CHAPTER PROBLEM

Are polygraph instruments really effective as "lie detectors"?

A polygraph instrument measures several physical reactions, such as blood pressure, pulse rate, and skin conductivity. Subjects are usually given several questions that must be answered and, based on physical measurements, the polygraph examiner determines whether or not the subject is lying. Errors in test results could lead to an individual being falsely accused of committing a crime or to a candidate being denied a job.

Based on research, the success rates from polygraph tests depend on several factors, including the questions asked, the test subject, the competence of the polygraph examiner, and the polygraph instrument used for the test.

Many experiments have been conducted to evaluate the effectiveness of polygraph devices, but we will consider the data in the table below, which includes results from experiments conducted by researchers Charles R. Honts (Boise State University) and Gordon H. Barland (Department of Defense Polygraph Institute). The table summarizes polygraph test results for 98 different subjects. In each case, it was known whether or not the subject lied. So, the table indicates when the polygraph was correct.

Analyzing the Results

When testing for a condition, such as lying, pregnancy, or disease, the result of the test is either positive or negative. However, sometimes errors occur during the testing process which can yield a *false positive* result or a *false negative* result. For example, a false positive result in a polygraph test would indicate that a subject lied when in fact he or she did not lie. A false negative would indicate that a subject did not lie when in fact he or she lied.

Incorrect Results

- False positive: Test **incorrectly** indicates the presence of a condition when the subject does not actually have that condition.
- False negative: Test **incorrectly** indicates that subject does not have the condition when the subject actually does have that condition.

Correct Results

- True positive: Test **correctly** indicates that the condition is present when it really is present.
- True negative: Test **correctly** indicates that the condition is not present when it really is not present.

Measures of Test Reliability

- Test sensitivity: The probability of a true positive.
- Test specificity: The probability of a true negative.

In this chapter we study the basic principles of **probability** theory. These principles will allow us to address questions related to the reliability (or unreliability?) of polygraph tests, such as these: Given the sample results below, what is the probability of a false positive or a false negative? Are those probabilities low enough to support the use of polygraph tests in making judgements about a test subject?

	Did the Subject Actually Lie?	
	No (Did Not Lie)	**Yes (Lied)**
Positive test result (Polygraph test indicated that the subject lied)	15 (false positive)	42 (true positive)
Negative test result (Polygraph test indicated that the subject did not lie)	32 (true negative)	9 (false negative)

MATH 103 CHAPTER 1 HOMEWORK	
4.2	1, 3, 5-13, 15, 17, 20, 23, 25, 28, 29, 31, 33, 35, 37, 39
4.3	1-12, 13, 14, 15, 17-20, 27-32, 33-38, 39
4.4	1-12, 13-16, 21, 24, 25, 27, 29, 32
4.5	1-7, 9, 11, 12, 15, 18, 23-26, 28, 29
4.6	1-5,7, 9, 11, 12, 13, 17, 18, 19, 21, 24, 27, 31, 33, 38

4.1 REVIEW AND PREVIEW

The previous chapters discussed the necessity of sound

_______________________ methods and common measures of

__________________________ of data, including the ________________

and __________________________ ___________________________. The

main objective of this chapter is to develop a sound understanding of

_______________________________ values, because those values

constitute the underlying ____________________________ on which the

methods of _______________________________ statistics are built.

RARE EVENT RULE FOR INFERENTIAL STATISTICS

If, under a given assumption, the __________________________ of a particular observed is extremely ________________________, we conclude that the ______________________________ is probably not _______________________.

4.2 BASIC CONCEPTS OF PROBABILITY

Key Concept…

In this section, we present three different approaches to finding the

________________________________ of an event. The most important

objective of this section is to learn to ___________________________

probability values, which are expressed as values between ____ and ____.

We also discuss expressions of ______________ and how probability is used

to determine the odds of an event __________________________.

PART 1: BASICS OF PROBABILITY

In considering _____________________________, we deal with procedures that

produce _____________________________.

DEFINITION

An **event** is any ______________________________ of ______________________ or

__________________________ of a _______________________________.

A **simple event** is an ______________________________ or ______________________

that cannot be further broken down into simpler ____________________________.

