h. Find the IQ score separating the top 37% from the others.

$z_{0.37} = 0.33$

$z = \frac{x-\mu}{\sigma}$

$0.33 = \frac{x-100}{15}$

$15(0.33)+100 = x$

$\boxed{x = 105.0}$

FINDING VALUES FROM KNOWN AREAS

1. Don't confuse __z-scores__ and __areas__. Remember, __z-scores__ are __distances__ along the __horizontal__ scale, but __areas__ are __regions__ under the __curve__.
2. Choose the correct __(right/left) side__ of the __graph__. A value separating the top 10% from the others will be located on the __right__ side of the graph, but a value separating the bottom 10% will be located on the __left__ side of the graph.
3. A __z-score__ must be __negative__ whenever it is located in the __left__ half of the __standard normal__ distribution.
4. Areas (or __probabilities__) are __positive__ or __0__ values, but they are never __negative__.

Always use graphs to __visualize__!!!

STEPS FOR FINDING VALUES USING TABLE A-2:

1. Sketch a __normal__ distribution curve, enter the given __probability__ or __percentage__ in the appropriate __region__ of the __graph__, and identify the __z-score__ being sought.
2. Use Table A-2 to find the __z-score__ (boundary value) corresponding to the __cumulative__ __left__ area bounded by __z_α__. Refer to the __body__ of Table A-2 to

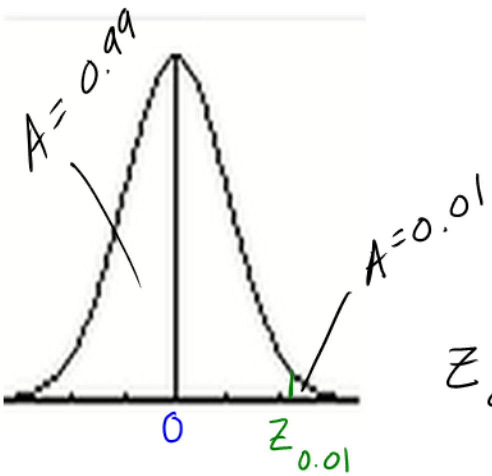
find the __closest__ area, then identify the corresponding __z-score__.

3. Solve for __x__ as follows:

$$z = \frac{x-\mu}{\sigma} \longleftrightarrow x = \sigma z + \mu$$

4. Refer to the __sketch__ of the __curve__ to make sure that the solution makes __sense__!

Example: Engineers want to design seats in commercial aircraft so that they are wide enough to fit 99% of all males. Men have hip breadths that are normally distributed with a mean of 14.4 inches and a standard deviation of 1.0 inch. Find the hip breadth for men that separates the smallest 99% from the largest 1 % (aka P_{99}).



$\mu = 14.4$ in

$\sigma = 1.0$ in

$z_{0.01} = 2.33$

$$z = \frac{x-\mu}{\sigma}$$

$$2.33 = \frac{x-14.4}{1.0}$$

$$x = 2.33 + 14.4$$

$$\boxed{x = 16.7 \text{ in}}$$

6.5 THE CENTRAL LIMIT THEOREM

Key Concept…

In this section, we introduce and apply the __central__ __limit__ __theorem__. The central limit theorem tells us that for a __population__ with __any__ distribution, the __distribution__ of the __sample__ __means__ approaches a __normal__ distribution as the sample size increases. This means that if the sample size is large enough, the distribution of sample means can be approximated by a normal distribution, even if the original population is NOT normally distributed. If the original population has mean μ and standard deviation σ, the mean of the sample means will also be μ, but the standard deviation of the sample means will be $\sigma/\sqrt{n}$, where n is the sample size.

It is essential to know the following principles:

1. For a population with any distribution, if $n > 30$, then the sample means have a distribution that can be approximated by a normal distribution, with mean μ and standard deviation $\sigma/\sqrt{n}$.
2. If $n \leq 30$ and the original population has a normal distribution, then the sample means have a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.
3. If $n \leq 30$ and the original population does not have a normal distribution, then the methods of this section DO NOT APPLY.

