ELEMENTARY ALGEBRA GUIDED NOTEBOOK

FOR USE WITH ROBERT BLITZER'S TEXTBOOK INTRODUCTORY AND INTERMEDIATE ALGEBRA FOR COLLEGE STUDENTS, 3RD ED.

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Section 1.1: Introduction to Algebra: Variables and Mathematical ModelsWhen you are done with your homework you should be able to…

- π Evaluate algebraic expressions
- π Translate English phrases into algebraic expressions
- π Determine whether a number is a solution of an equation
- π Translate English sentences into algebraic equations
- π Evaluate formulas

WARM-UP:

Perform the indicated operation and simplify.

1. $\dfrac{-(-5)^3-5+2}{8(2-11)}$

2. $16 \div 5 - 1$

EVALUATING ALGEBRAIC EXPRESSIONS

We can ________________ a _________________ that appears in an ________________ __________________ by a _______________. The ________________ is called __________________ the ________________.

A First Look at Order of Operations

1. Perform all operations ________________ ______________ ______________, such as _________________.
2. Do all ____________________ in the ____________ in which they occur from _____________ to _____________.

3. Do all __________________ and ___________________ in the _____________ in which they ______________ from ______________ to _____________.

Example 1: Find the mistake!

$$\begin{aligned} 3+2\div 5\cdot 10 &= 5\div 5\cdot 10 \\ &= 1\cdot 10 \\ &= 10 \end{aligned}$$

Example 2: Evaluate the following algebraic expressions at the given value(s):

1. $\dfrac{2x+25}{x-1}$, $x=-2$

2. $\dfrac{6x-9y+1}{y-x}$, $x=10$, $y=-4$

KEY WORDS FOR ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION

ADDITION	SUBTRACTION	MULTIPLICATION	DIVISION

Example 3: Write each English phrase as an algebraic expression.

1. Six more than a number

2. Twelve less a number

3. Two times the sum of a number and five increased by nine

EQUATIONS

An _______________ is a _________________ that two ________________ __________________ are _______________. What symbol does an equation always contain?

__________________ of an _________________ are ______________ of the _________________ that make the __________________ a _____________ statement. To determine whether a number is a _________________, ___________________ that number for the _______________ and __________________ each side of the equation. If the values on ____________ sides of the ___________________ are the _____________, the _______________ is a ___________________.

Example 4: Determine whether the given number is a solution of the equation.

1. $x+17=22;\ 5$

2. $5z=30;\ 8$

Example 5: Write each equation as an English sentence.

1. $9-3x=7$

2. $2(x+5)=x-4$

Example 6: Write each sentence as an equation.

1. The difference between forty and a number is ten.

2. The product of six and a number increased by three is thirty-three.

FORMULAS AND MATHEMATICAL MODELS

One aim of ____________ is to provide a compact, ______________ description of the world. These descriptions involve the use of ________________. A ________________ is an ________________ that expresses a ________________ between two or more __________________. The process of finding formulas to describe ___________ ______________ phenomena is called ________________ ________________. Such formulas, together with the ________________ assigned to the __________________, are called _______________ ___________.

Example 7:

A bowler's handicap, *H,* is often found using the following formula:

$H=0.8(200-A)$, where *A* denotes the bowler's average score.

1. If your average bowling score is 145, what is your handicap?

2. What would your final score be if you bowled 120 in a game?

Section 1.2: FRACTIONS IN ALGEBRA

When you are done with your homework you should be able to…

- π Convert between mixed numbers and improper fractions
- π Write the prime factorization of a composite number
- π Reduce or simplify fractions
- π Multiply fractions
- π Divide fractions
- π Add and subtract fractions with identical denominators
- π Add and subtract fractions with unlike denominators
- π Solve problems involving fractions in algebra

WARM-UP:

Evaluate the following algebraic expressions at the given value(s):

2. $\frac{3x-8}{5(x-1)}$, $x=4$

2. $6x-2y+5$, $x=0$, $y=-2$

VOCABULARY

Numerator: The _____________ or ________________ expression that is written ________________ the ________________ bar.

Denominator: The _____________ or ________________ expression that is written ________________ the ________________ bar.

Natural Numbers: The __________________ that we use for __________________.

Mixed Numbers: A _____________ number consists of the _____________ of a

______________ number and a ________________, expressed ______________

the use of an _________________ _____________.

Improper Fractions: An _________________ _________________is a fraction

whose ___________________ is _______________ than its ________________.

such as _________________.

CONVERTING A MIXED NUMBER TO AN IMPROPER FRACTION

STEPS

1. ___________________ the _______________ of the ____________ by the

   ______________ number and __________ the ________________ to this

   ______________.

2. Place the _____________ from step 1 ____________ the ______________ of

   the _______________ mixed number.

Example 1: Convert the following mixed numerals to improper fractions

1. $5\frac{7}{8}$

2. $2\frac{5}{11}$

CONVERTING FROM AN IMPROPER FRACTION TO A MIXED NUMBER

STEPS

1. ____________ the ______________ into the _______________. Record the _______________ (result of the division) and the _______________.
2. Write the ________________ number using the following form:

Example 2: Convert the following improper fractions to mixed numerals

1. $\frac{15}{2}$

2. $\frac{24}{7}$

FACTORS AND PRIME FACTORIZATIONS

Fractions can be ________________ by first ________________ the natural numbers that make up the ________________ and ________________. To ______________ a natural number means to write it as two or more ________________ numbers being __________________.

VOCABULARY

<u>Prime number</u>: A ____________ number is a _____________ number greater than 1 that has only ______________ and ____ as _______________.

<u>Composite numbers</u>: A ____________ number is a _____________ number greater than 1 that is ______ a ______________.

EVERY COMPOSITE NUMBER CAN BE EXPRESSED AS THE ______________ OF ______________ ___________________!!!

Expressing a _________________ number as the ________________ of ____________ numbers is called the ________________ ___________________ of that composite number.

Example 3: Find the prime factorization of the following numbers

1. 128
2. 54

REDUCING FRACTIONS

Two fractions are called _______________ if they represent the ____________ ____________. Writing a fraction as an __________________ ______________ with a _______________ _________________ is called _____________ a______________. A fraction is ______________ to its ______________ ____________ when the ________________ and ________________ have ____ _______________ _______________ other than ____.

FUNDAMENTAL PRINCIPLE OF FRACTIONS

The ___________ of a _____________ __________ __________ ____________ if both the _________________ and _________________ are ______________ (or ________________) by the ______________ nonzero _____________.

________ = ____

STEPS

1. Write the ____________ ____________________ of the ______________ and the ___________________.
2. _________________ the ___________________ and the ______________ by the _________________ _________________ _______________ (the product of all factors common to both).

Example 4: Reduce each fraction to its lowest terms

1. $\frac{18}{27}$

2. $\frac{100}{45}$

MULTIPLYING FRACTIONS

The _______________ of two or more ______________ is the ______________ of their __________________ divided by the _______________ of their __________________.

$$\frac{}{} \cdot \frac{}{} = \frac{}{}$$

Example 5: Multiply and reduce each product to its lowest terms

1. $\frac{16}{11} \cdot \frac{33}{2}$

2. $\frac{5}{8} \cdot 12$

DIVIDING FRACTIONS

The _______________ of two ______________ is the ______________ fraction _________________ by the _________________ of the ______________ fraction.

$$\frac{}{} \div \frac{}{} = \frac{}{} \cdot \frac{}{}$$

Example 6: Divide and reduce each quotient to its lowest terms

1. $\frac{25}{32} \div \frac{3}{4}$

2. $\frac{144}{3} \div 12$

ADDING AND SUBTRACTING FRACTIONS WITH IDENTICAL DENOMINATORS

The _______________ or __________________ of two ______________ with _________________ ___________________ is the sum or difference of their __________________ over the ______________ __________________.

$$\text{—} + \text{—} = \text{———} \quad \text{and} \quad \text{—} - \text{—} = \text{———}$$

Example 7: Perform the indicated operations

1. $\frac{5}{6} + \frac{3}{6}$

2. $\frac{11}{13} - \frac{10}{13}$

ADDING AND SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

The value of a fraction ___________ __________ change if the ______________ and ________________ are __________________ by the ___________ nonzero _______________.

Example 8: Write $\frac{5}{8}$ as an equivalent fraction with a denominator of 32.

The **least common denominator** is the _________________ number that the numbers in each denominator _______________ into.

STEPS FOR ADDING AND SUBTRACTING FRACTIONS WITH UNLIKE DENOMINATORS

1. ______________the fractions as ________________ ______________ with the _____________ _________________ ________________.
2. ____________ or ______________ the __________________, putting this result over the ______________ __________________.

USING PRIME FACTORIZATIONS TO FIND THE LCD

1. Find the ____________ _______________ of each __________________.
2. The ____________ is obtained by using the ________________ number of times each _____________ occurs in ___________ ________________ factorization.

Example 9: Perform the indicated operations

1. $\frac{23}{7}+\frac{5}{14}$

2. $\frac{5}{12}-\frac{2}{15}$

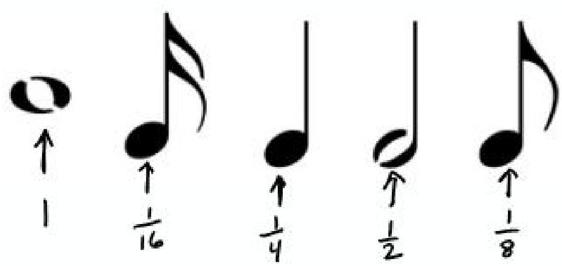
Example 10: Translate from English to an algebraic expression or equation. Let *x* represent the variable.

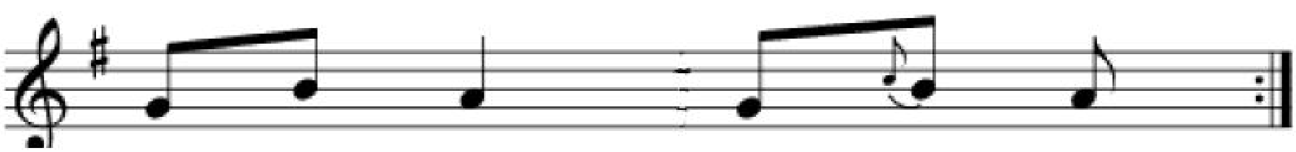
1. A number decreased by one third of itself.

2. The sum of one ninth of a number and one tenth of that number gives 15.

APPLICATIONS

Shown below is a line from the sheet music for "An Irish Lullaby". The time is $\frac{2}{4}$, which means that each measure must contain notes that add up to $\frac{2}{4}$. Use vertical lines to divide "An Irish Lullaby".





Section 1.3: THE REAL NUMBERS

When you are done with your homework you should be able to…

- π Define the sets that make up the real numbers
- π Graph numbers on a number line
- π Express rational numbers as decimals
- π Classify numbers as belonging to one or more sets of the real numbers
- π Understand and use inequality symbols
- π Find the absolute value of a real number

WARM-UP:

Perform the indicated operation and simplify:

1. $\frac{10}{27}\cdot\frac{3}{2}$

2. $\frac{28}{9}+\frac{2}{3}$

NATURAL NUMBERS AND WHOLE NUMBERS

A ____________ is a ______________ of objects whose contents can be clearly determined. The objects in a set are called the ________________ of the set.

Natural numbers: The ____________ of ______________ numbers is

Whole numbers: The ____________ of ______________ numbers is

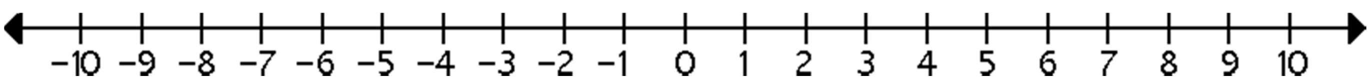
INTEGERS AND THE NUMBER LINE

The __________ consisting of the __________________ numbers, ____________, and the ________________ of the _________________ numbers is called the set of ___________________.

<u>Integers</u>: The _____________ of _______________ is

Example 1: Consider the following integers: 3, -3, 5, -5, 0

Graph each integer in the list on the same number line.



RATIONAL NUMBERS

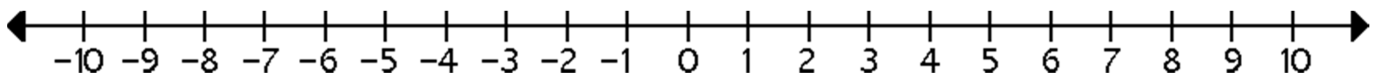
If two _______________ are added, subtracted, or multiplied, the result is always another ________________. Is this true when one integer is divided by another?

The set of __________________ numbers is the set of all numbers that can be expressed in the form __________, where _____ and _____ are _______________ and _____ is ________ equal to _____ (_____________). The integer _____ is called the _______________ and the integer _____ is called the ________________.

Are all integers rational numbers?

Example 2: Consider the following rational numbers: $-\frac{1}{2}$, $\frac{9}{4}$, -8, $-6\frac{2}{3}$

Graph each integer in the list on the same number line.



Example 3: Divide

1. $3 \div 8$

2. $3 \div 11$

RATIONAL NUMBERS AND DECIMALS

Any ____________ number can be expressed as a _____________. The resulting decimal will either _______________ (_________), or it will have a digit or block of digits that _______________.

IRRATIONAL NUMBERS

Any number that can be represented on the ______________ line that is ______ a __________________ number is called an __________________ number. In other words, the set of irrational numbers is the set of numbers whose _______ representations are neither ____________________ nor __________________.

THE SET OF REAL NUMBERS

All numbers that can be represented by ______________ on the number line are called ______________ numbers.

THE SETS THAT MAKE UP THE REAL NUMBERS

NAME	DESCRIPTION	EXAMPLES
NATURAL NUMBERS		
WHOLE NUMBERS		
INTEGERS		

RATIONAL NUMBERS		
IRRATIONAL NUMBERS		

Example 4: Consider the following set of numbers: $\left\{-\frac{4}{2}, 8, \frac{1}{3}, \sqrt{100}, 0, \pi, 0.3\right\}$

List the numbers in the set that are

1. Natural numbers
2. Whole numbers
3. Integers

4. Rational numbers

5. Irrational numbers

6. Real numbers

INEQUALITY SYMBOLS

On the real number line, the ___________ numbers ___________ from _______ to _________. The ___________ or two real numbers is the one farther to the __________ on a number line. The ______________ of two real numbers is the one farther to the _____________ on a number line.

NOTATION

Example 5: Insert $<$ or $>$ between each pair of integers to make the statement true.

1. 3 ____ 5
2. 3 ____ 0
3. -3____-5
4. -3____ 0
5. 0 ____-3
6. -5____ 5

ABSOLUTE VALUE

The ________________ ________________ of a real number ____, denoted __________, is the ________________ from ____ to ____ on a number line. Is the output of an absolute value expression ever negative?

Example 6: Find the absolute value:

1. $|2.5|$

2. $|-8|$

APPLICATIONS

The table below shows the amount spent on iPAD apps by Shannon's family during the months of May and July of 2011.

Name	Amount
Shannon	$48
Morgan	$67
Rory	$25
Erin	$32
Nicole	$12

1. Graph the five dollar amounts on a number line.

2. Write the names in order from the least spent on apps to the most spent on apps

Section 1.4: BASIC RULES OF ALGEBRA

When you are done with your homework you should be able to…

- π Understand and use the vocabulary of algebraic expressions
- π Use commutative properties
- π Use associative properties
- π Use distributive properties
- π Combine like terms
- π Simplify algebraic expressions

WARM-UP:

Perform the indicated operation and simplify:

1. $\frac{57}{4} \div \frac{3}{2}$

2. $\frac{3}{14} - \frac{1}{10}$

VOCABULARY OF ALGEBRAIC EXPRESSIONS

Terms: The ____________ of an ______________ expression are those parts that are ___________________ by ________________ or __________________. A ____________ is a ________________, a ________________, or a ______________ __________________ by one or more ___________________.

Coefficient: The ___________________ part of a ____________ is called its ___________________. What is the coefficient of a term which only has variables?

<u>Constant term</u>: A term that consists of just a _________________ is called a

__________________ ______________.

<u>Like terms</u>: Terms that have the _______________ the ___________

________________ _________________ are called ____________ __________.

Are constant terms like terms?

Example 1: Consider the following algebraic expression: $-12x+9+7x-8$

1. How many terms are there in the algebraic expression?

2. What is the coefficient of the first term?

3. List the constant term(s):

4. What are the like terms in the algebraic expression?

EQUIVALENT ALGEBRAIC EXPRESSIONS

Two ______________ expressions that have the ___________ value for _____

replacements are called _________________ _____________ _____________.