The **sample space** for a ______________________________ consists of all possible

_______________________ ___________________________.

NOTATION

P

$A,\ B,\ \text{and } C$

$P(A)$

1. **Relative Frequency Approximation of Probability**

   Conduct (or ______________________) a _______________________________,

   and count the number of times that event *A* ____________________________ occurs. Based on these actual results, *P(A)* is approximated as follows:

   P(A) = --

2. **Classical Approach to Probability (Requires ____________________________ ____________________________ Outcomes)**
   Assume that a given procedure has *n* different ________________________ events and that each of these simple events has an ____________________ chance of ____________________________. If an event *A* can occur in *s* of these *n* ways, then

   P(A) = -- = ------------

3. **Subjective Probabilities**
   P(A) is ___________________________ by using knowledge of the ____________________________ circumstances.

Example 1: Identifying Probability Values

a. What is the probability of an event that is certain to occur?

b. What is the probability of an impossible event?

c. A sample space consists of 10 separate events that are equally likely. What is the probability of each?

d. On a true/false test, what is the probability of answering a question correctly if you make a random guess?

e. On a multiple-choice test with five possible answers for each question, what is the probability of answering correctly if you make a random guess?

Example 2: Adverse Effects of Viagra

When the drug Viagra was clinically tested, 117 patients reported headaches, and 617 did not (based on data from Pfizer, Inc.).

a. Use this sample to estimate the probability that a Viagra user will experience a headache.

b. Is it unusual for a Viagra user to experience headaches?

c. Is the probability high enough to be of concern to Viagra users?

LAW OF LARGE NUMBERS

As a procedure is __________________________ again and again, the _________________________ ___________________________ probability of an event tends to approach the ______________________ probability. The _______ ______ ________________ ________________________ tells us that relative frequency approximations tend to get better with more _____________________.

PROBABILITY AND OUTCOMES THAT ARE NOT EQUALLY LIKELY

One common __________________________ is to _________________________ assume that outcomes are ____________________________ likely just because we know nothing about the likelihood of each outcome.

Example 3: Flip a coin 50 times and record your results.

a. What is the sample space?

b. What is the probability of getting a result of heads?

SIMULATIONS

Many procedures are so___________________________ that the classical approach is impractical. In such cases, we can more easily get good estimates by using the ____________________________ frequency approach. A ____________________________ of a procedure is a process that behaves in the same way as the ___________________________ itself, so that ____________________________ results are produced.

COMPLEMENTARY EVENTS

Sometimes we need to find the probability that an event *A* _________________ ____________________ occur.

DEFINITION

The **complement** of event *A*, denoted by $\bar{A}$, consists of all outcomes in which event *A* does not occur.

Example 4: Find the probability that you will select the incorrect answer on a multiple-choice item if you randomly select an answer.

ROUNDING OFF PROBABILITIES

When expressing the value of a probability, either give the ________________ fraction or decimal or round off final results to ________________ significant digits. All digits in a number are ________________________ except for the _______________ that are included for proper placement of the decimal point.

PART 2: BEYOND THE BASICS OF PROBABILITY: ODDS

Expressions of likelihood are often given as ________________, such as 50:1 (or 50 to 1). Because the use of odds makes many _________________________ difficult, statisticians, mathematicians, and scientists prefer to use ______________________________. The advantage of odds is that they make it easier to deal with money transfers associated with ______________________, so they tend to be used in ___________________, ___________________________, and _________________________.

DEFINITION

The **actual odds against** of event *A* occurring are the ratio ________________,

usually expressed in the form of __________________ or __________________,
where *a* and *b* are integers having no common factors.

The **actual odds in favor** of event *A* occurring are the ratio ________________,

which is the ________________________ of the actual odds against that event.

The **payoff odds** against event *A* occurring are the ratio of _________________

______________________ (if you win) to the amount ________________.

Example 4: Finding Odds in Roulette
A roulette wheel has 38 slots. One slot is 0, another is 00, and the others are numbered 1 through 36, respectively. You place a bet that the outcome is an odd number.

a. What is your probability of winning?

b. What are the actual odds against winning?

c. When you bet that the outcome is an odd number, the payoff odds are 1:1. How much profit do you make if you bet $18 and win?

