When you are done with your 4.5 homework you should be able to…

- π Verify the solution of a system of linear equations in three variables
- π Solve systems of linear equations in three variables
- π Identify inconsistent and dependent systems
- π Solve problems using systems in three variables

WARM-UP:

Solve the following system of linear equations. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

$$\begin{aligned} 5x-3y&=1 \\ y&=3x-7 \end{aligned}$$

SYSTEMS OF LINEAR EQUATIONS IN THREE VARIABLES AND THEIR SOLUTIONS

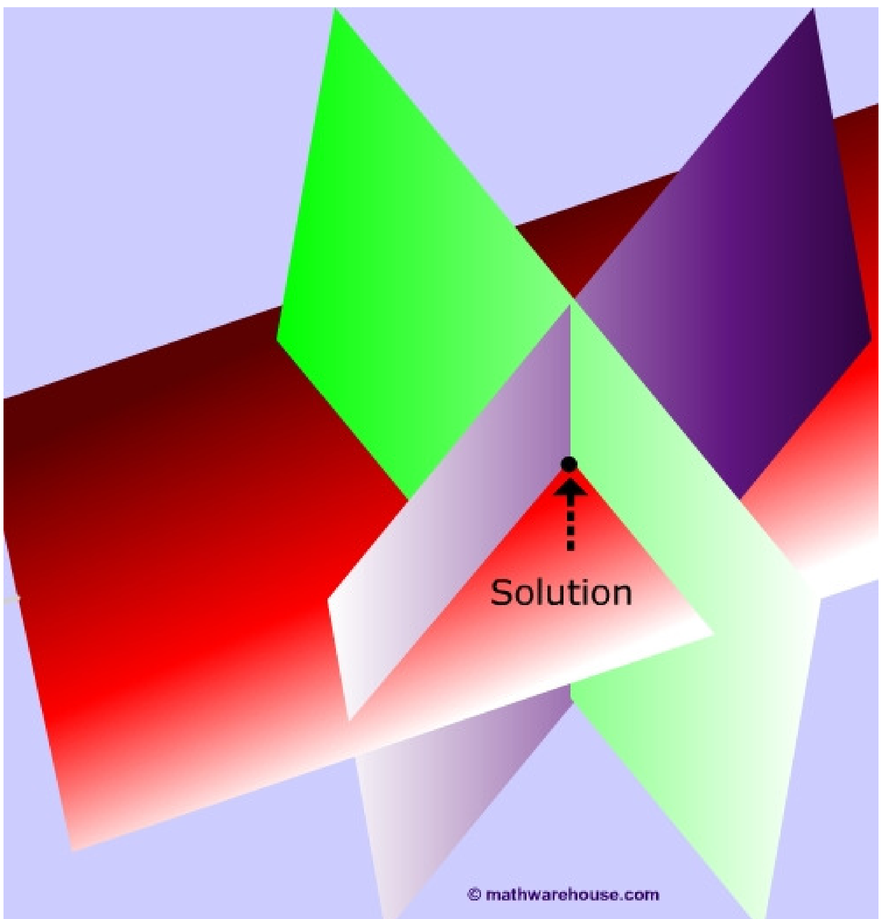
Any equation of the form ______________________, where ____, ____, ____, and ____ are real numbers such that ____, ____, and ____ are not ________ zero, is a _______________ ________________ in _________ _______________. The graph of this linear equation in three variables is a ______________ in

__________-_________________ space. The process of solving a ____________

of ____________ linear eqautions in __________ variables is geometrically equivalent to finding a ________________ of ____________________ (assuming that there is one) of three ______________ in space. A ________________ of a system of _______________ equations in ____________ variables is an _______________ _____________ of real numbers that _________________ ALL equations in the ________________. The _______________ _________ of the system is the ____________ of _______ its _______________.

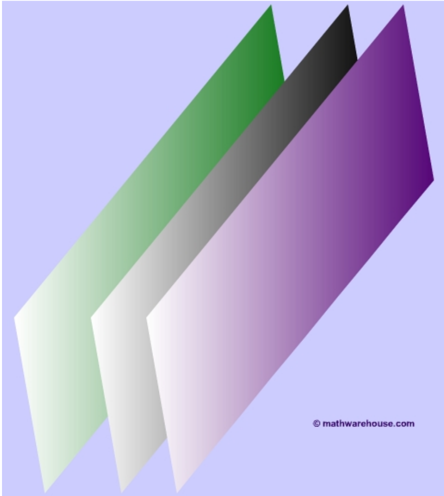
One Solution of three variable systems

If the three planes intersect as pictured below then the three variable system has 1 point in common, and a single solution represented by the black point below.