Example 2: Evaluate the following two algebraic expressions at $x=2$.

1. $-12x+9+7x-8$
2. $-5x+1$

Constant term: A term that consists of just a _________________ is called a

__________________ ______________.

Like terms: Terms that have the _______________ the ___________

________________ _________________ are called ____________ __________.

Are constant terms like terms?

Example 1: Consider the following algebraic expression: $-12x+9+7x-8$

5. How many terms are there in the algebraic expression?

6. What is the coefficient of the first term?

7. List the constant term(s):

8. What are the like terms in the algebraic expression?

EQUIVALENT ALGEBRAIC EXPRESSIONS

Two ______________ expressions that have the ___________ value for _____

replacements are called _________________ _____________ _____________.

Example 2: Evaluate the following two algebraic expressions at $x=2$.

1. $-12x+9+7x-8$

2. $-5x+1$

THE COMMUTATIVE PROPERTIES

Let *a* and *b* represent real numbers, variables, or algebraic expressions.

Commutative Property of Addition:

Changing _____________ when adding does not affect the ________.

Commutative Property of Multiplication:

Changing _____________ when multiplying does not affect the ____________.

Example 3: Use the commutative property to write an algebraic expression equivalent to each of the following:

1. $2x+4$
2. $x \cdot 13$

THE ASSOCIATIVE PROPERTIES

Let *a*, *b,* and *c* represent real numbers, variables, or algebraic expressions.

Associative Property of Addition:

Changing _____________ when adding does not affect the ________.

Associative Property of Multiplication:

Changing _____________ when multiplying does not affect the ____________.

Example 4: Use the associative property to simplify the algebraic expressions:

1. $4x+(7+x)$
2. $25(4x)$

THE DISTRIBUTIVE PROPERTY

Let *a*, *b,* and *c* represent real numbers, variables, or algebraic expressions.

Multiplication ________________ over _________________.

Example 5: Multiply:

1. $3(x+5)$
2. $-(4+x)$

OTHER FORMS OF THE DISTRIBUTIVE PROPERTY

PROPERTY	MEANING	EXAMPLES
$a(b-c)$ $= ab - ac$		
$a(b+c+d)$ $= ab + ac + ad$		
$(b+c)a$ $= ba + ca$		

COMBINING LIKE TERMS

The _________________ property lets us __________ and _______________ like terms.

Example 6: Combine like terms:

1. $3(4x)+(-x+21)$

2. $9x+(x+5)-2(-x+11+3y)$

STEPS FOR SIMPLIFYING ALGEBRAIC EXPRESSIONS

1. Use the _____________________ property to remove _________________.
2. Rearrange terms and ______________ ____________ terms using the __________________ and _____________________ properties. As you hone your skills, you'll be doing this step mentally!

3. Combine _____________ terms by combining the __________________ of the _________ and keeping the same ______________ ___________.

APPLICATIONS

The percentage of U.S. women, *W*, who used the internet *n* years after 2000 can be modeled by the formula $W = 2(2n + 25) + 0.5(n + 2)$.

1. Simplify the formula.

2. Use the simplified form of the mathematical model to find the percentage of U.S. women who used the internet in 2005.

Section 1.5: ADDITION OF REAL NUMBERS

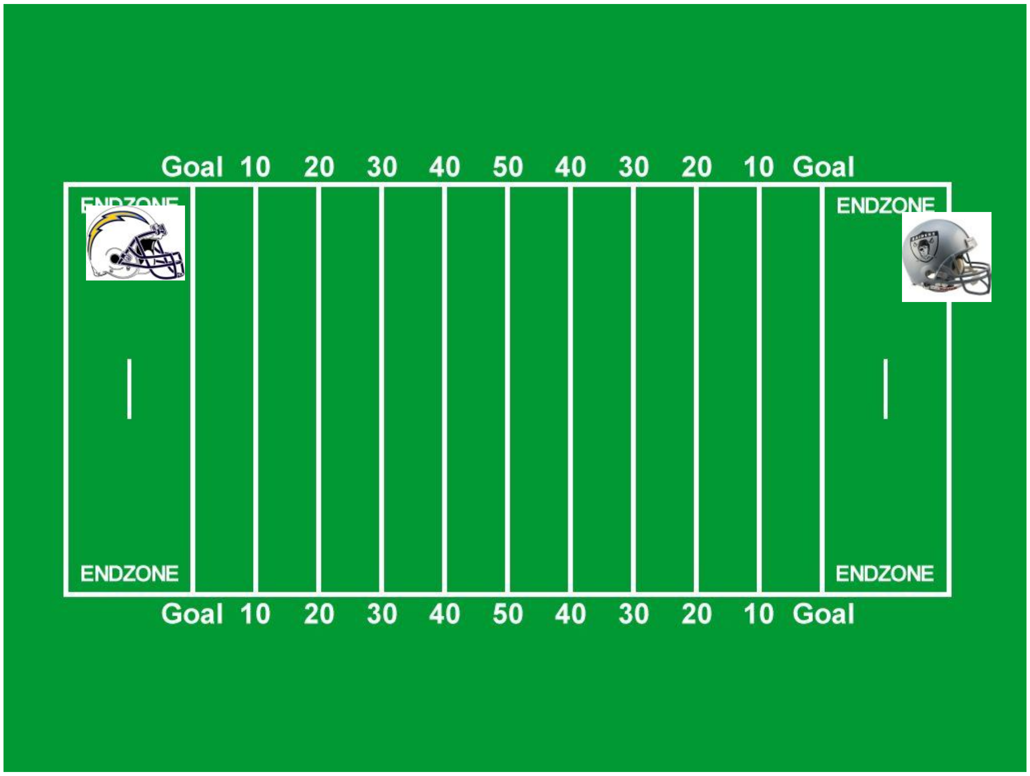
When you are done with your 1.5 homework you should be able to…

- π Add numbers with a number line
- π Find sums using identity and inverse properties
- π Add numbers without a number line
- π Use addition rules to simplify algebraic expressions
- π Solve applied problems using a series of addition

WARM-UP:

It is a Sunday during the fall semester. You are watching the Chargers/Raider game. The Chargers are currently on their own 30 yard line. During the first down that the Chargers have possession of the ball, the Chargers complete a pass for a gain of ten yards. On the second down, the Raiders sack the Chargers' quarterback, causing a loss of 10 yards.

a. Use the image of the football field shown below to model the gain and loss of yardage.

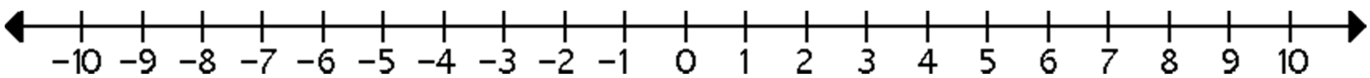


b. How would you use signed numbers to represent the ten yard gain?

c. How would you use signed numbers to represent the ten yard loss?

d. What is the net yardage gained?

e. How can we use the number line below to model 10 + (-10)?



f. Based on the information above, we can conclude that a number and its

___________________ sum to ___________.

IDENTITY AND INVERSE PROPERTIES OF ADDITION

PROPERTY	MEANING	EXAMPLES
IDENTITY PROPERTY OF ADDITION		
INVERSE PROPERTY OF ADDITION		

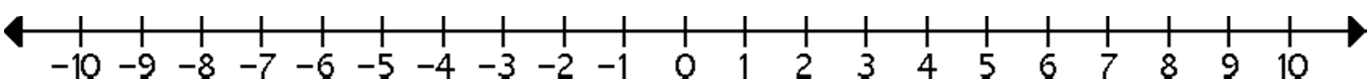
ADDING INTEGERS

NUMBER LINE MODELS

POSITIVE/NEGATIVE CHIP MODEL

Example 1: Illustrate $4+2$ using

a. A number line

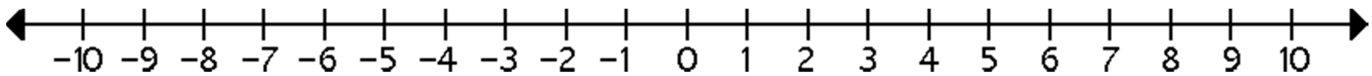


b. The positive/negative chip method

c. So the "rule" for a positive plus a positive is…

Example 2: Illustrate $-2+(-3)$ using

a. A number line

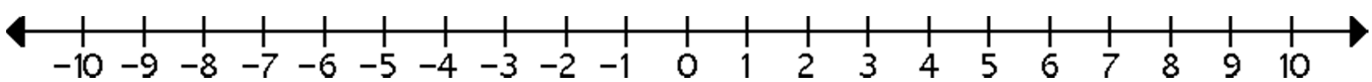


b. The positive/negative chip method

c. So the "rule" for a negative plus a negative is...

Example 3: Illustrate $-10+6$ using

a. A number line

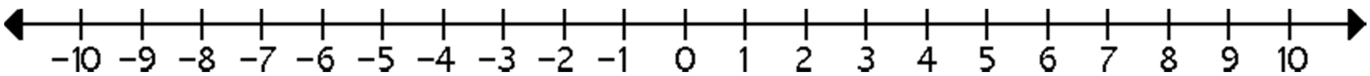


b. The positive/negative chip method

c. So the "rule" for a negative plus a positive is...

Example 4: Illustrate $1+(-7)$ using

a. A number line



b. The positive/negative chip method

c. So the "rule" for a positive plus a negative is…

SIMPLIFYING ALGEBRAIC EXPRESSIONS

Example 5: Simplify.

a. $-9x+(-4x)$

b. $-6y+22y-5$

APPLICATIONS

Example 6:

In a magic square, all the rows, all the columns and the 2 diagonals must add to the same number.

1. Complete the magic square, using only the positive integers 1 to 9:

	1	
4	9	2

2. Complete the magic square, using only the integers:

-10, -8, -6, -4, 0, 2, 4, 6

	-2	

Example 7: In high school, an elementary algebra class meets five hours a week for nine months. At MiraCosta College, an elementary algebra class meets five hours a week for 4 months. The class at MiraCosta College has how many fewer in-class hours?

MIXED PRACTICE

1. Fill in the blank.

   a. 5 is the _________________ of -5.

   b. On a number line, the greater number is to the ___________ of the lesser number.

   c. A number's distance from zero on a number line is the number's _____________

      ____________.

   d. Numbers less than zero are called ______________ numbers.

   e. When using an inequality symbol, the "arrow" points towards the

      ____________________ number.

2. Add.

   a. $-18+(-26)+100+34$

   c. $12^2-24\div 6$

   b. $-18+2(51-6)+100(-15+670)$

   d. $\dfrac{30+(-8)}{2(176-175)}$

Section 1.6: SUBTRACTION OF REAL NUMBERS

When you are done with your homework you should be able to…

- π Subtract real numbers
- π Simplify a series of additions and subtractions
- π Use the definition of subtraction to identify terms
- π Use the subtraction definition to simplify algebraic expressions
- π Solve problems involving subtraction

WARM-UP:

Simplify:

1. $\frac{1}{2}(2x-7)+3x$

2. $-(-x+5)+3(2)(5x-1)$

DEFINITION OF SUBTRACTION

For all real numbers *a* and *b,*

To subtract ______ from ______, _______ the _________________ (or additive inverse) of _____ to _____. The result of subtraction is called the ___________________.

A PROCEDURE FOR SUBTRACTING REAL NUMBERS

1. Change the subtraction operation to ______________________.
2. Change the _____________ of the number being __________________.
3. _________.

Example 1: Subtract.

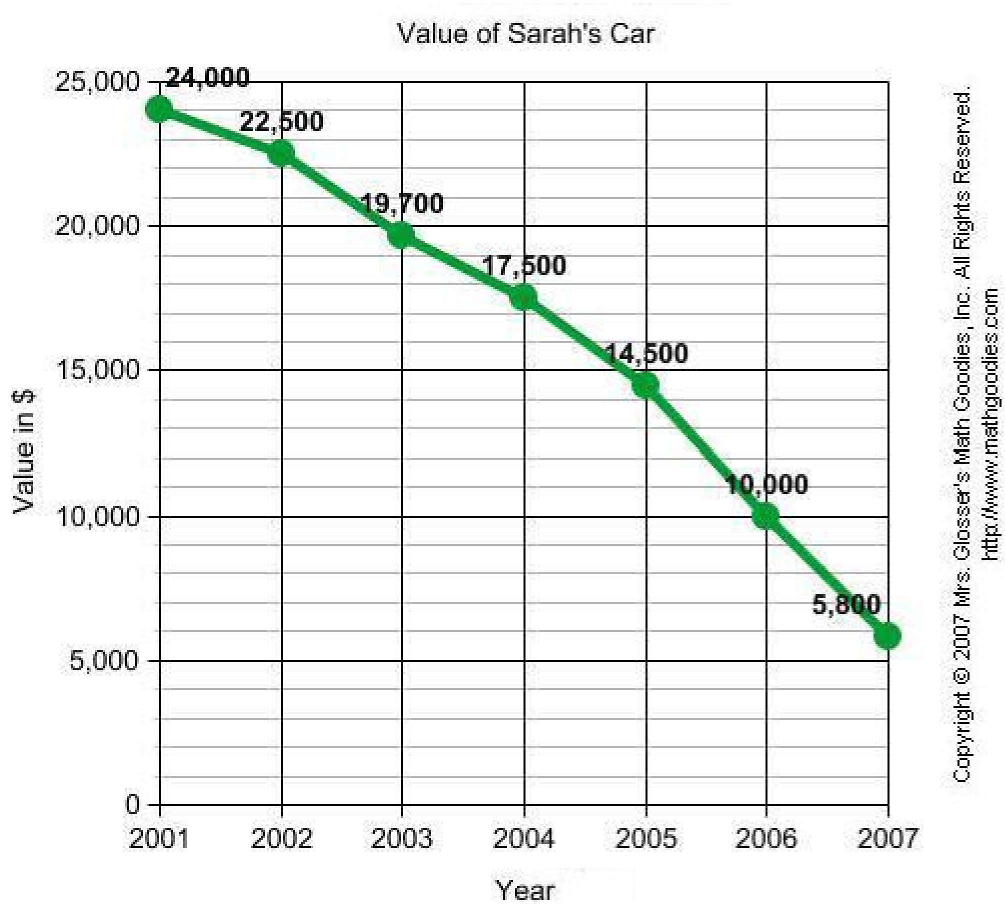
1. $-16-(-9)$
2. $16-20$
3. $-6-32$
4. $10.2-0.2-(-5.1)$

Example 2: Simplify.

1. $3(4x)-(2x-21)$
2. $9x+(x+5)-2(x-11+3y)$

APPLICATIONS

The line graph below illustrates the value of Sarah's car in dollars from the year 2001 to the year 2007.



1. How much was Sarah's car worth in 2005?

2. How much more was Sarah's car worth in 2002?

Section 1.7: MULTIPLICATION AND DIVISION OF REAL NUMBERS

When you are done with your homework you should be able to…

- π Multiply real numbers
- π Multiply more than two real numbers
- π Find multiplicative inverses
- π Use the definition of division
- π Divide real numbers
- π Simplify algebraic expressions involving multiplication
- π Determine whether a number is a solution of an equation
- π Use mathematical models involving multiplication and division

WARM-UP:

1. Find the value of each expression:
   a. $\frac{9}{10}-\left(\frac{1}{4}-\frac{7}{10}\right)$
   b. $-\left|-8-(-2)\right|-(-6)$

2. Write each English phrase as an algebraic expression. Let *x* represent the number:
   a. The difference between 9 times a number and -4 times a number
   b. The quotient of -7 and a number subtracted from the quotient of -12 and a number

THE PRODUCT OF TWO REAL NUMBERS

- The ____________ of two real numbers with ______________ signs is found by ___________________ their __________________ values. The product is ___________________.
- The ____________ of two real numbers with the ______________ sign is found by ___________________ their __________________ values. The product is ___________________.
- The ____________ of zero and any real number is ______.

Example 1: Multiply.

1. $-15(5)$

2. $8.3(-2)$

3. $(-11)(-12)$

4. $\frac{4}{3}\cdot 0$

MULTIPLYING MORE THAN TWO NUMBERS

1. Assuming that no factor is zero,
   - The ____________ of an _________ number of ______________ numbers is ___________________.
   - The ____________ of an _________ number of ______________ numbers is ___________________.
2. If any ______________ is _____, the product is _____.