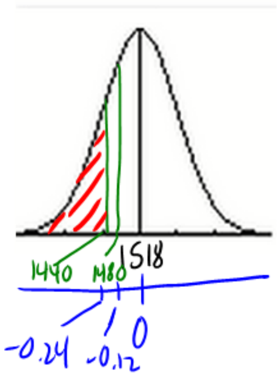
NOTATION

If all possible random samples of size n are selected from a population with mean μ and standard deviation σ, the mean of the sample means is denoted by $\mu_{\bar{x}}$, so $\mu_{\bar{x}}$ = μ. Also, the standard deviation of the sample means is denoted by $\sigma_{\bar{x}}$, so $\sigma_{\bar{x}}$ = $\sigma/\sqrt{n}$. $\sigma_{\bar{x}}$ is called the Standard error of the mean.

APPLYING THE CENTRAL LIMIT THEOREM

Example 1: Assume that SAT scores are normally distributed with mean $\mu = 1518$ and standard deviation $\sigma = 325$.

a. If 1 SAT score is randomly selected, find the probability that it is between 1440 and 1480.



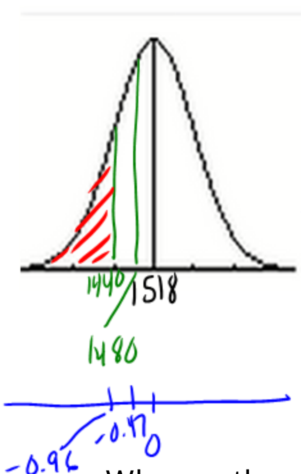
$$Z_1 = \frac{1440-1518}{325} \doteq -0.24$$

$$Z_2 = \frac{1480-1518}{325} \doteq -0.12$$

$$P(1440 < x < 1480) = P(-0.24 < Z < -0.12)$$
$$= P(Z < -0.12) - P(Z < -0.24)$$
$$= 0.4522 - 0.4052$$
$$= \boxed{0.0470}$$

b. If 16 SAT scores are randomly selected, find the probability that they have a mean between 1440 and 1480.

$n = 16$, $\mu_{\bar{x}} = \mu = 1518$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 325/\sqrt{16}$



$$Z_{1\bar{x}} = \frac{1440-1518}{325/\sqrt{16}} \doteq -0.96$$

$$Z_{2\bar{x}} = \frac{1480-1518}{325/\sqrt{16}} \doteq -0.47$$

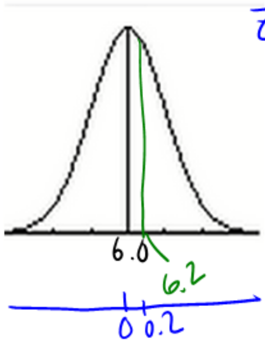
$$P(1440 < \bar{x} < 1480)$$
$$= P(-0.96 < Z_{\bar{x}} < -0.47)$$
$$= P(Z_{\bar{x}} < -0.47) - P(Z_{\bar{x}} < -0.96)$$
$$= 0.3192 - 0.1685$$
$$= \boxed{0.1507}$$

c. Why can the central limit theorem be used in part (b) even though the sample size does not exceed 30?

The original scores are normally distributed.

Example 2: Engineers must consider the breadths of male heads when designing motorcycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 inches and a standard deviation of 1.0 inch.

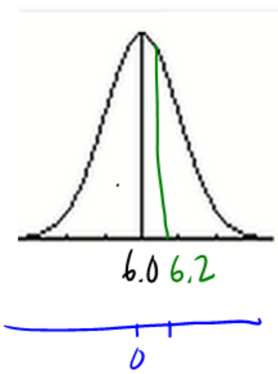
a. If one male is randomly selected, find the probability that his head breadth is less than 6.2 inches.



$$z = \frac{6.2-6.0}{1.0} = 0.2$$

$$P(x < 6.2) = P(z < 0.2)$$
$$= \boxed{0.5793}$$

b. The Safeguard Helmet company plans an initial production run of 100 helmets. Find the probability that 100 randomly selected men have a mean head breadth of less than 6.2 inches.



$$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

$$\mu_{\bar{x}} = \mu = 6.0$$
$$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 1/\sqrt{100}$$
$$n = 100$$

$$z_{\bar{x}} = \frac{6.2-6.0}{1.0/\sqrt{100}} = 2.00$$

$$P(\bar{x} < 6.2) = P(z_{\bar{x}} < 2.00)$$
$$= \boxed{0.9772}$$

c. The production manager sees the result from part (b) and reasons that all helmets should be made for men with head breadths less than 6.2 inches, because they would fit all but a few men. What is wrong with that reasoning?

The individual probability: $p(\text{man has a head breadth} > 6.2\text{ in}) = 1 - 0.5793$
$= 0.4207$

So approximately 42% of men have a head breadth greater than 6.2in.