EXAM 2/PART 2/CHAPTERS 5-6
36 POINTS POSSIBLE
CREDIT IS AWARDED BASED ON WORK SHOWN

NAME Key

1. (20 POINTS) Do you tailgate the car in front of you? About 35% of all drivers will tailgate before passing, thinking they can make the car in front of them go faster. Suppose that you are driving a considerable distance on a two-lane highway and are passed by four vehicles.
   a. (8 POINTS) Represent the probability distribution in the table below. Let *X* be the number of vehicles that tailgate before passing.

Binomial, $n=4$, $p=0.35$, $q=0.65$

$$\boxed{P(X=x)={}_nC_x\, p^x q^{n-x}}$$

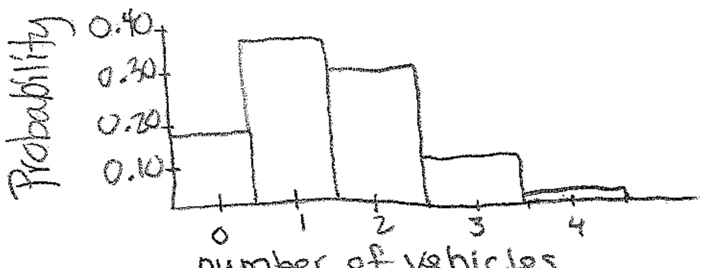
$$P(X=0)={}_4C_0(0.35)^0(0.65)^4\approx 0.179$$
$$P(X=1)={}_4C_1(0.35)^1(0.65)^3\approx 0.384$$
$$P(X=2)={}_4C_2(0.35)^2(0.65)^2\approx 0.310$$
$$P(X=3)={}_4C_3(0.35)^3(0.65)^1\approx 0.111$$
$$\rightarrow P(X=4)={}_4C_4(0.35)^4(0.65)^0\approx 0.0150$$

X	P(X)
0	0.179
1	0.384
2	0.310
3	0.111
4	0.0150

   b. (2 POINTS) Does the given information describe a probability distribution? Explain.

Yes. The sum of the probabilities is $0.999\approx 1$.

   c. (4 POINTS) Sketch the probability histogram which corresponds to the probability distribution from part (a).



   d. (2 POINTS) Compute the expected number of vehicles out of four that will tailgate.

$\mu=np$

$\mu=4(0.35) \rightarrow \boxed{\mu\approx 1.4}$

   e. (2 POINTS) Compute the standard deviation of this distribution.

$\sigma=\sqrt{npq}$

$\sigma=\sqrt{4(0.35)(0.65)} \rightarrow \boxed{\sigma\approx 1.0}$

   f. (2 POINTS) Would it be unusual if none of the four vehicles tailgated?

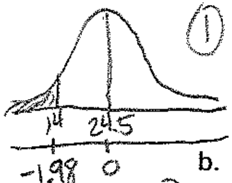
$\mu-2\sigma <$ usual # of vehicles that tailgate before passing $< \mu+2\sigma$

$-0.6 <$ usual # of vehicles that tailgate before passing < 3.4

$\boxed{\text{No since zero is within the range of usual values.}}$

2. (8 POINTS) Assume that the weights of healthy 10-week-old kittens are normally distributed with $\mu = 24.5$ ounces with a standard deviation of $\sigma = 5.3$ ounces. Use Table A-2 only. DO NOT use the STAT feature on your calculator. Include sketches of the indicated areas and/or boundary values.

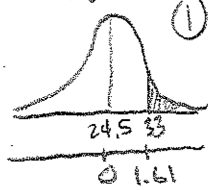
a. (2 POINTS)What is the probability that a healthy 10-week-old kitten will weigh less than 14 ounces?



① $z = \frac{14-24.5}{5.3} \approx -1.98$ ② $P(X<14) = P(Z<-1.98)$

$= \boxed{0.0239}$

b. (2 POINTS) What is the probability that a healthy 10-week-old kitten will weigh more than 33 ounces?

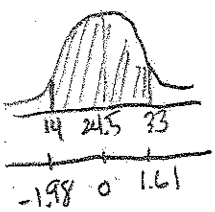


① $z = \frac{33-24.5}{5.3} \approx 1.61$ ② $P(X>33) = P(Z>1.60)$

$= 1 - P(Z<1.60)$

$= 1 - 0.9452$ $= \boxed{0.0548}$

c. (2 POINTS) What is the probability that a healthy 10-week-old kitten will weigh between 14 and 33 ounces?