Example 2: Multiply.

3. $-7(5)(-6)\cdot 2$

4. $(13)(-1)\left(-\frac{5}{2}\right)(-8)$

THE MEANING OF DIVISION

The result of _________________ the real number ____ by the nonzero real number ____ is called the ______________ of ____ and ____. We can write this ______________ as ______________ or ___________. We can define division in terms of ______________________ by using ____________________ inverse or __________________.

Example 3: Find the multiplicative inverse of each number.

1. 12

2. $-\frac{1}{4}$

3. $-\frac{7}{8}$

DEFINITION OF DIVISION

If *a* and *b* are real numbers and *b* is not equal to zero, then the ______________ of _____ and _____ is defined as

The __________________ of two real numbers is the ______________ of the _____________ number and the _________________ ________________ of the ______________ number.

Example 4: Divide using the definition of division.

1. $5 \div \frac{1}{5}$

2. $\frac{-123}{-3}$

THE QUOTIENT OF TWO REAL NUMBERS

- The ____________ of two real numbers with ______________ signs is found by __________________ their _________________ values. The quotient is __________________.
- The ____________ of two real numbers with the ______________ sign is found by __________________ their _________________ values. The quotient is __________________.
- Division of any real number by _____ is ____________________.
- Any nonzero number divided into _____ is ______.

Example 5: Divide.

1. $-\frac{2}{5} \div \frac{1}{10}$

3. $\frac{123}{-3}$

2. $\frac{0}{123}$

4. $-1.8 \div (-0.6)$

ADDITIONAL PROPERTIES OF MULTIPLICATION

PROPERTY	MEANING	EXAMPLES
IDENTITY PROPERTY OF MULTIPLICATION		
INVERSE PROPERTY OF MULTIPLICATION		
MULTIPLICATION PROPERTY OF -1		
DOUBLE NEGATIVE PROPERTY		

NEGATIVE SIGNS AND PARENTHESIS

If a _________________ sign precedes parentheses, _________________ the parentheses and ________________ the ____________ of ________________ ________________ within the parentheses.

Example 6: Simplify.

1. $-4(-3x+2)$

2. $5(3y-1)-(14y-2)$

APPLICATIONS

Use the formula $C=\frac{5}{9}(F-32)$ to express each Fahrenheit temperature, *F*, as its equivalent Celsius temperature, *C.*

1. $-13^{\circ}\mathrm{F}$

2. $5^{\circ}\mathrm{F}$

Section 1.8: EXPONENTS AND ORDER OF OPERATIONS

When you are done with your homework you should be able to...

- π Evaluate exponential expressions
- π Simplify algebraic expressions with exponents
- π Use the order of operations agreement
- π Evaluate mathematical models

WARM-UP:

1. Determine whether the given number is a solution of the equation.

$$\frac{5m-1}{6}=\frac{3m-2}{4};\ -4$$

2. Write a numerical expression for each phrase. Then simplify the numerical expression.
   a. 14 added to the product of 4 and -10

   b. The quotient of -18 and the sum of -15 and 12

DEFINITION OF A NATURAL NUMBER EXPONENT

If *b* is a real number and *n* is a natural number,

_________ is read "the ________ __________ of ____" or "____ to the ____ power. The expression _______ is called an _______________ _______________.

Example 1: Evaluate.

1. $(-5)^3$

2. $(-12)^2$

ORDER OF OPERATIONS

1. Perform all _________________ within ________________ symbols
2. Evaluate all _________________ expressions.
3. Do all _________________________ and ___________________ in the order in which they occur, working from _______________ to _____________.
4. Finally, do all _________________ and __________________ using one of the following procedures:
   - Work from ______________ to ________________ and do additions and subtractions in the ________________ in which they occur.

or

π Rewrite subtractions as __________________ of __________________.

Combine _______________ and __________________ numbers

separately, and then ______________ these results.

Example 2: Simplify.

1. $40 \div 4 \cdot 2$

2. $\dfrac{-5(7-2)-3(4-7)}{-13-(-5)}$

3. $(3 \cdot 5)^2 - 3 \cdot 5^2$

4. $\left[-\dfrac{4}{7}-\left(-\dfrac{2}{5}\right)\right]\left[-\dfrac{3}{8}+\left(-\dfrac{1}{9}\right)\right]$

Example 3: Simplify each algebraic expression.

1. $-6x^2+18x^2$

2. $4\left(7x^3-5\right)-\left[2\left(8x^3-1\right)+1\right]$

3. $6-5\left[8-(2y-4)\right]$

APPLICATIONS

In Palo Alto, CA, a government agency ordered computer-related companies to contribute to a pool of money to clean up underground water supplies. (The companies had stored toxic chemicals in leaking underground containers). The mathematical model $C = \frac{200x}{100 - x}$ describes the cost, *C*, in tens of thousands of dollars, for removing *x* percent of the contaminants.

1. Find the cost, in tens of thousands of dollars, for removing 50% of the contaminants.

2. Find the cost, in tens of thousands of dollars, for removing 60% of the contaminants.

3. Describe what is happening to the cost of the cleanup as the percentage of contaminant removed increases.

Section 2.1: THE ADDITION PROPERTY OF EQUALITY

When you are done with your homework you should be able to…

- π Identify linear equations in one variable
- π Use the addition property of equality to solve equations
- π Solve applied problems using formulas

WARM-UP:

Simplify:

1. $\frac{1}{2}-\frac{2}{3}\div\frac{5}{9}+\frac{3}{10}$

2. $-40\div 5\cdot 2$

LINEAR EQUATIONS IN ONE VARIABLE

In Chapter 1, we learned that an ____________________ is a statement that two ____________________ expressions are _________________.We determined whether a given number is an equation's ________________ by substituting that number for each occurrence of the _________________. When the ____________________ resulted in a true statement, that ________________ was a ____________________. When the substituted number resulted in a ______________ statement, that number was ___________ a ________________.

VOCABULARY

Solving an equation: The ____________ of finding the ______________ (or _________________) that make the equation a __________ statement. These numbers are called the ________________ or ______________ of the equation, and we say that they ________________ the equation.

DEFINITION OF A LINEAR EQUATION IN ONE VARIABLE

A _____________ _______________ in ___________ ______________ _____ is an equation that can be written in the form

where ____, ____, and ____ are real numbers, and _____________.

Example 1: Give three examples of a linear equation in one variable.

1.

2.

3.

Example 2: Give two examples of a nonlinear equation in one variable.

1.

2.

VOCABULARY

<u>Equivalent equations</u>: Equations that have the _______________ solution are _______________ _________________.

THE ADDITION PROPERTY OF EQUALITY

The ____________ real number or ________________ expression may be __________ to ___________ sides of an _________________ without changing the equation's _____________________. That is,

Example 3: Solve the following equations. Check your solutions.

1. $y-5=-18$

2. $18+z=14$

3. $x+10.6=-9$

4. $-\frac{1}{8}+x=-\frac{1}{4}$

5. $-3x-5+4x=9$

6. $7x+3=6(x-1)+9$

ADDING AND SUBTRACTING VARIABLE TERMS ON BOTH SIDES OF AN EQUATION

Our goal is to __________________ all the ______________ terms on one side of the equation. We can use the ________________ ________________ of ________________ to do this.

APPLICATIONS

1. The cost, *C,* of an item (the price paid by a retailer) plus the markup, *M*, on that item (the retailer's profit) equals the selling price, *S*, of the item. The formula is $C + M = S$.

   The selling price of a television is $650. If the cost to the retailer for the television is $520, find the markup.

2. What is the difference between solving an equation such as $5y+3-4y-8=6+9$ and simplifying an algebraic expression such as $5y+3-4y-8$?

Section 2.2: THE MULTIPLICATION PROPERTY OF EQUALITY

When you are done with your homework you should be able to…

- π Use the multiplication property of equality to solve equations
- π Solve equations in the form of $-x = c$
- π Use the addition and multiplication properties to solve equations
- π Solve applied problems using formulas

WARM-UP:

Solve:

1. $5z - 12 = z + 8$

2. $x = -7(2 - x) + 18$

THE MULTIPLICATION PROPERTY OF EQUALITY

The ____________ ______________ real number or _______________ expression may _______________ __________ sides of an ________________ without changing the ____________________. That is,

Example 1: Solve the following equations. Check your solutions.

1. $-5z = -20$

4. $-\frac{1}{8}x = 6$

2. $-51 = -y$

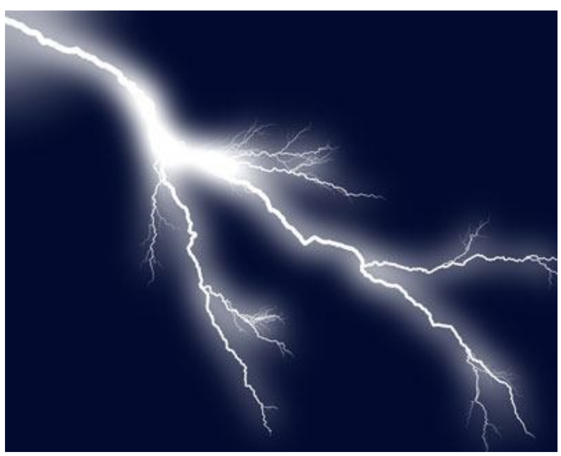
5. $6z - 3 = z + 2$

3. $8x - 3x = -45$

6. $5y + 6 = 3y - 6$

APPLICATIONS

The formula $M = \frac{n}{5}$ models your distance, *M,* from a lightning strike in a thunderstorm if it takes *n* seconds to hear thunder after seeing the lightning.



If you are three miles away from the lightning flash, how long will it take the sound of thunder to reach you?

Section 2.3: SOLVING LINEAR EQUATIONS

When you are done with your homework you should be able to…

- π Solve linear equations
- π Solve linear equations containing fractions
- π Identify equations with no solution or infinitely many solutions
- π Solve applied problems using formulas

WARM-UP:

Solve:

1. $-12z = 144$

2. $-x = -7x + 24$

A STEP-BY-STEP PROCEDURE FOR SOLVING LINEAR EQUATIONS

1. _____________ the _______________ ________________ on each side.

2. Collect all the _______________ terms on one side and all the _______________ terms on the other side.

3. _________________ the ___________________ and ______________.

4. _________________ the proposed solution in the ______________ equation.

Example 1: Solve the following equations. Check your solutions.

1. $-z - 34 + 10z = 2 + 10z - 54$

4. $3(x+2) = x + 30$

2. $20 = 44 - 8(2 - x)$

5. $2(x - 15) + 3x = (6 + 4x) - (9x - 2)$

3. $5x - 4(x + 9) = 2x + 3$

6. $100 = -(x - 1) + 4(x - 6)$

LINEAR EQUATIONS WITH FRACTIONS

Equations are ___________________ to solve when they do not contain ______________. To remove fractions, we can _________________ _________ sides of the equation by the ______________ ____________ _______________ of any fractions in the equation. Remember…the ___________ is the ___________________ number that all __________________ will _____________ into. This is often called "_________________ an equation of ________________".

Example 2: Solve the following equations. Clear the fractions first. Check your solutions.

1. $\frac{x}{2}+13=-22$

3. $\frac{3y}{4}-\frac{2}{3}=\frac{7}{12}$

2. $\frac{z}{5}-\frac{1}{2}=\frac{z}{6}$

4. $\frac{x-2}{3}-4=\frac{x+1}{4}$

RECOGNIZING INCONSISTENT EQUATIONS AND IDENTITIES

If you attempt to ______________ an equation with _______ ________________ or one that is _______________ for ______________ real number, you will _______________ the _______________.

- π An _________________ equation with _____ ____________________ results in a _______________ statement, such as _______________________.
- π An _________________ that is ____________ for ________________ real number results in a ______________ statement, such as _________________.

Example 3: Solve the following equations. Use words or set notation to identify equations that have no solution, or equations that are true for all real numbers. Check your solutions.

1. $2(x-5)=2x+10$

2. $5x-5=3x-7+2(x+1)$

3. $\frac{x}{2}+\frac{2x}{3}+3=x+3$

4. $\frac{x}{4}+3=\frac{x}{4}$

APPLICATIONS

The formula $p = 15 + \frac{5d}{11}$ describes the pressure of sea water, *p,* in pounds per square foot, at a depth of *d* feet below the surface.

1. The record depth for breath-held diving, by Francisco Ferreras (Cuba) off Grand Bahama Island, on November 14, 1993, involved pressure of 201 pounds per square foot. To what depth did Francisco descend on this venture? (He was underwater for 2 minutes and 9 seconds!)

2. At what depth is the pressure 20 pounds per square foot?

Section 2.4: FORMULAS AND PERCENTS

When you are done with your homework you should be able to…

- π Solve a formula for a variable
- π Express a percent as a decimal
- π Express a decimal as a percent
- π Use the percent formula
- π Solve applied problems involving percent change

WARM-UP:

Solve:

1. $4 = 0.25B$

2. $1.3 = P \cdot 26$

SOLVING A FORMULA FOR ONE OF ITS VARIABLES

Solving a formula for a variable means ___________________ the _____________ so that the ________________ is ____________________ on one side of the equation. To solve a formula for one of its variables, treat that ______________ as if it were the only _______________ in the ________________.

PERIMETER

The _______________ of a _________ ____________________ figure is the _________ of the _____________ of its _____________. Perimeter is measured in ____________ units, such as _______________, __________, ____________, or ___________________.

PERIMETER OF A RECTANGLE

The perimeter, _______, of a rectangle with length ____ and width ____ is given by the formula

SQUARE UNITS

A ______________ unit is a ____________, each of whose sides is __________ unit in length. The _____________ of a _______ _________________ figure is the number of _______________ ______________ it takes to fill the interior of the figure.

AREA OF A RECTANGLE

The area, _______, of a rectangle with length ____ and width ____ is given by the formula

Example 1: Solve the following formulas for the specified variable.

1. $d = rt;\ t$
2. $P = C + MC;\ C$

Example 2: Consider a rectangle which has an area of 15 square feet and a width of 3 feet.

1. Find the length.
2. Find the perimeter.

BASICS OF PERCENTS

_______________ are the result of _________________ numbers as _______ of ___________. The word ________________ means _______ ____________.

PERCENT NOTATION

______________ means _______________.

STEPS FOR EXPRESSING A PERCENT AS A DECIMAL NUMBER

1. Move the ______________ point ___________ places to the ____________.
2. Remove the _____________ sign.

Example 3: Express each percent as a decimal.

1. 9.5%
2. 235%

STEPS FOR EXPRESSING A DECIMAL NUMBER AS A PERCENT

1. Move the ______________ point ___________ places to the ____________.

2. Attach a _____________ sign.

Example 4: Express each decimal as a percent.

1. 1.75
2. 0.01

A FORMULA INVOLVING PERCENT

_________________ are useful in comparing two _____________. To _______________ the number _____ to the number _____ using a percent ____, the following formula is used:

Example 5: Solve.

1. What is 12% of 50?
2. 6 is 30% of what?
3. 200 is what percent of 20?

PERCENT INCREASE **PERCENT DECREASE**

APPLICATIONS

1. The average, or mean, *A*, of four exam grades, *x, y, z,* and *w,* is given by the formula $A = \dfrac{x+y+z+w}{4}$.

   a. Solve the formula for *w*.

   b. Use the formula in part (a) to solve this problem: On your first three exams, your grades are 76%, 78%, and 79%: $x = 76,\ y = 78,\ \text{and } z = 79$. What must you get on the fourth exam to have an average of 80%?

2. A charity has raised $225,000, with a goal of raising $500,000. What percent of the goal has been raised?

3. Suppose that the local sales tax rate is 7% and you buy a graphing calculator for $96.
   a. How much tax is due?

   b. What is the calculator's total cost?

4. The price of a color printer is reduced by 30% of its original price. When it still does not sell, its price is reduced by 20% of the reduced price. The salesperson informs you that there has been a total reduction of 50%. Is the salesperson using percentages properly? If not, what is the actual percent reduction from the original price?

Section 2.5: AN INTRODUCTION TO PROBLEM SOLVING

When you are done with your homework you should be able to…

- π Translate English phrases into algebraic expressions
- π Solve algebraic word problems using linear equations

WARM-UP:

Solve:

A fax machine regularly sells for $380. The sale price is $266. Find the percent decrease in the machine's price.

STEPS FOR SOLVING WORD PROBLEMS

1. Analysis: READ the problem. Then, ________________ the problem again!!!

   Draw a ____________________ and/or make a _____________. Identify

   and name all known and unknown __________________.

