DEFINITION

The power of a hypothesis test is the probability of rejecting a false null hypothesis. The value of the power is computed by using a particular significance level \( \alpha \) and a particular value of the population parameter that is an alternative to the value assumed true in the null hypothesis.

POWER AND THE DESIGN OF EXPERIMENTS

Just as \( 0.05 \) is a common choice for a significance level, a power of at least 0.80 is a common requirement for determining that a hypothesis test is effective. When designing an experiment, a goal of having a power value of at least 0.80 can often be used in determining the minimum sample size.

Example 6: Chantix tablets are used as an aid to help people stop smoking. In a clinical trial, 129 subjects were treated with Chantix twice a day for 12 weeks, and 16 subjects experienced abdominal pain. If someone claims that more than 8% of Chantix users experience abdominal pain, that claim is supported with a hypothesis test conducted with a 0.05 significance level. Using 0.18 as an alternative value of \( p \), the power of the test is 0.96. Interpret this value of the power of the test.

If the population proportion is actually equal to 0.18, there's a 96% chance of making the correct conclusion of rejecting the false null hypothesis that \( p = 0.08 \).
### Part 1: Basic Methods of Testing Claims About a Population Proportion \( p \)

**Objective**

Test a claim about a population proportion

**Notation**

\[
\begin{align*}
    n & = \text{sample size or \# of trials} \\
    \hat{p} & = \frac{x}{n} \quad \text{(sample proportion)} \\
    q & = 1 - \hat{p} \\
    p & = \text{population proportion} \\
    \end{align*}
\]
REQUIREMENTS

1. The __________ observations are a ______________ sample.
2. The ________________ for a ___________ __________ distribution are satisfied.
   - Fixed # of independent trials having constant probabilities, and each trial has 2 outcome categories → success and failure.
3. The conditions __________ and ______________ are both satisfied so the __________ distribution of __________ proportions can be approximated by a normal distribution with __________ and ______________. Note that __________ is the assumed proportion used in the claim.

TEST STATISTIC FOR TESTING A CLAIM ABOUT A PROPORTION

\[ z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \]

P - values:

Critical values:

FINDING THE NUMBER OF SUCCESSES \( x \)

Computer software and _______________ designed for ___________ tests of ___________ usually require _______________ consisting of the ___________ size \( n \) and the number of ___________ \( x \), but the ___________ proportion is often given instead of ___________.

\[ \hat{p} = \frac{x}{n} \rightarrow x = n\hat{p} \]
Example 1: Identify the indicated values. Use the normal distribution as an approximation to the binomial distribution. In a survey, 1864 out of 2246 randomly selected adults in the United States said that texting while driving should be illegal (based on data from Zogby International). Consider a hypothesis test that uses a 0.05 significance level to test the claim that more than 80% of adults believe that texting while driving should be illegal.

\[ p = 0.8 \rightarrow q = 0.2 \]
\[ \hat{p} = \frac{1864}{2246} \approx 0.830 \]
\[ n = 2246 \]

a. What is the test statistic?

\[ H_0: \ p = 0.8 \]
\[ H_A: \ p > 0.8 \]
\[ Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \]
\[ Z = \frac{0.830 - 0.8}{\sqrt{\frac{0.8 \times 0.2}{2246}}} \approx 3.55 \]

b. What is the critical value? (traditional method)

\[ \alpha = 0.05, \ 1\text{-tail test} \]
\[ Z_{0.05} = 1.645 \]

3.55 falls in the rejection region, so reject \( H_0 \).

c. What is the \( P \)-value?

\[ P\text{-value} = P(Z > 3.55) \]
\[ = 1 - P(Z < 3.55) \]
\[ = 1 - 0.9999 \]
\[ = 0.0001 \]

since 0.0001 < 0.05, reject \( H_0 \).

d. What is the conclusion?

Reject \( H_0 \). There is sufficient evidence to support the claim that the proportion of adults who believe that texting while driving should be illegal is greater than 80%.
Example 2: The company Drug Test Success provides a “1-Panel-THC” test for marijuana usage. Among 300 tested subjects, results from 27 subjects were wrong (either a false positive or a false negative). Use a 0.05 significance level to test the claim that less than 10% of the test results are wrong. Does the test appear to be good for most purposes?

a. Identify the null hypothesis

\[ H_0: p = 0.1 \]

b. Identify the alternative hypothesis

\[ H_A: p < 0.1 \]

c. Identify the test statistic

\[
Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.09 - 0.1}{\sqrt{\frac{0.1(0.9)}{300}}} = -0.58
\]

d. Identify the P-value or critical value(s)

- P-value method:

\[
P-value = P(Z < -0.58) = 0.2810
\]

- Critical value:

\[-Z_{0.05} = -1.645\]

-0.58 is not in the critical region so fail to reject \( H_0 \).

e. What is your final conclusion?

