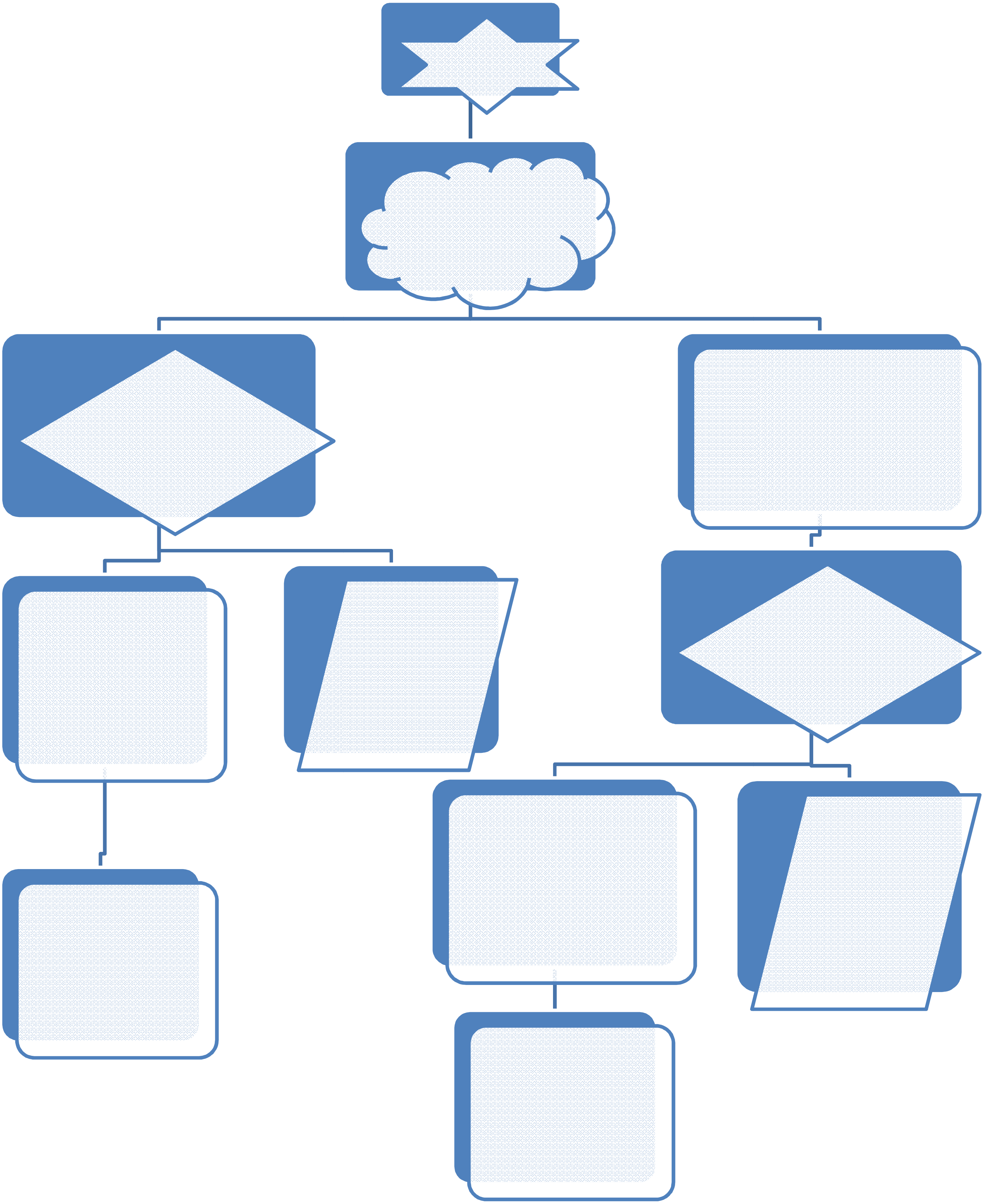
   _____________ area, then identify the corresponding ______________.