2. Translate to Mathese: Write an equation that translates, or ___________,

   the conditions of the problem.

3. Solve: __________________ the equation. Then ______________ your

   solution.

4. Conclusion: Write your result, in _________________.

Example 1: Solve the following word problems.

1. The sum of a number and 28 is 245. Find the number.

2. Three times the sum of five and a number is 48. Find the number.

3. Eight subtracted from six times a number is 298. Find the number.

4. If the quotient of three times a number and four is decreased by three, the result is nine. Find the number.

5. A car rental agency charges $180 per week plus $0.25 per mile to rent a car. How many miles can you travel in one week for $395?

6. A basketball court is a rectangle with a perimeter of 86 meters. The length is 13 meters more than the width. Find the width and length of the basketball court.

7. This year's salary, $42,074, is a 9% increase over last year's salary. What was last year's salary?

8. A repair bill on a sailboat came to $1603, including $532 for parts and the remainder for labor. If the cost of labor is $35 per hour, how many hours of labor did it take to repair the sailboat?

Section 2.6: PROBLEM SOLVING IN GEOMETRY

When you are done with your homework you should be able to…

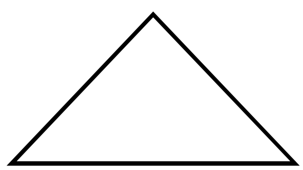
- π Solve problems using formulas for perimeter and area
- π Solve problems using formulas for a circle's area and circumference
- π Solve problems using formulas for volume
- π Solve problems involving the angles of a triangle
- π Solve problems involving complementary and supplementary angles

WARM-UP:

Solve:

After a 30% reduction, you purchase a DVD player for \$98. What was the selling price before the reduction?

COMMON FORMULAS FOR PERIMETER AND AREA

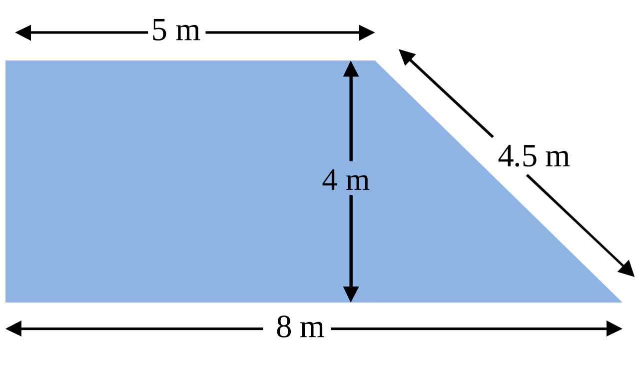


Example 1: Solve.

1. A triangle has a base of 6 feet and an area of 30 square feet. Find the triangle's height.

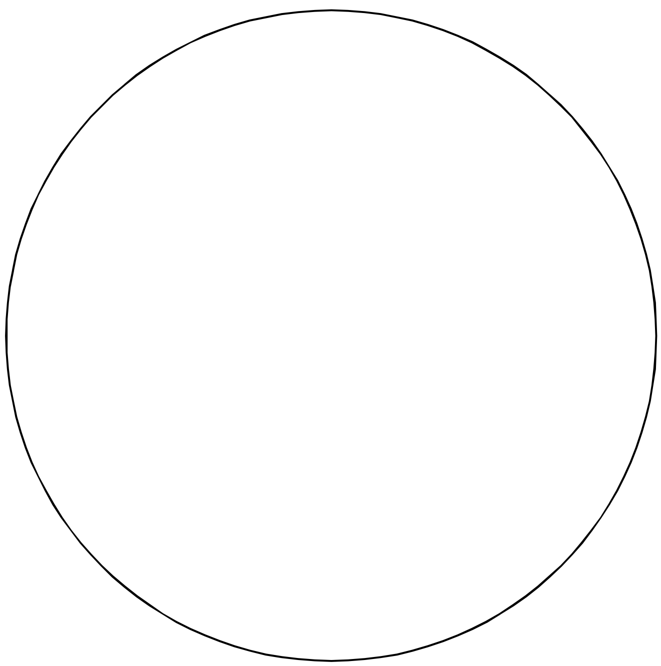
2. A rectangle has a width of 46 cm and a perimeter of 208 cm. What is the rectangle's length?

3. Find the area of the trapezoid.



GEOMETRIC FORMULAS FOR CIRCUMFERENCE AND AREA OF A CIRCLE

A ____________________ is the set of all _________________ in the __________ equally distant from a given point, its _________________. A ______________ (plural ________________), ______, is a line ________________ from the ________________ to any point on the _________________. For a given circle, ___________ radii have the same ________________. A __________________, ____, is a _________ segment through the _______________ whose endpoints both lie on the _______________. For a given circle, all _________________ have the _____________ length. In any circle, the length of a __________________ is ________________ the length of a _______________ and the length of a ________________ is _______________ the length of a ___________________.



Area **Circumference**

Example 2: Solve.

1. Find the area and circumference of a circle which has a diameter of 40 feet.

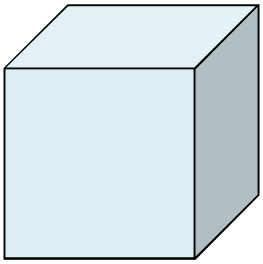
2. Which one of the following is a better buy: a large pizza with a 16-inch diameter for $12 or two small pizzas, each with a 10-inch diameter, for $12?

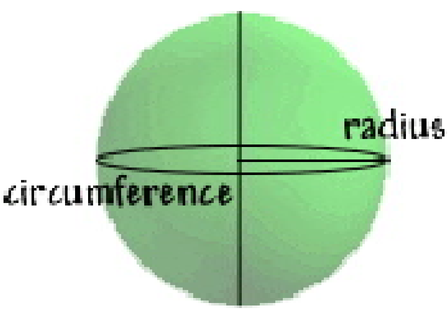
GEOMETRIC FORMULAS FOR VOLUME

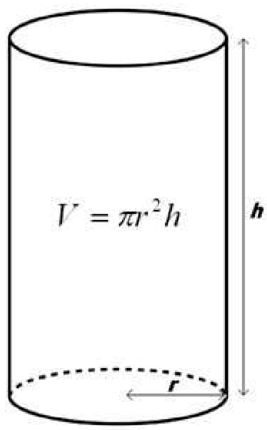
_______________ refers to the amount of _____________ occupied by a

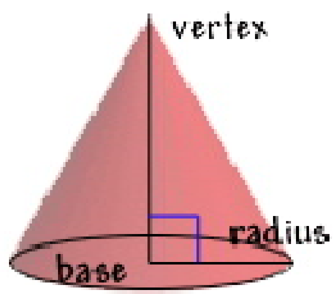
___________-___________________ figure. To measure this space, we use

____________ units.









Example 3: Solve.

1. Solve the formula for the volume of a cone for h.

2. A cylinder with radius 2 inches and height 3 inches has its radius quadrupled. How many times greater is the volume of the larger cylinder than the smaller cylinder?

3. Find the volume of a shoebox with dimensions 6 in x 12 in x 5 in.

THE ANGLES OF TRIANGLES

An _______________, symbolized by _________, is made up of two _________ that have a common ________________. The common endpoint is called the ________________. The two rays that form the angle are called its _________.

One way to ___________________ angles is in _______________, symbolized by a small, raised _____________ ______. There are ____________ in a circle. ______ is _____________ of a complete rotation.

THE ANGLES OF A TRIANGLE

The ____________ of the ________________ of the three angles of __________ triangle is ___________.

COMPLEMENTARY AND SUPPLEMENTARY ANGLES

Two angles with measures having a _________ of ________ are called _____________________ angles. Two angles with measures having a _________ of ____________ are called ___________________ ________________.

Example 4: Solve.

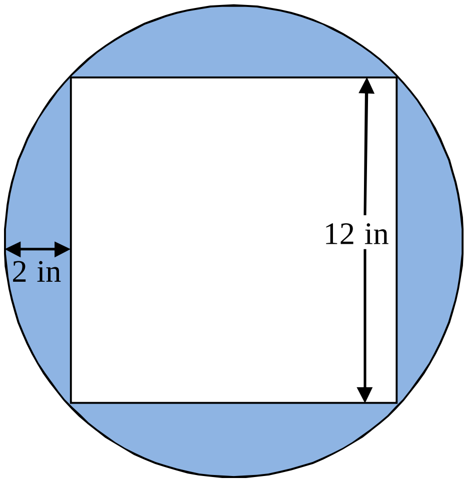
1. One angle of a triangle is three times as large as another. The measure of the third angle is 40º more than that of the smallest angle. Find the measure of each angle.

2. Find the measure of the complement of each angle.
   a. 56º
   b. 89.5º

3. Find the measure of the supplement of each angle.
   a. 177º
   b. 0.2º

4. Find the measure of the angle described.
   The measure of the angle's supplement is 52º more than twice that of its complement.

Example 5: Find the area of the shaded region.



Section 2.7: SOLVING LINEAR INEQUALITIES

When you are done with your homework you should be able to…

- π Graph the solutions of an inequality on a number line
- π Use interval notation
- π Understand properties used to solve linear inequalities
- π Solve linear inequalities
- π Identify inequalities with no solution of infinitely many solutions
- π Solve problems using linear inequalities

WARM-UP:

Solve:

Find the volume of a sphere with diameter 11 meters.

VOCABULARY

Linear inequality in one variable: An inequality in the form _________________,

_____________________, _____________________, or ____________________

is a linear inequality in one variable. ________ means ________ ________,

________ means ________ ________ or __________ ______, ________ means

________ ________, and ________ means ________ ________ or __________

______.

Solving an inequality: The ______________ of finding the ______ of ______________ that will make the inequality a __________ statement. These numbers are called the **solutions** of the ______________, and we say they **satisfy** the __________________. The ___________ of ________ solutions is called the **solution set** of the inequality.

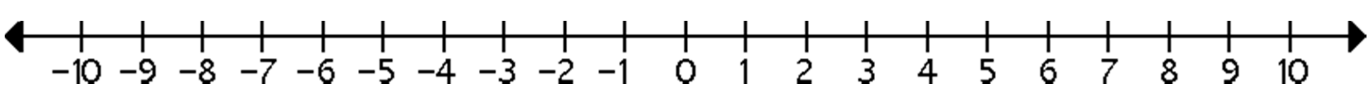
GRAPHS OF INEQUALITIES

There are ___________________ ______________ solutions to the inequality $x > 5$. In other words, the solution set for this inequality is all ___________ numbers which are ______________ ____________ ____________. Can we list all these numbers? What does the graph of the solution set look like? Hmmmm…

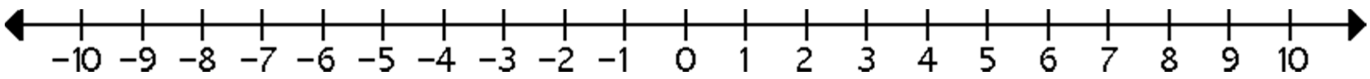
Graphs of __________________ to ______________ __________________ are shown on a _____________ __________ by shading _________ _____________ representing numbers that are __________________. _____________ _____________, ________, indicate ________________ that are _____________ and _________________, _______, indicate ______________ that are __________ __________________.

Example 1: Graph the solutions of each inequality.

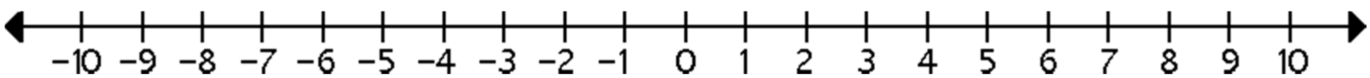
a. $x \leq 6$



b. $x > -\frac{3}{2}$



c. $-\frac{3}{2} < x \leq 6$



SOLUTION SETS OF INEQUALITIES

INEQUALITY	INTERVAL NOTATION	SET-BUILDER NOTATION	GRAPH
$x > a$			
$x \geq a$			
$x < b$			
$x \leq b$			
$a < x < b$			
$a \leq x \leq b$			
$a < x \leq b$			
$a \leq x < b$			

PARENTHESIS ARE ALWAYS USED WITH __________ OR ___________!!!

PROPERTIES OF INEQUALITIES

PROPERTY	THE PROPERTY IN WORDS	EXAMPLE
THE ADDITION PROPERTY OF INEQUALITY If ___________, then ____________. If ___________, then ____________.		
THE POSITIVE MULTIPLICATION PROPERTY OF INEQUALITY If ___________ and ___ is positive, then ____________. If ___________ and ___ is positive, then ____________.		
THE NEGATIVE PROPERTY OF INEQUALITY If ___________ and ___ is negative, then ____________. If ___________ and ___ is negative, then ____________.		

STEPS FOR SOLVING A LINEAR INEQUALITY

1. Simplify the _______________ _________________ on each side.

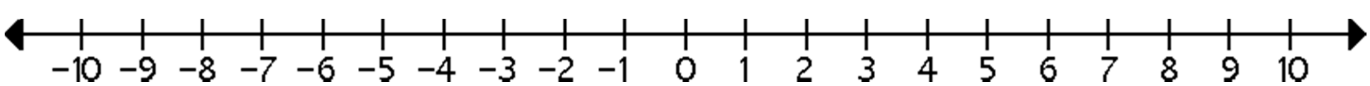
2. Use the ________________ property of __________________ to collect all the _________________ terms on one side and all the ________________ terms on the other side.

3. Use the ___________________ property of __________________ to _________________ the _________________ and ___________. _______________ the _______________ of the _________________ when _____________________ or ______________________ both sides by a __________________ number.

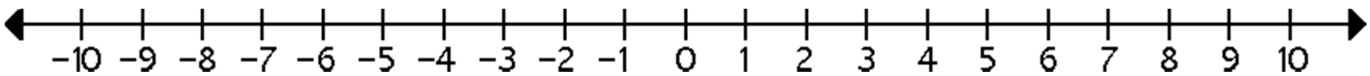
4. Express the _________________ set in ______________ or ______-______________ notation, and ______________ the solution set on a ________________ line.

Example 2: Solve each inequality and graph the solution.

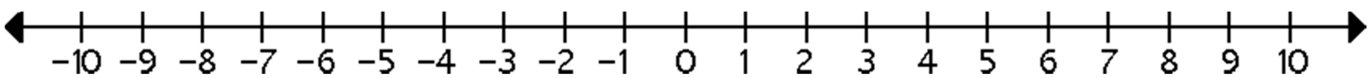
a. $x - 3 \leq 2$



b. $5x + 8 > 2x - 7$



c. $4(x+1) \geq 3x + 6$



RECOGNIZING INEQUALITIES WITH NO SOLUTION OR INFINITELY MANY SOLUTIONS

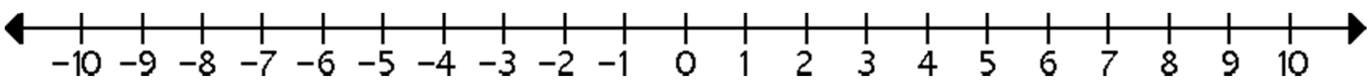
If you attempt to solve an inequality with _____ _________________ or one that is __________ for _______________ __________ number, you will _________________ the ____________________.

- An inequality with ______ _____________________ results in a _____________ statement, such as __________. The solution set is _______ or _______, the ____________ set, and the _______________ is an ________________ number line.

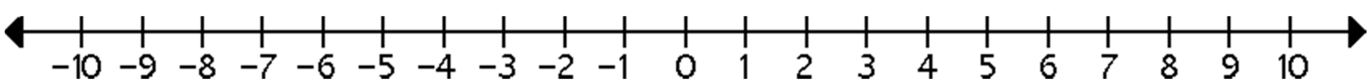
- π An inequality that is _______ for ___________ ______________ number results in a ________ statement, such as ___________. The solution set is ___________ or ______________________, and the graph is a ____________ _________________ ___________________line.

Example 3: Solve each inequality and graph the solution.

a. $2(x+1)-1<2x+1$



b. $5x>2(x-7)+3x$



APPLICATION

On three examinations, you have grades of 88, 78, and 86. There is still a final examination, which counts as one grade.

1. In order to get an A, your average must be at least 90. If you get 100 on the final, compute your average and determine if an A in the course is possible.

2. To earn a B in the course, you must have a final average of at least 80. What must you get on the final to earn a B in the course?