Fail to reject \( H_0 \). There is not sufficient evidence to support the claim that less than 10% of the test results are wrong. I don’t believe the test is good for most purposes as at least 10 per hundred persons receive incorrect results.
Example 3: In recent years, the town of Newport experienced an arrest rate of 25% for robberies (based on FBI data). The new sheriff compiles records showing that among 30 recent robberies, the arrest rate is 30%, so she claims that her arrest rate is greater than the 25% rate in the past. Is there sufficient evidence to support her claim that the arrest rate is greater than 25%?

a. Identify the null hypothesis

\[ H_0: \hat{p} = 0.25 \]

b. Identify the alternative hypothesis

\[ H_A: \hat{p} > 0.25 \]

c. Identify the test statistic

\[
Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \\
\hat{p} = 0.30, \quad \hat{q} = 0.70, \quad p = 0.25, \quad q = 0.75, \quad n = 30 \]

\[
Z = \frac{0.30 - 0.25}{\sqrt{\frac{0.25 \times 0.75}{30}}} = 0.63
\]

d. Identify the \( P \)-value or critical value(s)

**P-value method:**

\[
P-value = P(Z > 0.63) = 1 - P(Z < 0.63) = 1 - 0.7357 = 0.2643
\]

\[
0.2643 > 0.05
\]

**Traditional method:**

\[
Z = 0.63 \text{ lies outside the rejection region}
\]

\[
z_{0.05} = 1.645
\]

e. What is your final conclusion? Fail to reject \( H_0 \).

Fail to reject \( H_0 \). There is not sufficient evidence to support the sheriff's claim that the arrest rate is greater than 25%.
8.4 TESTING A CLAIM ABOUT A MEAN: SIGMA KNOWN

Key Concept...

In this section, we discuss hypothesis testing methods for claim made about a population mean, assuming the population standard deviation is a known value. Here we use the normal distribution with the same components of hypothesis tests that were introduced in Section 8.2.

<table>
<thead>
<tr>
<th>TESTING CLAIMS ABOUT A POPULATION MEAN (WITH $\sigma$ KNOWN)</th>
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<tbody>
<tr>
<td><strong>OBJECTIVE</strong></td>
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<tr>
<td>Test a claim about a population mean when we know the</td>
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<tr>
<td>population standard deviation</td>
</tr>
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<td><strong>NOTATION</strong></td>
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<tr>
<td>$n =$ sample size</td>
</tr>
<tr>
<td>$\mu_x =$ population mean of all sample means (assumed true under the null)</td>
</tr>
<tr>
<td>$\bar{x} =$ sample mean</td>
</tr>
<tr>
<td>$\sigma =$ population standard deviation</td>
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<td></td>
</tr>
<tr>
<td><strong>REQUIREMENTS</strong></td>
</tr>
<tr>
<td>1. The sample is a simple random sample (SRS).</td>
</tr>
<tr>
<td>2. The value of the population standard deviation $\sigma$ is known.</td>
</tr>
<tr>
<td>3. The population is normally distributed and/or $n &gt; 30$.</td>
</tr>
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</table>
Example 1: When a fair die is rolled many times, the outcomes of 1, 2, 3, 4, 5, and 6 are equally likely, so the mean of the outcomes should be 3.5. The author drilled holes into a die and loaded it by inserting lead weights, then rolled it 40 times to obtain a mean of 2.9375. Assume that the standard deviation of the outcomes is 1.7078, which is the standard deviation for a fair die. Use a 0.05 significance level to test the claim that outcomes from the loaded die have a mean different from the value of 3.5 expected with a fair die.

a. Identify the null hypothesis

\[ H_0: \mu_x = 3.5 \]

b. Identify the alternative hypothesis

\[ H_A: \mu_x \neq 3.5 \]

c. Identify the test statistic

\[
\bar{x} = 2.9375, \quad n = 40
\]

\[
\mu_x = 3.5, \quad \sigma = 0.6
\]

\[
\sigma_x = 1.7078
\]

\[
z = \frac{\bar{x} - \mu_x}{\sigma_x} = \frac{2.9375 - 3.5}{1.7078} = -2.08
\]

d. Identify the \( P \)-value or critical value(s)

Traditional Method:

\[
P = 2 \cdot P(Z < -2.08) = 2 \cdot 0.0188 = 0.0376
\]

\[
0.0376 < 0.05
\]

So, reject \( H_0 \)

e. What is your final conclusion?

Reject \( H_0 \). There is sufficient evidence at the 5% level that the mean of the loaded die is different than the mean of a fair die.