Example 4: In a sample of seven cars, each car was tested for nitrogen-oxide emissions (in grams per mile) and the following results were obtained: 0.06, 0.11, 0.16, 0.15, 0.14, 0.08, 0.15 (based on data from the EPA).

a. Assuming that this sample is representative of the cars in use, construct a 98% confidence interval estimate of the mean amount of nitrogen-oxide emissions for all cars.

loosely normal

$n=7$, $d.f.=6$

$\alpha=0.02$, $\alpha/2=0.01$

$t_{6,0.01}=3.143$

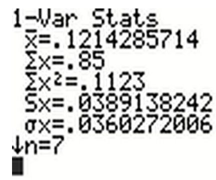
$\bar{x}=0.121$, $s=0.039$

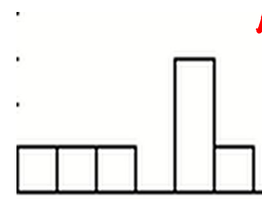
$E=3.143\cdot\frac{0.039}{\sqrt{7}}$

$E\approx 0.0463$

$\bar{x}-E<\mu<\bar{x}+E$

$\boxed{0.075<\mu<0.167}$





b. If the EPA requires that nitrogen-oxide emissions be less than 0.165 g/mi, can we safely conclude that this requirement is being met?

No, since the likely emission amounts include amounts greater than 0.165 g/mi.

Example 5: Listed below are 12 lengths (in minutes) of randomly selected movies from Data Set 9 in Appendix B.

110 96 125 94 132 120 136 154 149 94 119 132

a. Construct a 99% confidence interval estimate of the mean length of all movies.

See movie on mathchick.net

b. Assuming that it takes 30 minutes to empty a theater after a movie, clean it, allow time for the next audience to enter, and show previews, what is the minimum time that a theater manager should plan between start times of movies, assuming that this time will be sufficient for typical movies?

8.1 REVIEW AND PREVIEW

DEFINITION

In statistics, a **hypothesis** is a claim or statement about a property of the population.

A **hypothesis test (aka test of significance)** is a procedure for testing a claim about a property of a population.

8.2 BASICS OF HYPOTHESIS TESTING

PART 1: BASICS CONCEPTS OF HYPOTHESIS TESTING

The methods presented in this chapter are based on the rare event rule for inferential statistics.

RARE EVENT RULE FOR INFERENTIAL STATISTICS

If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.

WORKING WITH THE STATED CLAIM: NULL AND ALTERNATIVE HYPOTHESES

The **null hypothesis** denoted by H_0 is a statement that the value of a population parameter is equal to some claimed value. The term null is used to indicate no change or no effect or no difference.

The **alternative hypothesis** denoted by H_A or H_1 or H_a is the statement that the parameter has a value that somehow differs from the null hypothesis.

For the methods of this chapter, the Symbolic form of the alternative hypothesis must use one of these symbols: <, >, ≠.

IDENTIFYING H_0 **AND** H_1

START

1

- Identify the specific claim or hypothesis to be tested
- Express it in symbolic form

2

- Give the symbolic form that must be true when the original claim is false

3

- Using the two symbolic expressions obtained so far, identify the null hypothesis H_0 and the alternative hypothesis H_1
- H_1 is the symbolic expression that does not contain equality
- H_0 is the symbolic expression that the parameter equals the fixed value being considered

Example 1: Examine the given statement, then express the null hypothesis and the alternative hypothesis in symbolic form.

a. The majority of college students have credit cards.

$H_0: p = \frac{1}{2}$

$H_1: p > \frac{1}{2}$

b. The mean weight of plastic discarded by households in one week is less than 1 kg.

$H_0: \mu = 1\text{kg}$

$H_1: \mu < 1\text{kg}$

CONVERTING SAMPLE DATA TO A TEST STATISTIC

Test statistic for proportion:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

p is the pop. proportion we assume to be true in the null hypothesis.

Test statistic for mean:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad \text{OR} \quad t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

μ is the value we assume to be true in the null hypothesis

Example 2: Find the value of the test statistic. The claim is that less than ½ of adults in the United States have carbon monoxide detectors. A KRC Research survey of 1005 adults resulted in 462 who have carbon monoxide detectors.

① $H_0: p = \frac{1}{2}$

$H_1: p < \frac{1}{2}$

② Test. Stat.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$n = 1005, \ x = 462$

$\hat{p} = \frac{x}{n} = \frac{462}{1005} \approx 0.460$

$p = \frac{1}{2}, \ q = 1 - p = \frac{1}{2}$

$$z = \frac{0.460 - 0.5}{\sqrt{\frac{(.5)(.5)}{1005}}} \approx \boxed{-2.54}$$

TOOLS FOR ASSESSING THE TEST STATISTIC: CRITICAL REGION, SIGNIFICANCE LEVEL, CRITICAL VALUE, AND *P*-VALUE

The test Statistic alone usually does not give us enough information to make a decision about the claim being tested. The following tools can be used to understand and interpret the test statistic.