4.3 ADDITION RULE

Key Concept…

In this section, we present the addition rule as a device for finding probabilities that can be expressed as ________________________, which denotes the probability that either event *A* occurs ____________ event *B* occurs. In the previous section we presented the basics of probability and considered events categorized as _______________________ events. In this and the following section we consider ________________________ events.

DEFINITION

A **compound event** is any event combining _________________ or more ________________________ events.

NOTATION

$$P(A \text{ or } B) =$$

FORMAL ADDITION RULE

The **formal addition rule**:
$P(A \text{ or } B) =$ ____

where $P(A \text{ and } B)$ denotes the probability that ________ and __________

both occur at the _________________ time as an _____________________

in a _________________ or __________________________.

INTUITIVE ADDITION RULE

The **intuitive addition rule**: To find $P(A \text{ or } B)$, find the __________ of the

_________________ of ways that event _________ can occur and the number

of ways that event __________ can occur, adding in such a way that every

__________________ is counted only ______________. $P(A \text{ or } B)$ is equal

to that _____________, ________________________ by the total number of

_______________________ in the _____________________ space.

DEFINITION

Events *A* and *B* are **disjoint (aka mutually exclusive)** if they cannot

_________________ at the same __________________.

COMPLEMENTARY EVENTS

Recall that the complement of event *A* is denoted _____________, and consists

of all the _______________________ in which event *A* _____________ _______

occur. An event and its complement must be _______________________, because

it is _____________________________ for an event and its complement to occur at the same time. Also, we can be sure that *A* either does or does not occur, which

implies that either ___________ or _____________ must occur.

Example 1: Sobriety Checkpoint

When the author observed a sobriety checkpoint conducted by the Dutchess County Sheriff Department, he saw that 676 drivers were screened and 6 were arrested for driving while intoxicated. Based on those results, we can estimate the $P(I) = 0.00888$, where I denotes the event of screening a driver and getting someone who is intoxicated. What does $P(\overline{I})$ denote and what is its value?

RULES OF COMPLEMENTARY EVENTS

$$P(A) + P(\overline{A}) = 1$$

$$P(\overline{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\overline{A})$$

Example 2: Use the data in the table below, which summarizes challenges by tennis players (based on the data reported in USA Today). The results are from the first U.S. Open that used the Hawk-Eye electronic system for displaying an instant replay used to determine whether the ball is in bounds or out of bounds. In each case, assume that one of the challenges is randomly selected.

	Was the challenge to the call successful?	
	Yes	No
Men	201	288
Women	126	224

a. If S denotes the event of selecting a successful challenge, find $P\left(\bar{S}\right)$.

b. If M denotes the event of selecting a challenge made by a man, find $P\left(\bar{M}\right)$.

c. Find the probability that the selected challenge was made by a man or was successful.

d. Find the probability that the selected challenge was made by a woman or was successful.

e. Find $P(\text{challenge was made by a man or was not successful})$.

f. Find $P(\text{challenge was made by a woman or was not successful})$.

4.4 MULTIPLICATION RULE: BASICS

Key Concept…

In section 4-3 we presented the ______________________ rule for finding $P(A \text{ or } B)$, the probability that a ____________________ trial has an outcome of ___________ or ___________ or both. In this section we present the basic _______________________________ rule, which is used for finding $P(A \text{ and } B)$, the probability that event ________ occurs in a first trial and event ___________ occurs in a second trial. If the ___________________________ of the first event *A* somehow ____________________ the probability of the second event *B*, it is important to ___________________ the probability of *B* to reflect the occurrence of event *A*.

NOTATION

$P(A \text{ and } B) =$

$P(B \mid A) =$

DEFINITION

Two events A and B are **independent** if the occurrence of one does not

____________________ the _________________________ of the occurrence of

the other. If A and B are not ___________________________, they are said to be **dependent**.

Example 1: Give an example of

a. Two independent events

b. Two dependent events

FORMAL MULTIPLICATION RULE

The **formal multiplication rule**:

$P(A \text{ and } B) =$ ____.

If A and B are ___________________________ events, $P(B \mid A)$ is the same

as_________________________.