Section 3.1: GRAPHING LINEAR EQUATIONS IN TWO VARIABLES

When you are done with your homework you should be able to…

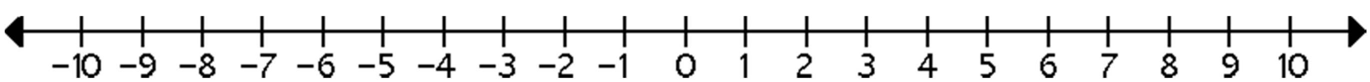
- π Plot ordered pairs in the rectangular coordinate system
- π Find coordinates of points in the rectangular coordinate system
- π Determine whether an ordered pair is a solution of an equation
- π Find solutions of an equation in two variables
- π Use point plotting to graph linear equations
- π Use graphs of linear equations to solve problems

WARM-UP:

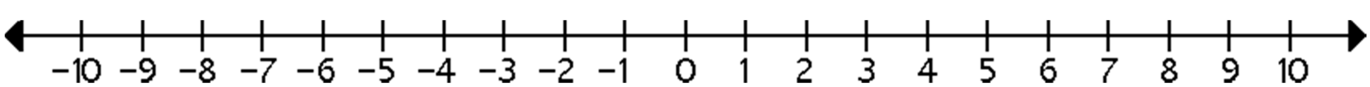
1. Find the volume of a box with dimensions ½ ft by 3 ft by 8 ft.

2. Solve the following inequalities and graph the solution sets.

   a. $x \le 6(3x-5)$

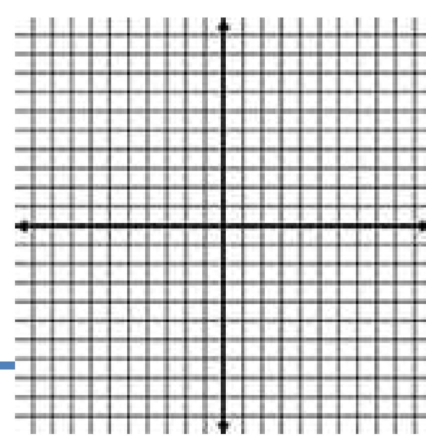


   b. $2x-1 \le 2x$



POINTS AND ORDERED PAIRS

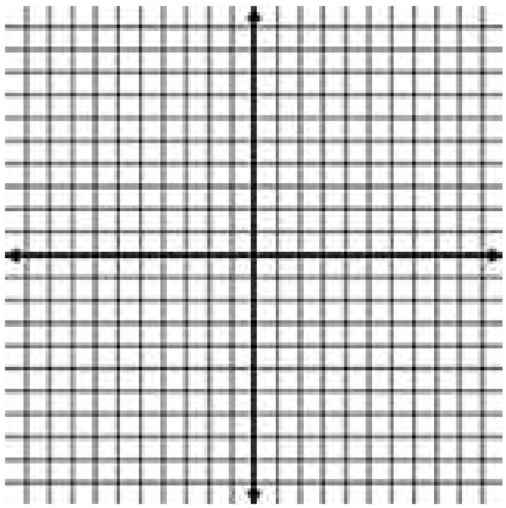
The idea of visualizing equations as geometric figures was developed by the French philosopher and mathematician _____________ ________________. This idea is the _________________ __________________ system or the ______________________ coordinate system. The rectangular coordinate system consists of ________ ______________ lines that _______________ at right ____________ at their ____________ points. The horizontal number line is the ______________ and the vertical number line is the _____________. The point of intersection is a ________________ called the ______________. Positive numbers are to the ______________ and _______________ the origin. Negative numbers are to the _____________ and _____________ the origin. The ________ divide the ___________ into ___________ regions, called _______________. The points located on the ______________ are __________ in any quadrant. Each ______________ in the rectangular coordinate system ________________ to an ______________ ___________ of real numbers, __________. The ________ number in each pair, called the ________________, denotes the _____________ and _______________ from the _____________ along the ______________. The second number, called the ____________________, denoted the ______________ distance along a ________ __________________ to the _______________ or along the ________________ itself.



Example 1: Plot the following ordered pairs.

$(2,5),\ (-3,7),\ (-2,-4)$

(2,5)	
(-3,7)	
(-2,-4)	



SOLUTIONS OF EQUATIONS IN TWO VARIABLES

A ____________________ of an __________________ in __________ variables, _____ and _____, is an ________________ __________ of real numbers with the following property: When the __________________ is substituted for ______ and the __________________ is substituted for ______ in the equation, we obtain a __________ statement.

Example 2: Determine whether each of the given points is a solution of the equation $8x + y = 1$.

a. $(0,1)$ b. $(-1,3)$ c. $(2,-15)$

Example 3: Find three solutions of $2y = -x - 1$.

GRAPHING LINEAR EQUATIONS IN THE FORM $y = mx + b$

The ___________ of the _______________ is the ________ of all _____________ whose _________________ satisfy the equation.

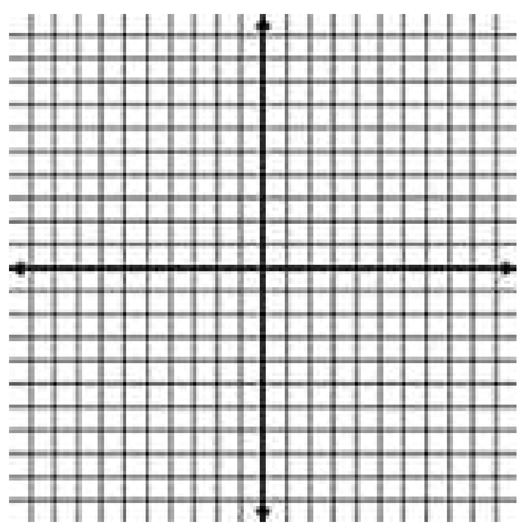
STEPS FOR USING THE POINT-PLOTTING METHOD FOR GRAPHING AN EQUATION IN TWO VARIABLES

1. Find several ______________ ___________ that are ________________ of the equation.
2. Plot these ordered pairs as _______________ in the ___________________ coordinate system.
3. _________________ the points with a ________________ curve or ________, depending on the type of equation.

Example 3: Graph the following equations by plotting points.

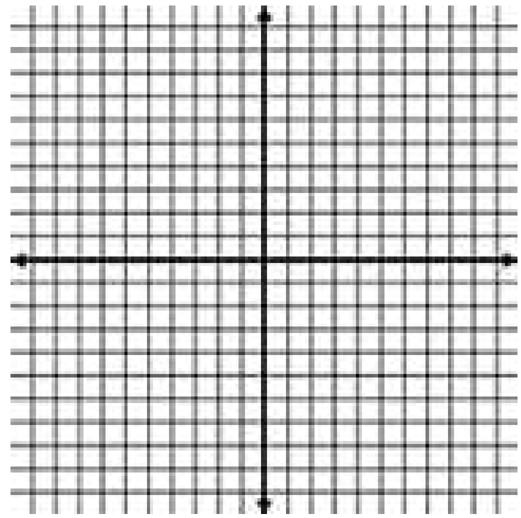
a. $y = 2x$

x	$y = 2x$	(x, y)



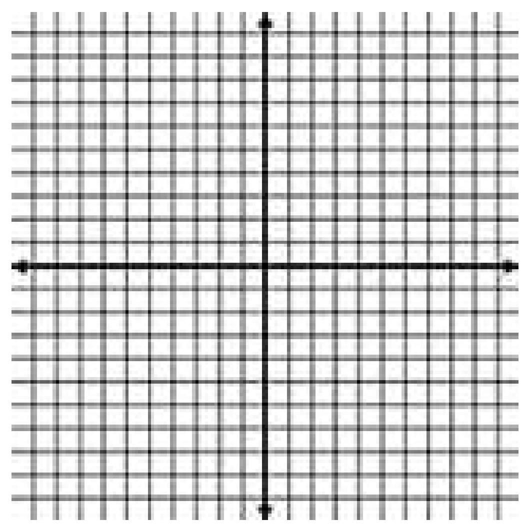
b. $y = -3x + 9$

x	$y = -3x + 9$	(x, y)



c. $y = \frac{2}{5}x + 3$

x	$y = \frac{2}{5}x + 3$	(x, y)



COMPARING GRAPHS OF LINEAR EQUATIONS

If the value of ______ does not change,

- The graph of ________________ is the graph of __________ shifted ____ units ____ when ____ is a positive number.
- The graph of ________________ is the graph of __________ shifted ____ units ____ when ____ is a positive number.

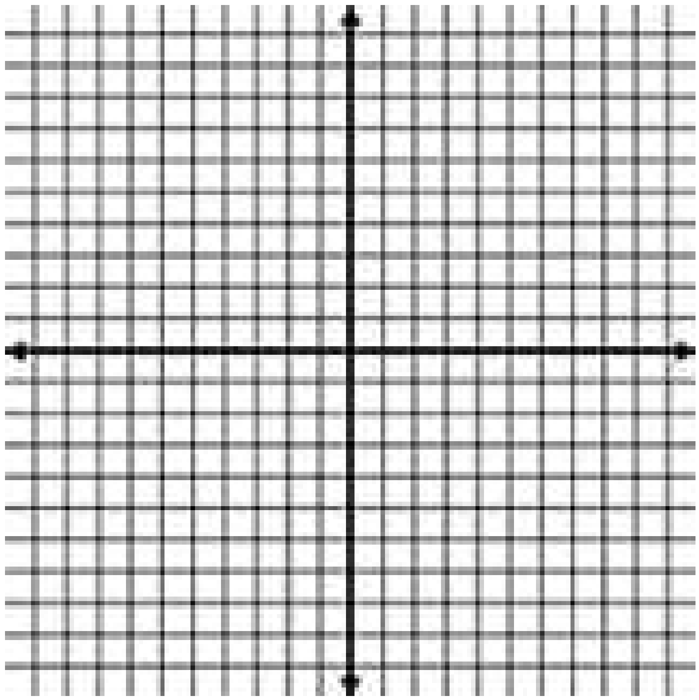
APPLICATION

In 1960, per capita fish consumption was 10 pounds. This increased by approximately 0.15 pound per year from 1960 through 2005. These conditions can be described by the mathematical model $F = 0.15n + 10$, where *F* is per capita fish consumption *n* years after 1960.

a. Let *n* = 0, 10, 20, 30, and 40. Make a table of values showing five solutions of the equation.

n	$F = 0.15n + 10$	(n, F)

b. Graph the formula in a rectangular coordinate system.



c. Use the graph to estimate per capita fish consumption in 2020.

d. Use the formula to project per capita fish consumption in 2020.

Section 3.2: GRAPHING LINEAR EQUATIONS USING INTERCEPTS

When you are done with your homework you should be able to…

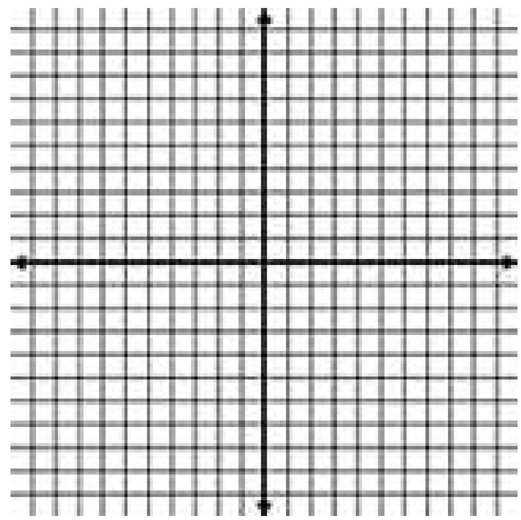
- π Use a graph to identify intercepts
- π Graph a linear equation in two variables using intercepts
- π Graph horizontal or vertical lines

WARM-UP:

Graph the following equations by plotting points.

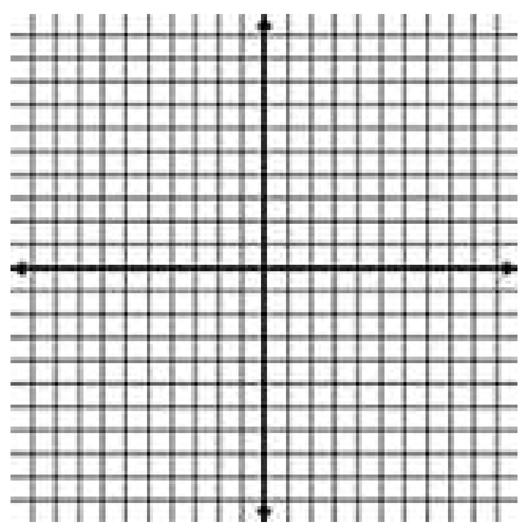
a. $y=-x$

x	$y=-x$	(x,y)



b. $y=\frac{2}{3}x-7$

x	$y=\frac{2}{3}x-7$	(x,y)



INTERCEPTS

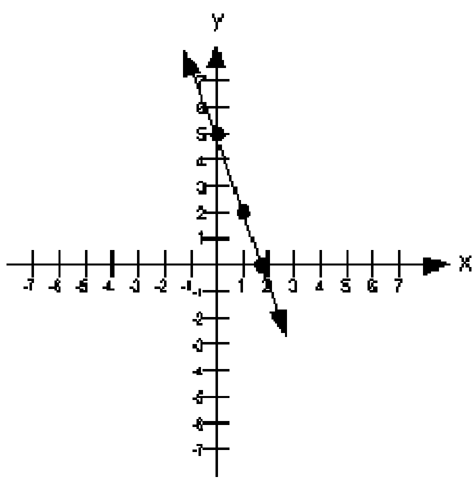
An _________________ of a graph is the __________________ of a point where the graph __________________ the _____________. **The _________________ corresponding to an _________________ is always ________!!!**

A _________________ of a graph is the __________________ of a point where the graph __________________ the _____________. **The _________________ corresponding to a _________________ is always ________!!!**

Example 1: Use the graph to identify the

a. *x*-intercept

b. *y*-intercept



GRAPHING USING INTERCEPTS

An equation of the form __________________, where ____, ____, and ____ are integers, is called the _______________ form of a line.

STEPS FOR USING INTERCEPTS TO GRAPH $Ax+By=C$

1. Find the ____________________. Let ________________ and solve for ____.

2. Find the ____________________. Let ________________ and solve for ____.

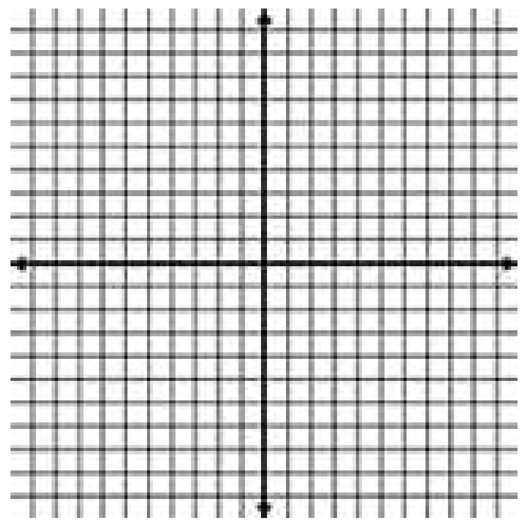
3. Find a checkpoint, a _____________ ordered-pair __________________.

4. Graph the equation by drawing a ______________ through the ______ points.

Example 2: Graph using intercepts and a checkpoint.

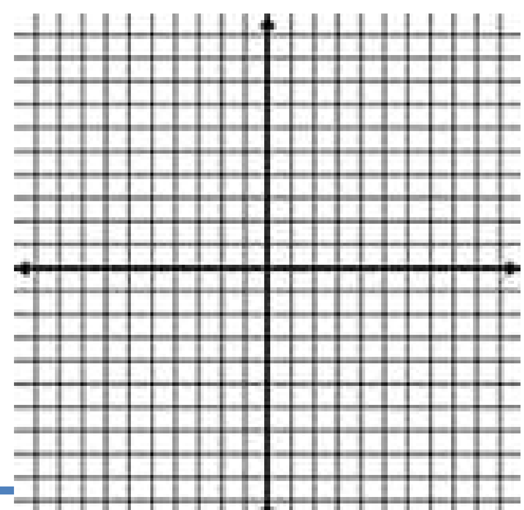
a. $x+y=6$

x	$x+y=6$	(x,y)



b. $3x-2y=-7$

x	$3x-2y=-7$	(x,y)



EQUATIONS OF HORIZONTAL AND VERTICAL LINES

We know that the graph of any equation of the form ___________________ is a _____________ as long as ____ and ____ are not both _______. What happens if ____ or ____, but not both, is zero?