① $z_1 = \frac{14-24.5}{5.3} \approx -1.98$ ② $P(14 < X < 33) = P(-1.98 < Z < 1.61)$

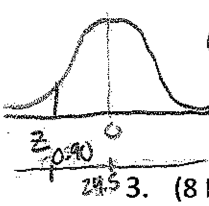
$z_2 = \frac{33-24.5}{5.3} \approx 1.61$

$= P(Z<1.61) - P(Z<-1.98)$

$= 0.9452 - 0.0239$

$= \boxed{0.9213}$

d. (2 POINTS) A kitten whose weight is in the bottom 10% of the probability distribution of weights is called undernourished. What is the weight that serves as a cutoff for the weight of an undernourished kitten.



① $z_{0.90} = -1.28$ ② $x = \mu + z\sigma$

$x = 24.5 + (-1.28)(5.3)$

$x \approx 18.0$

The cutoff weight of an undernourished kitten is 18.0 ounces.

3. (8 POINTS) Let *x* represent the dollar amount spent on supermarket impulse buying in a 10-minute unplanned shopping interval. Based on a *Denver Post* article the mean of the *x* distribution is about \$20 and the estimated standard deviation is about \$7. Use Table A-2 only. DO NOT use the STAT feature on your calculator. Include sketches of the indicated areas and/or boundary values.

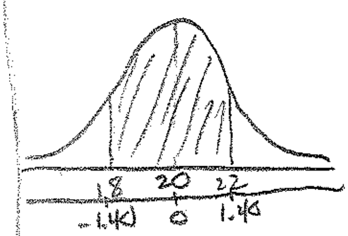
a. (2 POINTS) Consider a random sample of $n = 100$ customers, each of whom has 10 minutes of unplanned shopping time in a supermarket. From the central limit theorem, what can you say about the probability distribution of $\bar{x}$, the average amount spent by these customers due to impulse buying?

The distribution will be normal.

b. What are the mean and standard deviation of the $\bar{x}$ distribution?

(1 POINT) mean of $\bar{x}$ $\mu_{\bar{x}} = \mu = \boxed{\$20}$

(1 POINT) standard deviation of $\bar{x}$: $\sigma_{\bar{x}} = \sigma/\sqrt{n} = \frac{7}{\sqrt{100}} = \boxed{\$0.70}$



c. (2 POINTS) What is the probability that $\bar{x}$ is between \$18 and \$22?

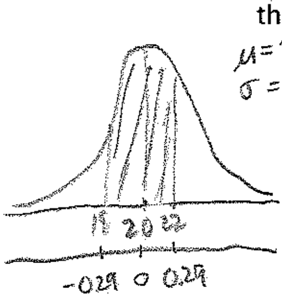
① $z_{1_{\bar{x}}} = \frac{18-20}{0.7} \approx -2.86$ ② $P(18 < \bar{x} < 22) = P(-2.86 < z_{\bar{x}} < 2.86)$ $= \boxed{0.9958}$

$z_{2_{\bar{x}}} = \frac{22-20}{0.7} \approx 2.86$

$= P(z_{\bar{x}} < 2.86) - P(z_{\bar{x}} < -2.86)$

$= 0.9979 - 0.0021$

d. (2 POINTS) Let us assume that *x* has a distribution that is approximately normal. What is the probability that *x* is between \$18 and \$22?



$\mu = 20$
$\sigma = 7$

① $z_1 = \frac{18-20}{7} \approx -0.29$ ② $P(18 < x < 22) = P(-0.29 < z < 0.29)$

$= P(z < .29) - P(z < -.29)$

$z_2 = \frac{22-20}{7} \approx 0.29$

$= 0.6141 - 0.3859$

$= \boxed{0.2282}$

Answer Key
Testname: M103_E2P1_SP12

1) A
2) A
3) B
4) C
5) C
6) A
7) A
8) C
9) D
10) B
11) A
12) D
13) A
14) C

White exam

Answer Key
Testname: M103_E2P1_SP12

blue
exam

1) B
2) B
3) A
4) C
5) C
6) D
7) A
8) A
9) B
10) D
11) A
12) A
13) B
14) C