b. Find the probability of selecting a subject with a negative test result, given that the subject lied.

$P(-|L) = \frac{9}{51}$ or $P(-|L) = \frac{P(L \text{ and } -)}{P(L)} = \frac{9/98}{51/98} = \boxed{\frac{9}{51}}$

c. Find $P(\text{negative test result} \mid \text{subject did not lie})$.

$P(-|\bar{L}) = \boxed{\frac{32}{47}}$

d. Find $P(\text{subject did not lie} \mid \text{negative test result})$.

$P(\bar{L}|-) = \boxed{\frac{32}{41}}$

e. Are the results from (c) and (d) equal?

HECK NO!!!

Example 4: The Orange County Department of Public Health tests water for contamination due to the presence of *E. coli* bacteria. To reduce the laboratory costs, water samples from six public swimming areas are combined for one test, and further testing is done only if the combined sample fails. Based on past results, there is a 2% chance of finding *E. coli* bacteria in a public swimming area. Find the probability that a combined sample from six public swimming areas will reveal the presence of *E. coli* bacteria.

$$P(\text{sample fails}) = P(\text{at least 1 fails})$$
$$= 1 - P(\text{none fail})$$
$$= 1 - (0.98)^6$$
$$\approx \boxed{0.114}$$

4.6 COUNTING

FUNDAMENTAL COUNTING RULE

For a ___sequence___ of two ___events___ in which the first event can occur ___n___ ways and the second event can occur ___m___ ways, the events together can occur a total of ___$n \cdot m$___ ways.

Example 1: How many different California vehicle license plates (not specialized plates) are possible if the first, fifth, sixth, and seventh digits consist of a number from 1-9, and the second, third, and fourth digits have letters?

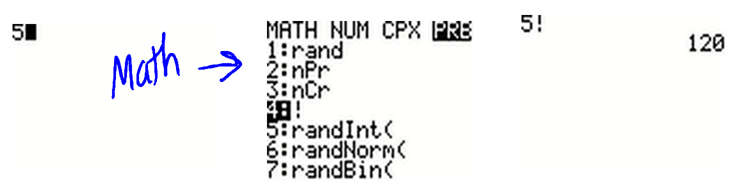
$\underline{9} \cdot \underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{9} \cdot \underline{9} \cdot \underline{9}$

$\boxed{115,316,136}$

NOTATION $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

The **factorial symbol(!)** denotes the product of decreasing positive whole numbers.

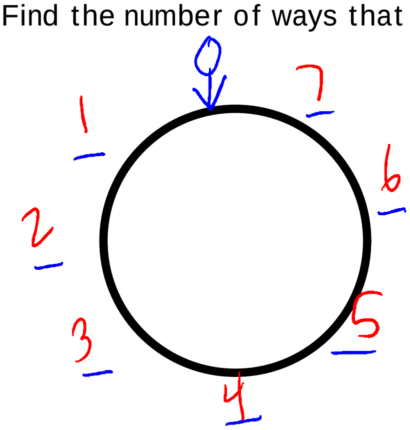
Example 2: Evaluate 5!



FACTORIAL RULE

A collection of ___n___ different items can be ___arranged___ in order ___n!___ different ways.

Example 3: Find the number of ways that 8 people can be seated at a round table.



$(8-1)! = 7!$

$= \boxed{5040}$

PERMUTATIONS RULE (WHEN ITEMS ARE ALL DIFFERENT)

Requirements:

1. There are __n__ __different__ items available.
2. We select __r__ of the __n__ items (without replacement).
3. We consider __rearrangement__ of the same items to be __different__ sequences. This would mean that ABC is different from CBA and is counted separately.

If the preceding requirements are satisfied, the number of __permutations__ (aka __sequences__) of __r__ items selected from __n__ different available items (without replacement) is

$$_nP_r = \frac{n!}{(n-r)!}$$

Example 4: A political strategist must visit state capitols, but she has time to visit only three of them. Find the number of different possible routes.

$n = 50$

$r = 3$

$$_{50}P_3 = \frac{50!}{(50-3)!} = \frac{50 \cdot 49 \cdot 48 \cdot \cancel{47!}}{\cancel{47!}} = \boxed{117{,}600}$$

```
50        50 nPr 3▮        50 nPr 3
                                    117600
```

PERMUTATIONS RULE (WHEN SOME ITEMS ARE IDENTICAL TO OTHERS)

Requirements:

1. There are __n__ items available, and some items are __identical__ to others.
2. We select __all__ of the __n__ items (without replacement).
3. We consider __rearrangements__ of distinct items to be __different__ sequences.

If the preceding requirements are satisfied, and if there are __n_1__ alike, __n_2__ alike, ..., __n_k__ alike, the number of __permutations__ or __sequences__ of all items selected without replacement is

$$\frac{n!}{n_1!\,n_2!\cdots n_k!}$$

Example 5: In a preliminary test of the MicroSort gender-selection method, 14 babies were born and 13 of them were girls.

a. Find the number of different possible sequences of genders that are possible when 14 babies are born.

2 2 2 2 2 2 2 2 2 2 2 2 2 2

$2^{14} = 16,384$

b. How many ways can 13 girls and 1 boy be arranged in a sequence?

$n = 14$

$n_1 = 13$

$n_2 = 1$

$$\frac{14!}{13! \cdot 1!} = \boxed{14}$$

c. If 14 babies are randomly selected, what is the probability that they consist of 13 girls and 1 boy?

$$P(13G, 1B) = \frac{\text{\# of ways to get 13 girls and 1 Boy}}{\text{total \# of ways to get 14 babies}}$$

$$= \frac{14}{16384} \rightarrow \approx 0.000854$$

d. Does the gender-selection method appear to yield a result that is significantly different from a result that might be expected from random chance?

Yes. P(13 G, 1B) is very small. We would expect to see half boys and half girls so this method seems to work.

COMBINATIONS RULE

Requirements:

1. There are __n__ __different__ items available.
2. We select __r__ of the __n__ items (without replacement).
3. We consider __rearrangements__ of the same items to be the __same__. This would mean that ABC is the same as CBA.

If the preceding requirements are satisfied, the number of __combinations__ of __r__ items selected from __n__ different items is

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

Example 6: Find the number of different possible five-card poker hands.

$n = 52$

$r = 5$

$${}_{52}C_5 = \frac{52!}{(52-5)! \cdot 5!}$$

$$= \boxed{2{,}598{,}960}$$

Example 7: The Mega Millions lottery is run in 12 states. Winning the jackpot requires that you select the correct five numbers between 1 and 56, and, in a separate drawing, you must also select the correct single number between 1 and 46. Find the probability of winning the jackpot.

Let W_1 denote winning the 1st drawing

Let W_2 denote winning the 2nd drawing

$$P(\text{winning}) = P(W_1) \cdot P(W_2)$$

$$= \frac{1}{{}_{56}C_5} \cdot \frac{1}{46}$$

$$= \frac{1}{3819816} \cdot \frac{1}{46}$$

$$\approx \boxed{\begin{array}{l} 0.0000000569 \\ = 5.69 \times 10^{-9} \end{array}}$$

```
1/56 nCr 5*1/46
     5.69114597E-9
```