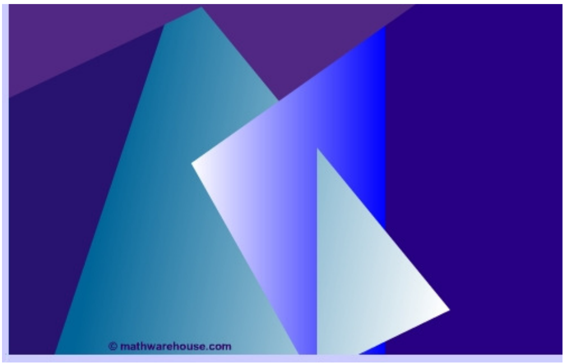


No Solution of three variable systems

Below is a picture of three planes that have no solution.There is no single point at which **all three** planes intersect, therefore this system has no solution.

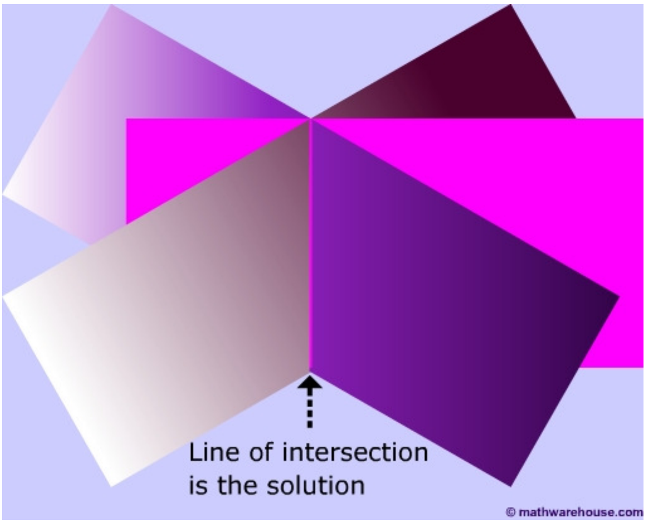


The other common example of systems of three variables equations that have no solution is pictured below. In the case below, each plane intersects the other two planes. **However, there is no single point at which all three planes meet.** Therefore, the system of 3 variable equations below has no solution.



Infinite Solutions of three variable systems

If the three planes intersect as pictured below then the three variable system has a line of intersection and therefore an infinite number of solutions.



SOLVING LINEAR SYSTEMS IN THREE VARIABLES BY ELIMINATING VARIABLES

1. Reduce the _______________ to ________ equations in _______ variables. This is usually accomplished by taking __________ ________________ _____________ of equations and using the _______________ method to _________________ the SAME VARIABLE from BOTH _____________.
2. ________________ the resulting _______________ of two equations. The result is an equation in _______ variable that gives the __________ of that variable.
3. ____________-________________ the __________ of the variable found in step 2 into either of the equations in _______________ variables to find the value of the _______________ variable.
4. Use the values of the _________ variables from steps ____ and ____ to find the value of the ____________ variable by __________-__________________ into one of the ________________ equations.
5. _____________ the proposed solution in _____________ of the

Example 1: Determine if the given ordered triple is a solution of the system.

a.

$(5,-3,-2)$

$$\begin{aligned} x+y+z&=0 \\ x+2y-3z&=5 \\ 3x+4y+2z&=-1 \end{aligned}$$

b.

$(2,-1,3)$

$$\begin{aligned} x+y+z&=4 \\ x-2y-z&=1 \\ 2x-y-z&=-1 \end{aligned}$$

Example 2: Solve each system. If there is no solution or if there are infinitely many solutions and a system's equations are dependent, so state. Use set notation to express solution sets.

a.

$$\begin{aligned} 2x+y-2z&=-1 \\ 3x-3y-z&=5 \\ x-2y+3z&=6 \end{aligned}$$

b.

$$\begin{aligned} 2x+4y+5z&=8 \\ x-2y+3z&=-6 \\ 2x-4y+6z&=8 \end{aligned}$$

c.

$$\begin{aligned} x+2y+z&=4 \\ 3x-4y+z&=4 \\ 6x-8y+2z&=8 \end{aligned}$$