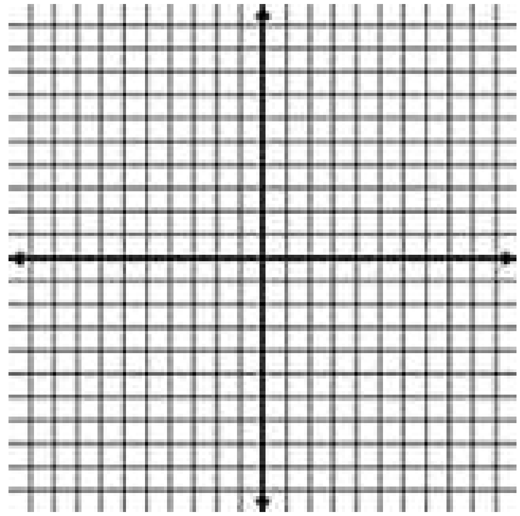
HORIZONTAL AND VERTICAL LINES

The graph of _____________ is a ________________ line. The _____________ is ______________.

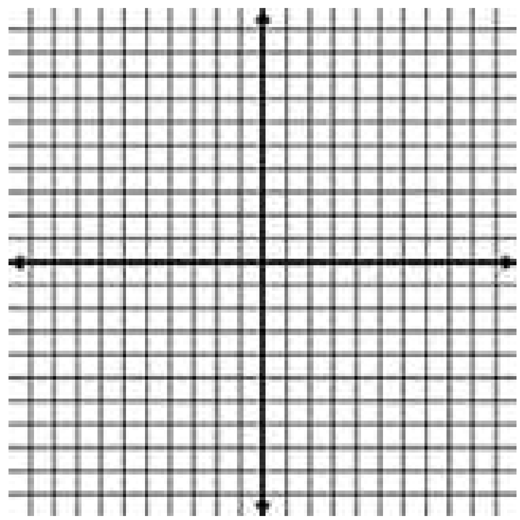
The graph of _____________ is a ________________ line. The _____________ is ______________.

Example 3: Graph.

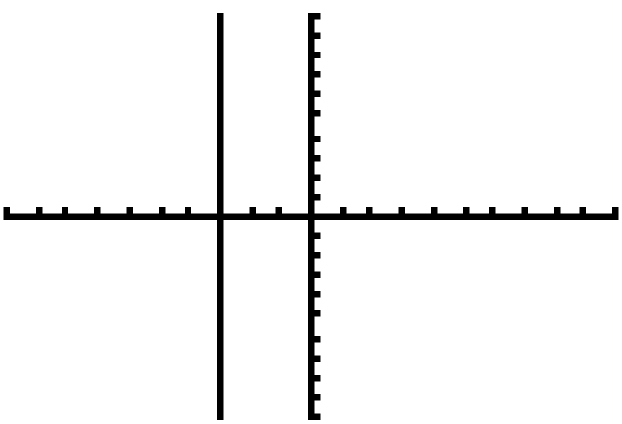
a. $y = 8$



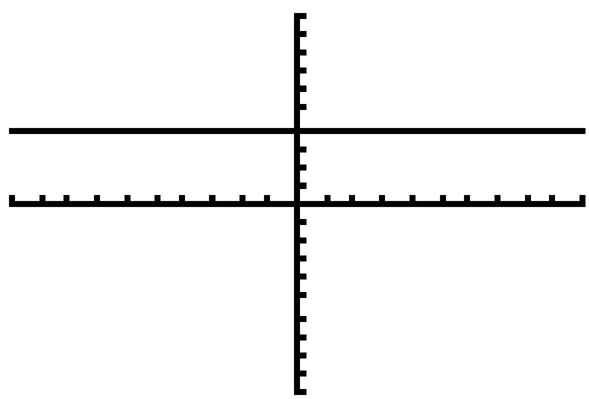
b. $12x = -60$



Example 4: Write an equation for each graph.



a.



b.

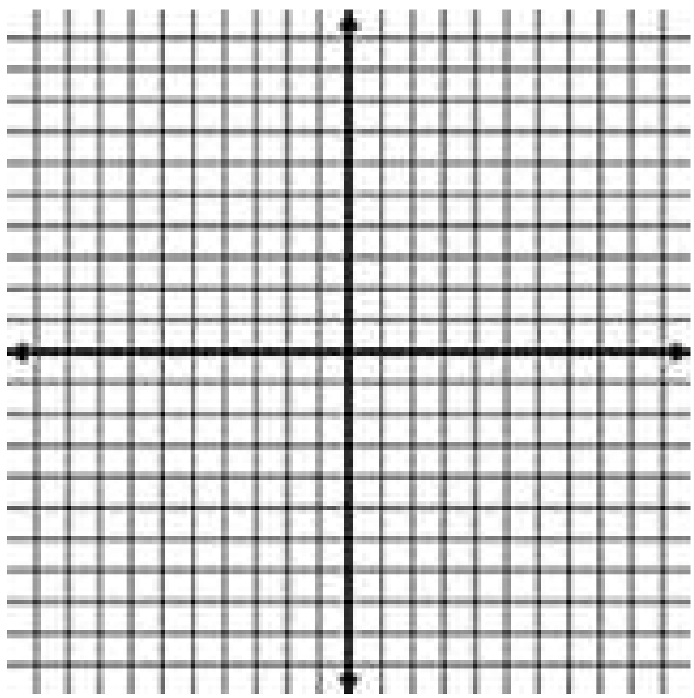
APPLICATION

A new car worth $24,000 is depreciating in value by $3000 per year. The mathematical model $y = -3000x + 24000$ describes the car's value, *y,* in dollars, after *x* years.

a. Find the *x*-intercept. Describe what this means in terms of the car's value.

b. Find the *y*-intercept. Describe what this means in terms of the car's value.

c. Use the intercepts to graph the linear equation.



d. Use your graph to estimate the car's value after five years.

Section 3.3: SLOPE

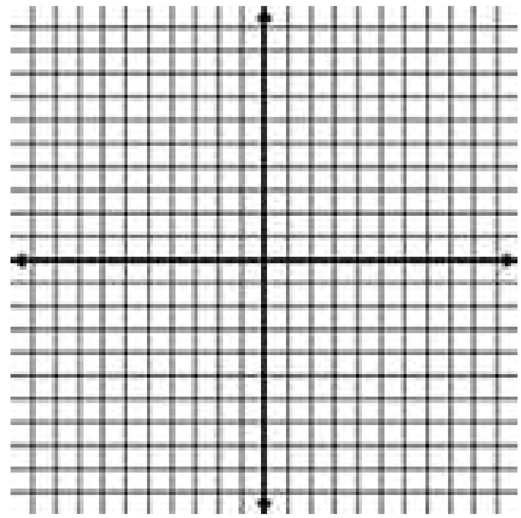
When you are done with your homework you should be able to…

- π Compute a line's slope
- π Use slope to show that lines are parallel
- π Use slope to show that lines are perpendicular
- π Calculate rate of change in applied situations

WARM-UP:

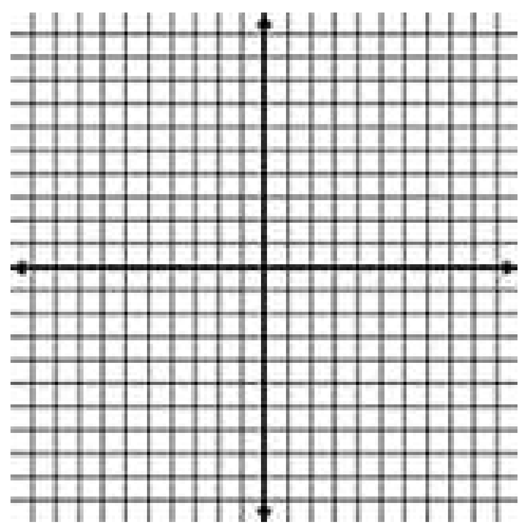
Graph each equation.

a. $y-2=0$



b. $-2x-3y=9$

x	$-2x-3y=9$	(x, y)



THE SLOPE OF A LINE

Mathematicians have developed a useful _______________ of the _________________ of a line, called the ______________ of the line. Slope compares the ____________________ change (the __________) to the ___________________ change (the ________) when moving from one __________ point to another along the line.

DEFINITION OF SLOPE

The ______________ of the line through the distinct points ___________ and ____________ is

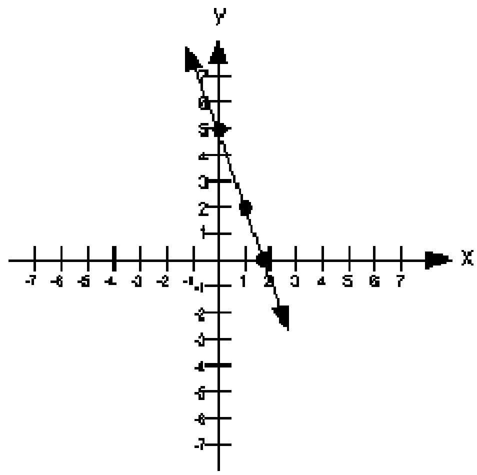
where _________________. It is common to use the letter _____ to represent the slope of a line. This letter is used because it is the first letter of the French verb monter, meaning to rise, or to ascend.

Example 1: Find the slope of the line passing through each pair of points:

a. $(-1,4)$ and $(3,-6)$

b. $\left(8,\frac{3}{2}\right)$ and $\left(-\frac{5}{2},7\right)$

Example 2: Use the graph to find the slope of the line



POSSIBILITIES FOR A LINE'S SLOPE

POSITIVE SLOPE	NEGATIVE SLOPE	ZERO SLOPE	UNDEFINED SLOPE

SLOPE AND PARALLEL LINES

Two _______________________ lines that lie in the same plane are _______________. If two lines do not ________________, the ____________ of the ________________ change to the ___________________ change is the _____________ for each __________. Because two parallel lines have the same _________________, they must have the same ________________.

1. If two nonvertical lines are ________________, then they have the same ______________.
2. If two distinct nonvertical lines have the same _____________, then they are ___________________.
3. Two distinct vertical lines, each with ________________ slope, are _________________.

SLOPE AND PERPENDICULAR LINES

Two lines that _______________________ at a ___________ _____________ (_____) are said to be _______________________.

1. If two nonvertical lines are ________________, then the ______________ of their ______________ is __________.
2. If the _________________ of the ______________ of two lines is _____, then the lines are ______________________.

3. A ______________________ line having __________ slope is

______________________ to a vertical line having ______________ slope.

Example 3: Determine whether the lines through each pair of points are parallel, perpendicular, or neither.

a. $(-2,-15)$ and $(0,-3)$; $(-12,6)$ and $(6,3)$

b. $(-2,-7)$ and $(3,13)$; $(-1,-9)$ and $(5,15)$

c. $(-1,-11)$ and $(0,-5)$; $(0,-8)$ and $(12,-6)$

APPLICATION

Construction laws are very specific when it comes to access ramps for the disabled. Every vertical rise of 1 foot requires a horizontal run of 12 feet. What is the grade of such a ramp? Round to the nearest tenth of a percent.

Section 3.4: THE SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

When you are done with your homework you should be able to…

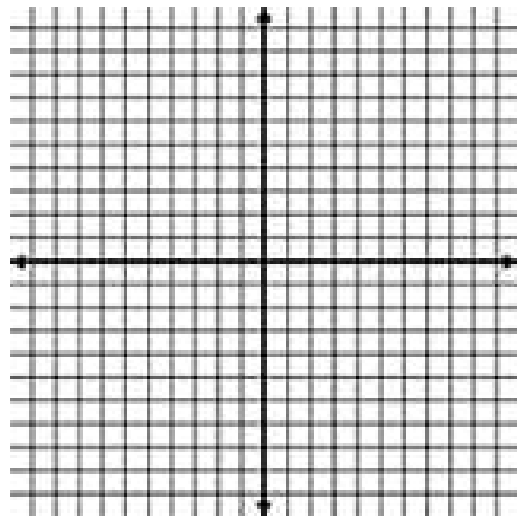
- Find a line's slope and *y*-intercept from its equation
- Graph lines in slope-intercept form
- Use slope and *y*-intercept to graph $Ax + By = C$
- Use slope and *y*-intercept to model data

WARM-UP:

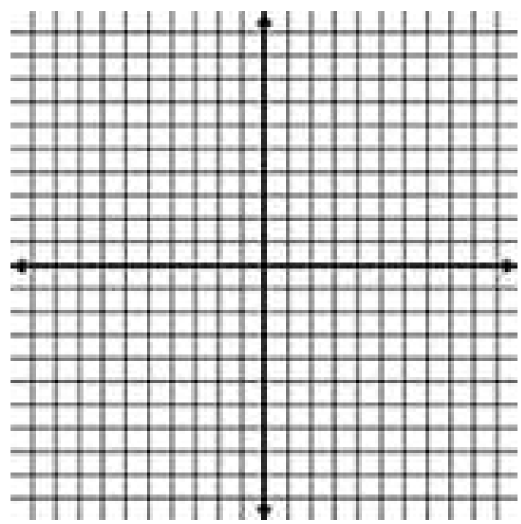
Graph each equation.

a. $4x - 8y - 2 = 0$

x	$4x - 8y - 2 = 0$	(x, y)



b. The line which passes through the points $(-1, 2)$ and $(3, 0)$.



SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

The ______________ - _________________ form of the __________________ of a nonvertical line with slope _______ and ________________ _____ is

Example 1: Find the slope and the *y*-intercept of the line with the given equation:

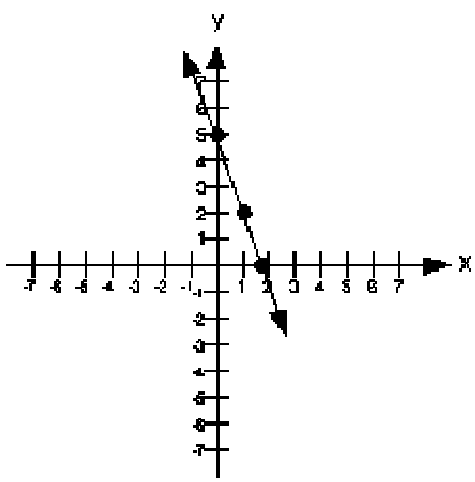
a. $y = -4x - 1$

b. $6x - y = -1$

c. $y = \frac{5}{7}x + 2$

d. $y = -\frac{x}{3} + \frac{2}{3}$

Example 2: Use the graph to find the equation of the line in slope-intercept form.



GRAPHING BY USING $y = mx + b$ SLOPE AND *Y*-INTERCEPT

1. Plot the point containing the ___________________ on the _____ axis.

   This is the point ___________.

2. Obtain a second ______________ using the _____________, _____. Write

   _____ as a _______________, and use ___________ over _________,

   starting at the ________________.

3. Use a __________________ to draw a ______________ through the two

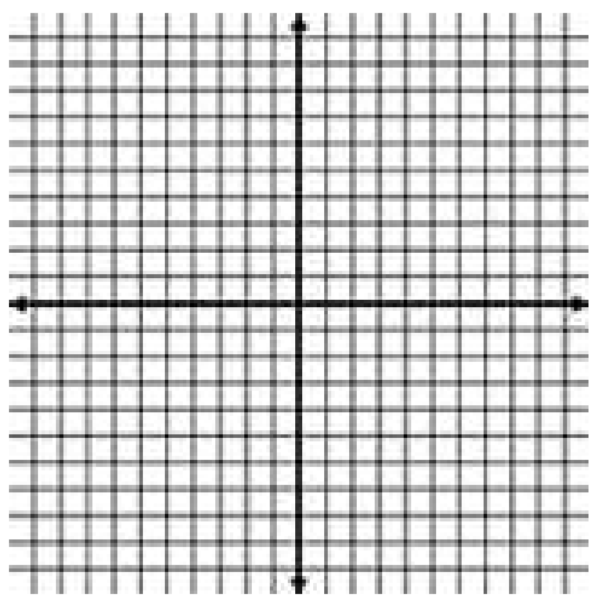
   _____________. Draw _________________________ at the ___________

   of the line to show that the line continues __________________ in both

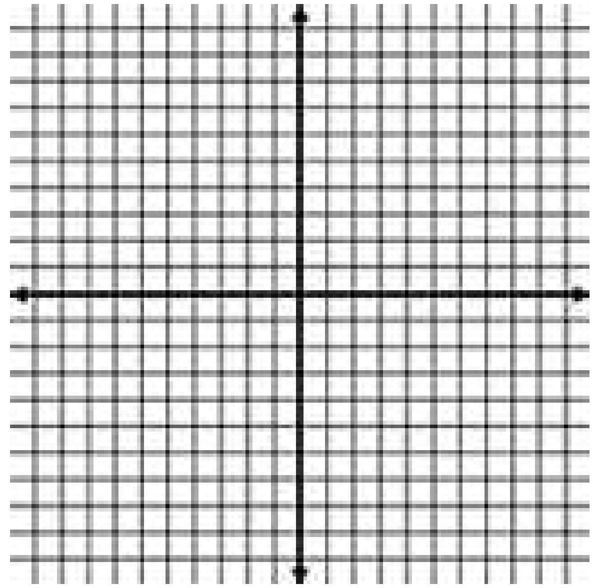
   directions.