INTUITIVE ADDITION RULE

When finding the probability that event *A* occurs in one trial and event *B* occurs in the next trial, ___________________________ the probability of event *A* by the probability of event *B*, but be sure that the ____________________________ of event *B* takes into account the previous ______________________________ of event *A*.

Example 2: Use the data in the table below, which summarizes blood groups and Rh types for 100 subjects.

	O	A	B	AB
Rh^+	39	35	8	4
Rh^-	6	5	2	1

a. If 2 of the 100 subjects are randomly selected, find the probability that they are both group O and type Rh^+.

i. Assume that the selections are made with replacement.

ii. Assume that the selections are made without replacement.

b. People with blood that is group O and type Rh⁻ are considered to be universal donors, because they can give blood to anyone. If 4 of the 100 subjects are randomly selected, find the probability that they are all universal recipients.
   i. Assume that the selections are made with replacement.

   ii. Assume that the selections are made without replacement.

Example 3: Suppose that you are married and want to have 3 children. Assume that the probability for you to give birth to a girl is equal to the probability for you to give birth to a boy, and that you only give birth to one child at a time.

a. Make a tree diagram and list the sample space.

b. What is the probability that you have all girls?

c. What is the probability that you have 2 boys?

d. What is the probability that you have at least one girl?

TREATING DEPENDENT EVENTS AS INDEPENDENT: THE 5% GUIDELINE FOR CUMBERSOME CALCULATIONS

If calculations are very cumbersome and if a ____________________ size is no more than ____________ of the size of the population, treat the selections as being ______________________________ (even if the selections are made without _________________________, so they are technically ___________________________).

Example 4: A quality control analyst randomly selects three different car ignition systems from a manufacturing process that has just produced 200 systems, including 5 that are defective.

a. Does this selection process involve independent events?

b. What is the probability that all three ignition systems are good? (Do not treat the events as independent).

c. Use the 5% guideline for treating the events as independent, and find the probability that all three ignition systems are good.

d. Which answer is better: The answer from part (b) or the answer from part (c)? Why?

4.5 MULTIPLICATION RULE: COMPLEMENTS AND CONDITIONAL PROBABILITY

Key concept…

In section 4.4 we introduced the basic ______________________________ rule. In this section we extend our use of the multiplication rule to the following two applications:

1. Probability of "at least one": Find the probability that among

   ___________________ trials, we get at least one of some specified event.
2. Conditional probability: Find the probability of an event when we

   have _____________________________ information that some

   other event has already _______________________________.

COMPLEMENTS: THE PROBABILITY OF "AT LEAST ONE"

- At least one is equivalent to ________________ or ________________.
- The _________________________of getting at least one item of a particular

  type is that you get _____________ items of that type.

STEPS FOR FINDING THE PROBABILITY OF AT LEAST ONE OF SOME EVENT

1. Use the symbol *A* to denote the event of getting at _______________ one.
2. Let $\overline{A}$ represent the event of getting _________ of the items being considered.
3. Calculate the probability that none of the outcomes results in the event being considered.
4. _________________ the result from _________. So you have the expression

$$P(\text{at least one}) = 1 - P(\text{none})$$

Example 1: Provide a written description of the complement of the following event:
When Brutus asks five different women for a date, at least one of them accepts.

Example 2: If a couple plans to have 8 children what is the probability that there will be at least one girl?

CONDITIONAL PROBABILITY

A ______________________ probability is used when the probability is affected by the knowledge of other _________________________.

DEFINITION

A **conditional probability** of an event is a ______________________ obtained with the additional _________________________ that some other event has already __________________________. $P(B|A)$ denotes the _____________ probability of an event *B* occurring, given that event *A* has already ___________.

$$P(B|A) = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

INTUITIVE APPROACH TO CONDITIONAL PROBABILITY

The ______________________________ probability of *B* __________________ *A* can be found by _________________________ that event *A* has occurred, and then calculating the probability that event *B* will ___________________________.

Example 3: Use the table below to find the following probabilities.