3. Solve for _____ as follows:

4. Refer to the _______________ of the _______________ to make sure

   that the solution makes ____________________!

Example: Engineers want to design seats in commercial aircraft so that they are wide enough to fit 99% of all males. Men have hip breadths that are normally distributed with a mean of 14.4 inches and a standard deviation of 1.0 inch. Find the hip breadth for men that separates the smallest 99% from the largest 1 % (aka P_{99}).



6.5 THE CENTRAL LIMIT THEOREM

Key Concept…

In this section, we introduce and apply the ______________________ ________________________ ________________________. The central limit theorem tells us that for a ___________________________ with ____________ distribution, the _________________________ of the _______________________ ______________________ approaches a _________________________ _________________________ as the sample size ___________________________. This means that if the sample size is _____________ enough, the _________________________ of _________________ _____________________ can be approximated by a _____________________ _______________________, even if the original population is _____________ normally distributed. If the original population has ___________________ _______ and ______________________ _____________________________ _______, the ______________ of the ____________________ ____________________ will also be ______, but the ________________________ _________________________ of the _____________________ _______________________ will be _________, where _____ is the ______________________ size.

It is essential to know the following principles:

1. For a ____________________________ with any

   _____________________, if _______________, then the sample means

   have a ____________________________ that can be approximated by a

   ____________________________distribution, with mean _____ and

   standard deviation ___________.

2. If _____________ and the original population has a _______________

   distribution, then the __________________ ____________________ have

   a ________________________ distribution with mean ____ and standard

   deviation ___________.

3. If _____________ and the original population does not have a

   _____________________ distribution, then the methods of this section

   ___________ ____________ ___________________.

NOTATION

If all possible ______________________ _____________________ of size ____

are selected from a population with mean ____ and standard deviation _____, the

mean of the ____________________ ___________________ is denoted

by______, so ______________ = ______. Also, the standard deviation of the

sample means is denoted by ______, so ____________ = ____________.

________ is called the _________________ ________________ of the mean.

____________.

APPLYING THE CENTRAL LIMIT THEOREM

Example 1: Assume that SAT scores are normally distributed with mean $\mu = 1518$ and standard deviation $\sigma = 325$.

a. If 1 SAT score is randomly selected, find the probability that it is between 1440 and 1480.

b. If 16 SAT scores are randomly selected, find the probability that they have a mean between 1440 and 1480.

c. Why can the central limit theorem be used in part (b) even though the sample size does not exceed 30?

Example 2: Engineers must consider the breadths of male heads when designing motorcycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 inches and a standard deviation of 1.0 inch.

a. If one male is randomly selected, find the probability that his head breadth is less than 6.2 inches.

b. The Safeguard Helmet company plans an initial production run of 100 helmets. Find the probability that 100 randomly selected men have a mean head breadth of less than 6.2 inches.

c. The production manager sees the result from part (b) and reasons that all helmets should be made for men with head breadths less than 6.2 inches, because they would fit all but a few men. What is wrong with that reasoning?

CORRECTION FOR A FINITE POPULATION

In applying the central limit theorem, our use of __________ assumes that the ________________________ has _________________________ many members. When we sample with ______________________, the population is effectively ________________________. Many applications involve ___________________ without ________________________, so ____________________________ samples depend on ________________________ outcomes. Here is a common rule of thumb:

When sampling ______________________ replacement andf the sample size _____ is _________________ than __________ of the ______________ population size _____ (that is, _____________), adjust the standard deviation of _______________________ means ________ by multiplying it by the ______________________ ________________________ _____________________ _______________________:

Example 3: In a study of Reye's Syndrome, 160 children had a mean age of 8.5 years, a standard deviation of 3.96 years, and ages that approximated a normal distribution. Assume that 36 of those children are to be randomly selected for a follow-up study.

a. When considering the distribution of the mean ages for groups of 36 children, should $\sigma_{\bar{x}}$ be corrected by using the finite population correction factor? Explain.

b. Find the probability that the mean age of the follow-up sample group is greater than 10.0 years.