Example 3: Graph using the slope and *y*-intercept.

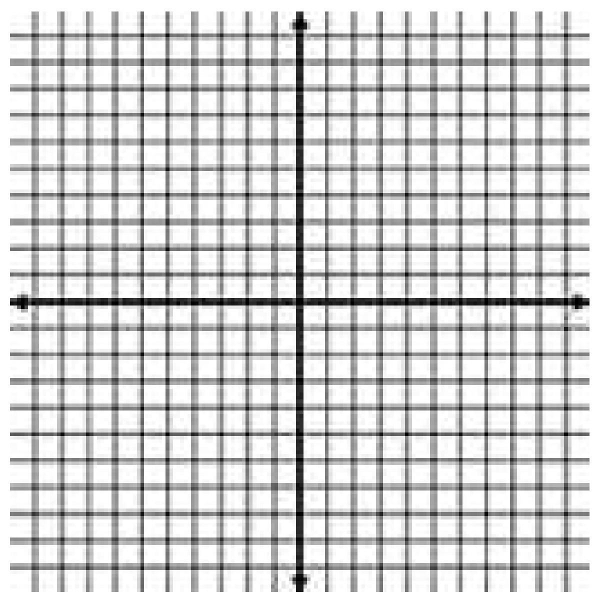
a. $y = -5x + 3$



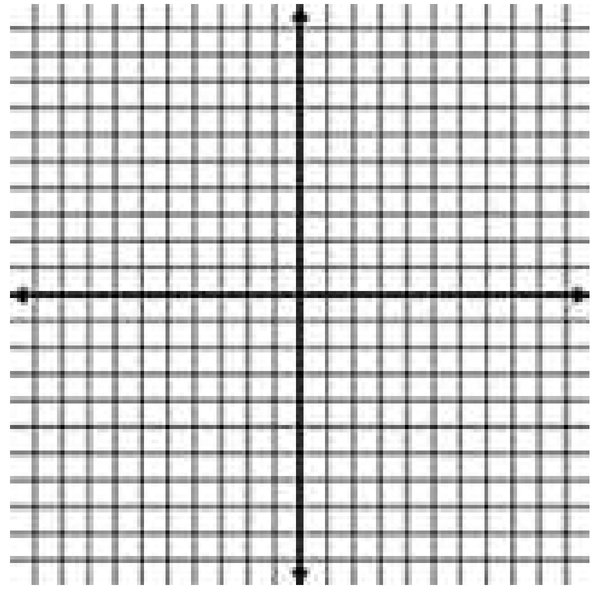
b. $10x-5y=25$



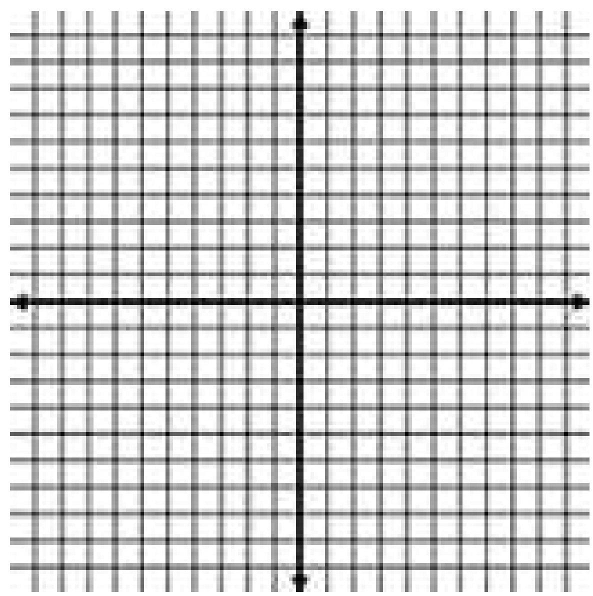
c. $x=2y-3$



d. $-y = x - 1$



e. $y = -\frac{6}{7}x + 4$



APPLICATION

Write an equation in the form of $y = mx + b$ of the line that is described.

1. The *y*-intercept is -4 and the line is parallel to the line whose equation is $2x + y = 8$.

2. The line falls from left to right. It passes through the origin and a second point with opposite *x*- and *y*-coordinates.

Section 3.5: THE POINT-SLOPE FORM OF THE EQUATION OF A LINE

When you are done with your homework you should be able to…

- π Use the point-slope form to write equations of a line
- π Find slopes and equations of parallel and perpendicular lines
- π Write linear equations that model data and make predictions

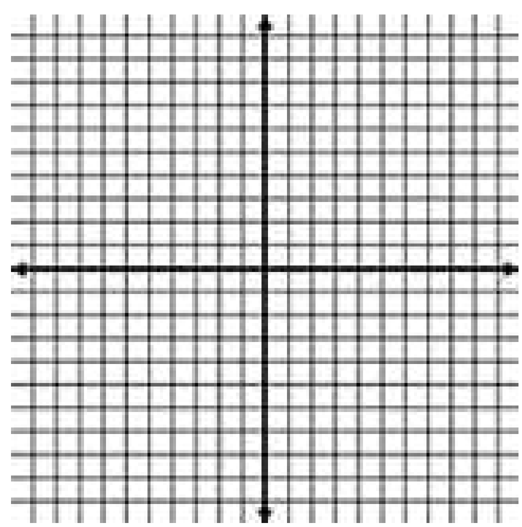
WARM-UP:

1. Simplify.

$$2-5\left[2-\left(7x+2\right)\right]$$

2. Graph the equation using the slope and *y*-intercept.

$$-\frac{x}{3}-\frac{y}{4}=1$$



POINT-SLOPE FORM

We can use the ________________ of a line to obtain another useful form of the line's equation. Consider a nonvertical line that has slope _____ and contains the point ______________. Now let _________ represent any other ____________ on the ______________. Keep in mind that the point _____________ is _________________ and is ______________ in _____________ _____________ position. The point ________________ is ________________.

POINT-SLOPE FORM OF THE EQUATION OF A LINE

The ______________ - _________________ form of the __________________ of a nonvertical line with slope _______ that passes through the point _________ is

Example 1: Write the point-slope form of the equation of the line with the given slope that passes through the given point.

a. $m = -2;\ (5, -11)$

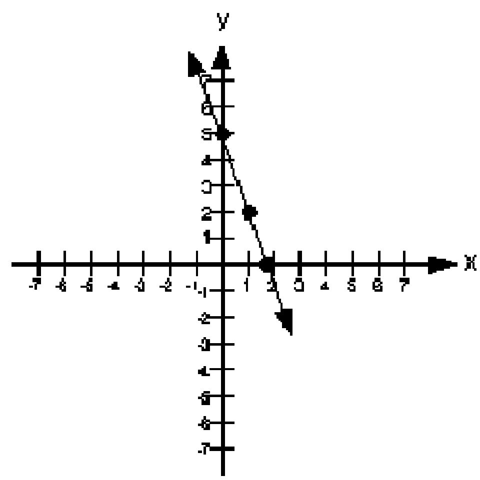
b. $m = \frac{5}{8};\ \left(\frac{1}{4}, 7\right)$

c. $m = 0;\ (-21, 5)$

d. $m = \text{undefined};\ (0, 0)$

Example 2: Use the graph to find two equations of the line in point-slope form.

1.



2.

Now write the slope-intercept form:

1.

2.

EQUATIONS OF LINES

FORM	WHAT YOU SHOULD KNOW
Standard Form	Graph equations in this form using __________________ and a ______________________.
$y = b$	Graph equations in this form as __________________ lines with _____ as the _____________________.
$x = a$	Graph equations in this form as __________________ lines with _____ as the _____________________.
Slope-Intercept Form	Graph equations in this form using the__________________, _____ and the slope, ______. *Start with this form when writing a ______________ equation if you know a line's _______________ and ____________________.
Point-Slope Form	Start with this form when writing a linear equation if you know the ______________ of the line and a _________ on the ____________ NOT containing the __________________ OR ____________ points on the line, _____________ of which contains the ___________________. Calculate the _____________ using

PARALLEL AND PERPENDICULAR LINES

Recall that parallel lines have the _____________ ________________ and

perpendicular lines have ________________ which are _________________

________________.

Example 3: Use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

a. Passing through $(-2,-7)$ and parallel to the line whose equation is $y=-5x+4$.

b. Passing through $(-4,2)$ and perpendicular to the line whose equation is $y=-\frac{1}{3}x+7$.

c. Passing through $(5,-9)$ and parallel to the line whose equation is $x+7y=12$.

Section 4.1: SOLVING SYSTEMS OF LINEAR EQUATIONS BY GRAPHING

When you are done with your homework you should be able to…

- Decide whether an ordered pair is a solution of a linear system
- Solve systems of linear equations by graphing
- Use graphing to identify systems with no solution or infinitely many solutions
- Use graphs of linear systems to solve problems

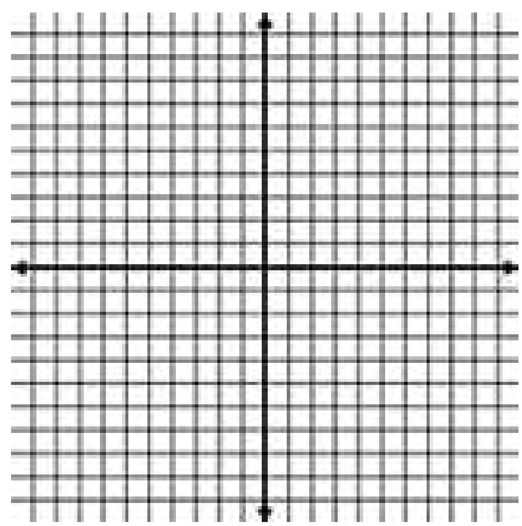
WARM-UP:

1. Determine if the given number or ordered pair is a solution to the given equation.

a. $5x+3=21;\ \dfrac{18}{5}$

b. $-x+2y=0;\ (4,1)$

2. Graph the line which passes through the points (0,1) and (-5,3).



SYSTEMS OF LINEAR EQUATIONS AND THEIR SOLUTIONS

We have seen that all __________________ in the form _________________ are straight ______________ when graphed. ___________ such equations are called a __________________ of _________________ ___________________ or a __________________ ___________________. A __________________ to a system of two _________________ equations in two ________________ is an __________________ ______________ that ________________ ________________ equations in the _________________.

Example 1: Determine whether the given ordered pair is a solution of the system.

a.

$(-2,-5)$

$6x-2y=-2$

$3x+y=-11$

b.

$(10,7)$

$6x-5y=25$

$4x+15y=13$

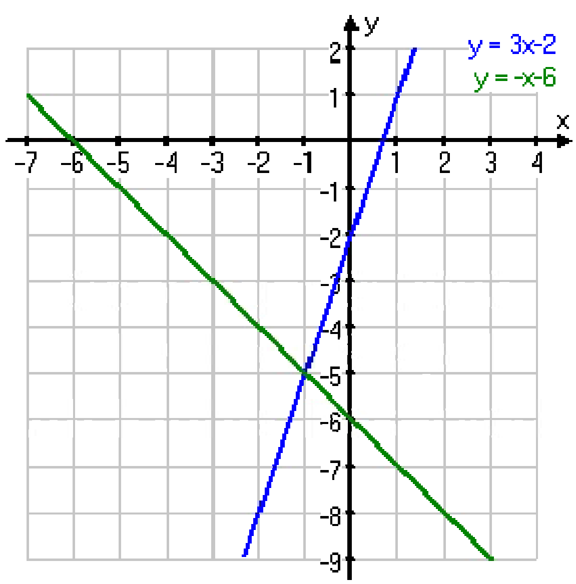
SOLVING LINEAR SYSTEMS BY GRAPHING

The _______________ of a __________________ of two linear equations in ________ variables can be found by _________________ _______________ of the __________________ in the ________________ rectangular __________________ system. For a system with _____________ solution, the ____________________ of the point of ____________________ give the _____________________ solution.

STEPS FOR SOLVING SYSTEMS OF TWO LINEAR EQUATIONS IN TWO VARIABLES, *x* AND *y*, BY GRAPHING

1. Graph the first ______________________.
2. ________________ the second equation on the _______________ set of ____________.
3. If the _______________ representing the __________ graphs ______________ at a _______________, determine the __________________ of this point of intersection. The ______________ ___________ is the __________________ of the ________________.
4. __________________ the __________________ in ______________ equations.

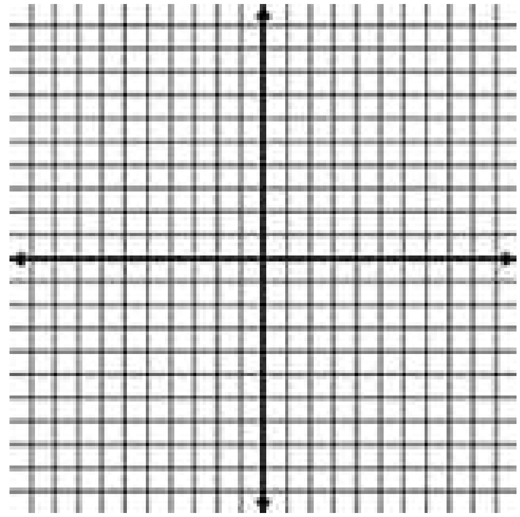
Example 2: Use the graph below to find the solution of the system of linear equations.



Example 3: Solve each system by graphing. Use set notation to express solution sets.

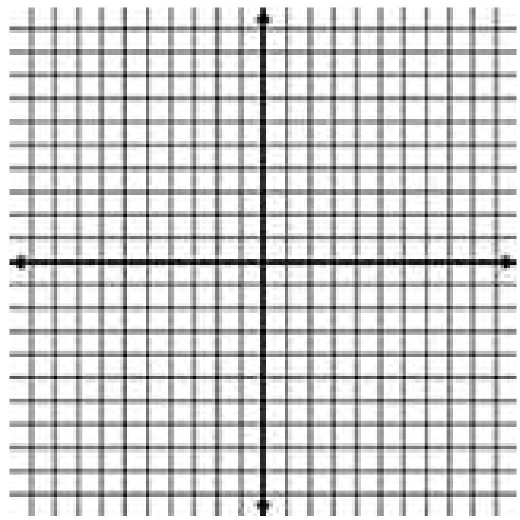
a.

$$x + y = 2$$
$$x - y = 4$$



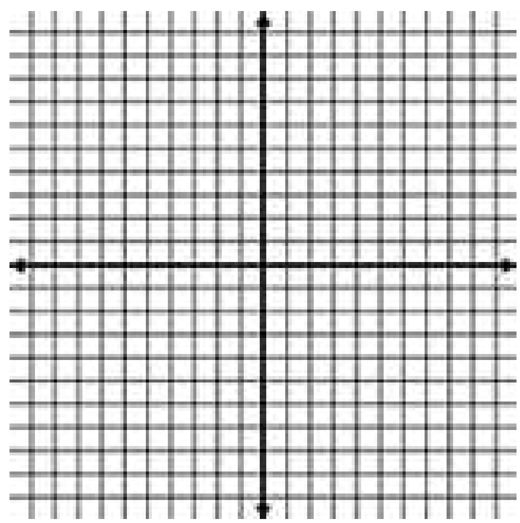
b.

$$y = 3x - 4$$
$$y = -2x + 1$$



c.

$$x + y = 6$$
$$y = -3$$



LINEAR SYSTEMS HAVING NO SOLUTION OR INFINITELY MANY SOLUTIONS

We have seen that a ____________________ of linear equations in __________ variables represents a _______________ of _______________. The lines either _________________ at ___________ point, are ______________, or are ______________________. Thus, there are _______________ possibilities for the ______________ of solutions to a system of two linear equations.