	Did the Subject Actually Lie?	
	No (Did Not Lie)	**Yes (Lied)**
Positive test result (Polygraph test indicated that the subject lied)	15 (false positive)	42 (true positive)
Negative test result (Polygraph test indicated that the subject did not lie)	32 (true negative)	9 (false negative)

a. Find the probability of selecting a subject with a positive test result, given that the subject did not lie.

b. Find the probability of selecting a subject with a negative test result, given that the subject lied.

c. Find $P(\text{negative test result} \mid \text{subject did not lie})$.

d. Find $P(\text{subject did not lie} \mid \text{negative test result})$.

e. Are the results from (c) and (d) equal?

Example 4: The Orange County Department of Public Health tests water for contamination due to the presence of *E. coli* bacteria. To reduce the laboratory costs, water samples from six public swimming areas are combined for one test, and further testing is done only if the combined sample fails. Based on past results, there is a 2% chance of finding *E. coli* bacteria in a public swimming area. Find the probability that a combined sample from six public swimming areas will reveal the presence of *E. coli* bacteria.

CONFUSION OF THE INVERSE

To incorrectly believe that _________________ and _________________ are

the same, or to incorrectly use one value for the other, is often called

______________________________ of the _______________________.

4.6 COUNTING

Key concept…

In this section we present methods for ________________________ the

number of __________________________ ways in a variety of different

situations. Probability problems typically require that we know the total

number of possible _________________________, but finding that total often requires the methods of this section.

FUNDAMENTAL COUNTING RULE

For a ___________________________ of two _____________________ in which the first event can occur _________ ways and the second event can occur __________ ways, the events together can occur a total of _________________ ways.

Example 1: How many different California vehicle license plates (not specialized plates) are possible if the first, fifth, sixth, and seventh digits consist of a number from 1-9, and the second, third, and fourth digits have letters?

NOTATION

The **factorial symbol(!)** denotes the product of decreasing positive whole numbers.

Example 2: Evaluate 5!

FACTORIAL RULE

A collection of _______ different items can be _______________________ in order ______________ in different ways.

Example 3: Find the number of ways that 8 people can be seated at a round table.

PERMUTATIONS RULE (WHEN ITEMS ARE ALL DIFFERENT)

Requirements:

1. There are ________ ____________________________ items available.
2. We select ________ of the ________ items (without replacement).
3. We consider _______________________________ of the same items to be ______________________________ sequences. This would mean that ABC is different from CBA and is counted separately.

If the preceding requirements are satisfied, the number of ______________________________ (aka __________________________) of ______ items selected from ________ different available items (without replacement) is

$$_nP_r = \underline{\hspace{6cm}}$$

Example 4: A political strategist must visit state capitols, but she has time to visit only three of them. Find the number of different possible routes.

PERMUTATIONS RULE (WHEN SOME ITEMS ARE IDENTICAL TO OTHERS)

Requirements:

1. There are _______ items available, and some items are_______________ to others.
2. We select _________ of the _____ items (without replacement).
3. We consider ___________________________ of distinct items to be

   _____________________________ sequences.

If the preceding requirements are satisfied, and if there are ______ alike, ________ alike, …, ________ alike, the number of ______________________

or _________________________ of all items selected without replacement is

____

Example 5: In a preliminary test of the MicroSort gender-selection method, 14 babies were born and 13 of them were girls.

a. Find the number of different possible sequences of genders that are possible when 14 babies are born.

b. How many ways can 13 girls and 1 boy be arranged in a sequence?

c. If 14 babies are randomly selected, what is the probability that they consist of 13 girls and 1 boy?

d. Does the gender-selection method appear to yield a result that is significantly different from a result that might be expected from random chance?

COMBINATIONS RULE

Requirements:

1. There are _______ _________________________ items available.
2. We select _______ of the _______ items (without replacement).
3. We consider ____________________________ of the same items to be the ___________________________. This would mean that ABC is the same as CBA.

If the preceding requirements are satisfied, the number of

___________________________ of _____ items selected from _______ different items is

$$ {}_nC_r = \underline{\hspace{5cm}} $$

Example 6: Find the number of different possible five-card poker hands.

Example 7: The Mega Millions lottery is run in 12 states. Winning the jackpot requires that you select the correct five numbers between 1 and 56, and, in a separate drawing, you must also select the correct single number between 1 and 46. Find the probability of winning the jackpot.