- The **critical region (aka rejection region)** is the set of all values of the test statistic that cause us to reject the null hypothesis Ho
- The **significance level (denoted by α)** is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true. If the test statistic falls in the critical region, we reject the null hypothesis, so α is the probability of making the mistake of rejecting the null hypothesis when it is true.
- A **critical value** is any value that separates the critical region from the values of the test statistic that do not lead to rejection of the null hypothesis. The critical values depend on the nature of the null

hypothesis, the sampling distribution that applies, and the significance level ~~of~~ α. The procedure can be summarized as follows:

Critical region in the left tail:

$-z_\alpha$

$H_0: p = p_0$
$H_1: p < p_0$

OR

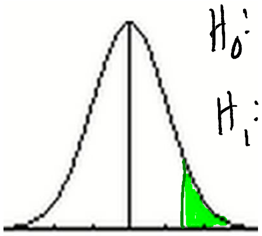
$H_0: \mu = \mu_0$
$H_1: \mu < \mu_0$
(if σ is known)

$-t_{df,\alpha}$

$H_0: \mu = \mu_0$
$H_1: \mu < \mu_0$
(σ is not known, α is the area in one tail)

ex: $n=30, \alpha=.05$
$-t_{29,0.05} = -1.699$

Critical region in the right tail:

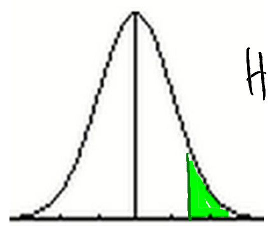


z_α

$H_0: p = p_0$
$H_1: p > p_0$

or

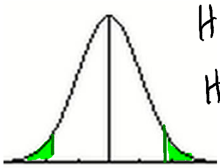
$H_0: \mu = \mu_0$
$H_1: \mu > \mu_0$
(σ is known)



$t_{d.f.,\alpha}$

$H_0: \mu = \mu_0$
$H_1: \mu > \mu_0$
(σ is not known)

Critical region in two tails:

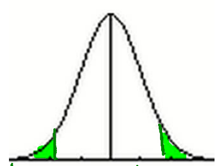


$-z_{\alpha/2}$ $z_{\alpha/2}$

$H_0: p = p_0$
$H_1: p \neq p_0$

OR

$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$
(σ is known)

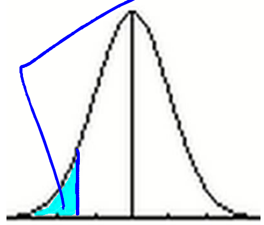


$-t_{d.f.,\alpha/2}$ $t_{d.f.,\alpha/2}$

$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$
(σ is not known)

The ***P*-value (aka *p*-value or probability value)** is the probability of getting a value of the test statistic that is at least as extreme as the one representing the sample data, assuming that the null hypothesis is true. *P*-values can be found after finding the area beyond the test statistic.

$<$



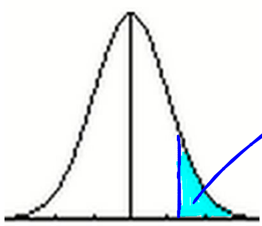
test statistic

$P(z < \text{test statistic})$

OR

$P(t < \text{test statistic})$

$>$



test statistic

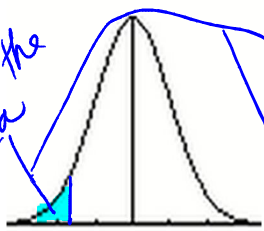
$P(z > \text{test statistic})$

or

$P(t > \text{test statistic})$

$\neq$

half the area

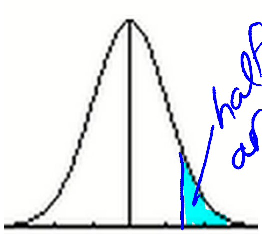


test statistic

$2 \cdot P(z < \text{test statistic})$

or

$2 \cdot P(t < \text{test statistic})$



half the area

test statistic

$2 \cdot P(z > \text{test statistic})$

or

$2 \cdot P(t > \text{test statistic})$

DECISIONS AND CONCLUSIONS

***P*-value method:** Using the significance level α:

If *P*-value $\leq$ α, reject H_0

If *P*-value $>$ α, fail to reject H_0

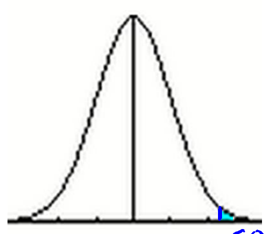
Traditional method: If the test statistic falls within the rejection (critical) region, reject H_0. If the test statistic does not fall within the rejection region, fail to reject H_0.

Confidence intervals: A confidence interval estimate of a population parameter contains the likely values of that parameter. If a confidence interval does not include a claimed value of a population parameter, reject that claim.

Example 3: Use the given information to find *P*-value.

a. The test statistic in a right-tailed test is $z = 2.50$

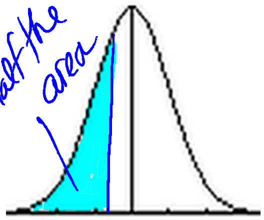
$$P\text{-value} = P(Z > 2.50)$$
$$= 1 - P(Z < 2.50)$$
$$= 1 - 0.9938$$
$$= \boxed{0.0062}$$



$z = 2.50$

b. The test statistic in a two-tailed test is z = -0.55

half the area

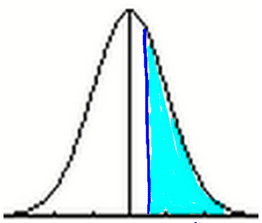


$z=-0.55$

$$\text{P-value} = 2\cdot P(z<-0.55)$$
$$= 2\cdot 0.2912$$
$$= \boxed{0.5824}$$

c. With $H_1: p \neq \frac{3}{4}$, the test statistic is z = 0.35

2-tailed



$z=0.35$

$$\text{P-value} = 2\cdot P(z>0.35)$$
$$= 2\cdot(1-P(z<0.35))$$
$$= 2\cdot(1-0.6368)$$
$$= 2(0.3632)$$
$$= \boxed{0.7264}$$

d. With $H_1: p < 0.777$, the test statistic is z = -2.95

Example 4: State the final conclusion in simple non-technical terms. Be sure to address the original claim. Original claim: The percentage of on-time U.S. airline flights is less than 75%. Initial conclusion: Reject the null hypothesis.