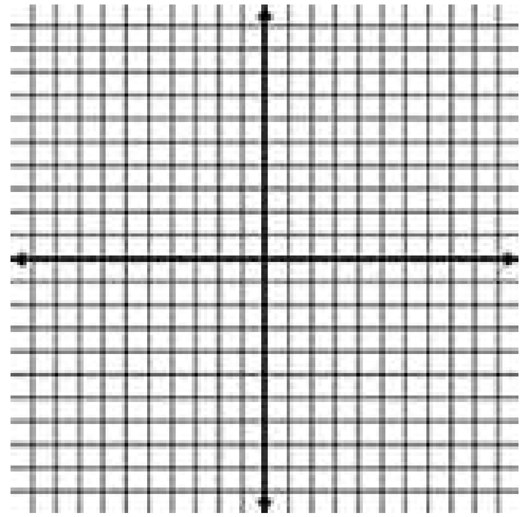
THE NUMBER OF SOLUTIONS TO A SYSTEM OF TWO LINEAR EQUATIONS

NUMBER OF SOLUTIONS	WHAT THIS MEANS GRAPHICALLY
Exactly _____________ ordered pair solution.	The two lines _________________ at _________ point. This is a __________________ system.
________ Solution	The two lines are __________________. This is an __________________ system.
___________________ many solutions	The two lines are __________________. This is a system with ________________ equations.

Example 4: Solve each system by graphing. If there is no solution or infinitely many solutions, so state. Use set notation to express solution sets.

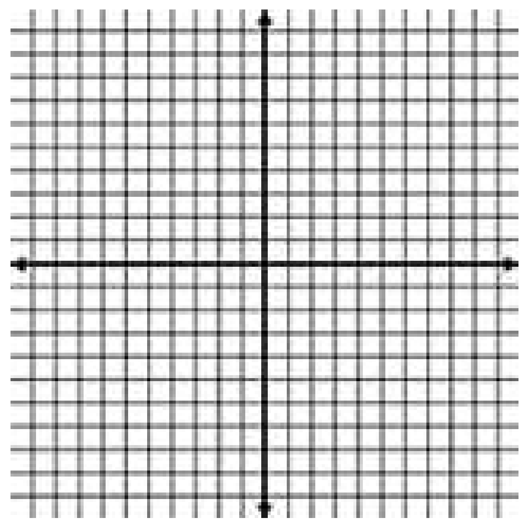
a.

$$x + y = 4$$
$$2x + 2y = 8$$



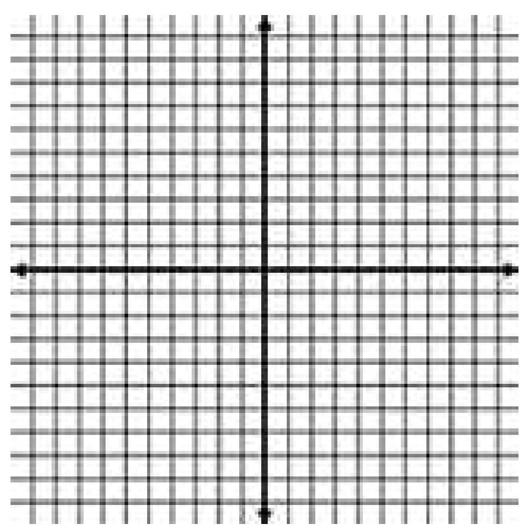
b.

$$y = 3x - 1$$
$$y = 3x + 2$$



c.

$$2x - y = 0$$
$$y = 2x$$

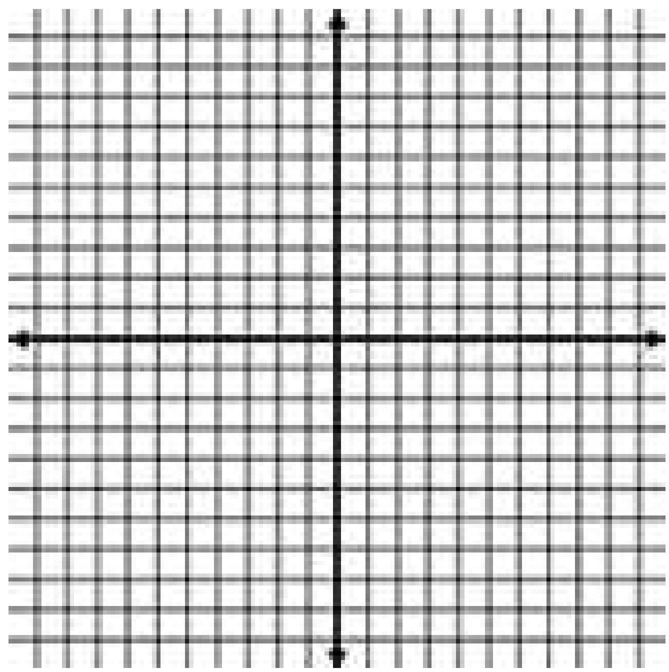


APPLICATION

A band plans to record a demo. Studio A rents for $100 plus $50 per hour. Studio B rents for $50 plus $75 per hour. The total cost, *y*, in dollars, of renting the studios for *x* hours can be modeled by the linear system

$$y = 50x + 100$$
$$y = 75x + 50$$

a. Use graphing to solve the system. Extend the *x*-axis from 0 to 4 and let each tick mark represent 1 unit (one hour in a recording studio). Extend the *y*-axis from 0 to 400 and let each tick mark represent 100 units (a rental cost of $100).



b. Interpret the coordinates of the solution in practical terms.

When you are done with your 4.2 homework you should be able to…

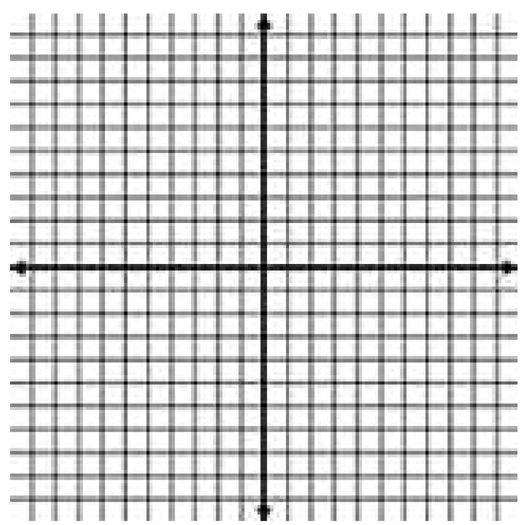
- π Solve linear systems by the substitution method
- π Use the substitution method to identify systems with no solution or infinitely many solutions
- π Solve problems using the substitution method

WARM-UP:

1. Solve.
   $-5x+3(2x-7)=x-21$

2. Solve the following system of linear equations by graphing. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

   $y=-4x+6$
   $y=-2x$



Steps for Solving a System of Two Linear Equations Containing Two Variables by Substitution

1. Solve one of the equations for one of the unknowns.
2. Substitute the expression solved for in Step 1 into the **other** equation. The result will be a _________________ equation in _______ variable.
3. ______________ the linear equation in one variable found in Step 2.
4. _____________________ the value of the variable found in Step 3 into one of the **original** equations to find the _________________ of the other ________________.
5. Check your answer by ___________________________ the ________________ ______________ into ________________ of the original equations.

Example 1: Solve the following systems of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

a.

$$\begin{aligned} 5x + 2y &= -5 \\ 3x - \ \ y &= -14 \end{aligned}$$

b.

$$y = 5x - 3$$
$$y = 2x - \frac{21}{5}$$

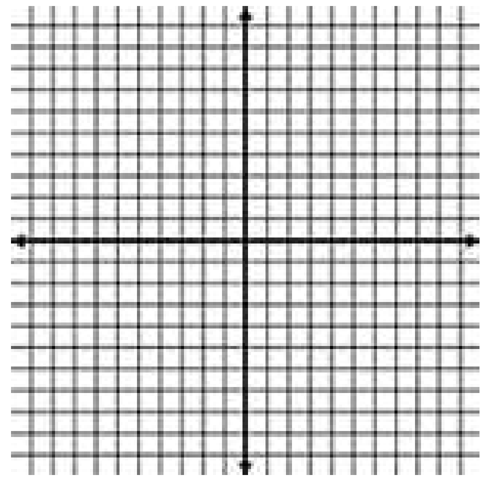
π Suppose you are solving a system of equations and you end up with 5 = 0. This is a ________________________ and yields a result of ________ or ________. This system consists of two ___________________ lines which never ____________________.

π Suppose you are solving a system of equations and you end up with 5 = 5 or x = x. This is an ________________ and yields a result of all ______________ _____________ which are on the ______________. In other words, the system would have ______________________ ________________ solutions. This system consists of two lines which are ________________________.

Example 2: Solve the following systems of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Graph the system.

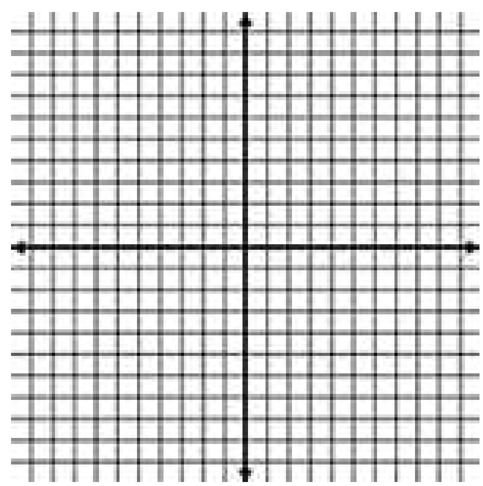
a.

$$-x+3y=4$$
$$2x-6y=-8$$



b.

$$x - 5y=3$$
$$-2x+10y=8$$



Example 3: Write a system of equations that has infinitely many solutions.

APPLICATIONS

1. Christa is a waitress and collects her tips at the table. At the end of the shift she has 68 bills in her tip wallet, all ones and fives. If the total value of her tips is $172, how many of each bill does she have?

2. Melody wishes to enclose a rectangular garden with fencing, using the side of her garage as one side of the rectangle. A neighbor gave her 30 feet of fencing, and Melody wants the length of the garden along the garage to be 3 feet more than the width. What are the dimensions of the garden?

When you are done with your 4.3 homework you should be able to…

- π Solve linear systems by the addition method
- π Use the addition method to identify systems with no solution or infinitely many solutions
- π Determine the most efficient method for solving a linear system

WARM-UP:

1. Solve the following system of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

$$y = \frac{7}{2}x - 3$$
$$y = -4x + 2$$

ELIMINATING A VARIABLE USING THE ADDITION METHOD

The ____________________ method is most useful if one of the equations has an _______________ variable. A third method for solving a linear system is the ___________________ method. The addition method _________________ a variable by _______________ the equations. When we use the addition method, we want to obtain two equations whose ____________ is an equation containing only __________ variable. The key step is to obtain, for one of the variables, __________________ that differ only in _____________.

Steps for Solving a System of Two Linear Equations Containing Two Variables by Addition

1. If necessary, ______________ both equations in the form _____________.
2. If necessary, ______________ either equation or both equations by appropriate nonzero numbers so that the ___________ of the *x*-coefficients or *y*-coefficients is __________.
3. __________ the equations in step 2. The __________ is an ______________ in ___________ variable.
4. _____________ the equation in one variable.
5. ____________-______________ the value obtained in step 4 into either of the ____________ equations and ____________ for the other variable.
6. ____________ the solution in __________ of the original equations.

Example 1: Solve the following systems of linear equations by the addition method.

State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Use set notation to express solution sets.

a.

$$x + y = 6$$
$$x - y = -2$$

b.

$$3x - y = 11$$
$$2x + 5y = 13$$

COMPARING SOLUTION METHODS

METHOD	**ADVANTAGES**	**DISADVANTAGES**
GRAPHING	You can _________ the _________________.	If the solutions do not involve ____________ or are too __________ or ___________ to be ___________ on the graph, it's impossible to tell exactly what the ________________ are.
SUBSTITUTION	Gives ____________ solutions. Easy to use if a _______________ is on _________ side by itself.	Solutions cannot be __________. Can introduce extensive work with ______________ when no variable has a coefficient of _____ or ______.
ADDITION	Gives ___________ solutions. Easy to use!	Solutions cannot be ____________.

Example 2: Solve the following systems of linear equations by any method. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Use set notation to express solution sets.

a.

$$x + y = 6$$
$$x - y = -2$$

b.

$$3x - y = 11$$
$$2x + 5y = 13$$

c.

$$4x - 2y = 2$$
$$2x - y = 1$$

d.

$$3x = 4y + 1$$
$$4x + 3y = 1$$

e.

$$2x + 4y = 5$$
$$3x + 6y = 6$$

Section 4.4: PROBLEM USING SOLVING SYSTEMS OF EQUATIONS

When you are done with your homework you should be able to…

- π Solve problems using linear systems
- π Solve simple interest problems
- π Solve mixture problems
- π Solve motion problems

WARM-UP:

1. Solve the system of linear equations using the substitution or the addition method. Determine if the system is consistent or inconsistent, and if the equations are dependent or independent. Give your result in set notation.

a.

$$2x-3y=4$$
$$3x+4y=0$$

b.

$$x-y=3$$
$$2x=4+2y$$

A STRATEGY FOR SOLVING WORD PROBLEMS USING SYSTEMS OF EQUATIONS

When we solved problems in chapter 2, we let x represent a ________________

that was ____________________. Problems in this section involve ___________

unknown ____________________. We will let _______ and _______ represent

the ________________ quantities and _________________ the English words

into a _________________ of ________________ equations.

Example 1: The sum of two numbers is five. If one number is subtracted from the other, their difference is thirteen. Find the numbers.

Example 2: Each day, the sum of the average times spent on grooming for 15- to 19-year-old women and men is 96 minutes. The difference between grooming times for 15- to 19-year-old women and men is 22 minutes. How many minutes per day do 15- to 19-year-old women and men spend on grooming?

Example 3: A rectangular lot whose perimeter is 1600 feet is fenced along three sides. An expensive fencing along the lot's length costs $20 per foot. An inexpensive fencing along the two side widths costs only $5 per foot. The total cost of the fencing along the three sides comes to $13000. What are the lot's dimensions?

Example 4: On a special day, tickets for a minor league baseball game cost $5 for adults and $1 for students. The attendance that day was 1281 and $3425 was collected. Find the number of each type of ticket sold.

Example 5: You invested $11000 in stocks and bonds, paying 5% and 8% annual interest. If the total interest earned for the year was $730, how much was invested in stocks and how much was invested in bonds?

Example 6: A jeweler needs to mix an alloy with a 16% gold content and an alloy with a 28% gold content to obtain 32 ounces of a new alloy with a 25% gold content. How many ounces of each of the original alloys must be used?

A FORMULA FOR MOTION

Distance equals ________________ times _______________.

Example 7: When a plane flies with the wind, it can travel 4200 miles in 6 hours. When the plane flies in the opposite direction, against the wind, it takes 7 hours to fly the same distance. Find the rate of the plane in still air and the rate of the wind.

Example 8: With the current, you can row 24 miles in 3 hours. Against the same current, you can row only 2/3 of this distance in 4 hours. Find your rowing rate in still water and the rate of the current.

Section 5.1: ADDING AND SUBTRACTING POLYNOMIALS

When you are done with your homework you should be able to…

- π Understand the vocabulary used to describe polynomials
- π Add polynomials
- π Subtract polynomials
- π Graph equations defined by polynomials of degree 2

WARM-UP:

Simplify:

$-6x+5y-2x^2-2y+x^2$

DESCRIBING POLYNOMIALS

A ____________________ is a ___________ term or the ____________ of two or more ________________ containing _________________ with ____________ number ________________. It is customary to write the ______________ in the order of _________________ powers of the __________________. This is the _________________ form of a _____________________. We begin this chapter by limiting discussion to polynomials containing __________ variable. Each term of such a _____________________ in _____ is of the form _______________. The ______________ of ______________ is ______.

THE DEGREE OF ax^n

If ____________ and _____ is a _____________ number, the _____________ of ____________ is _________. The _______________ of a nonzero constant term is _________. The constant zero has no defined degree.

Example 1: Identify the terms of the polynomial and the degree of each term.

a. $-4x^5 - 13x^3 + 5$

b. $-x^2 + 3x - 7$

A polynomial is _________________ when it contains no _____________ symbols and no ____________ _________________. A simplified polynomial that has exactly ___________ term is called a ___________________. A simplified polynomial that has _____________ terms is called a ______________ and a simplified polynomial with _____________ terms is called a ________________. Simplified polynomials with _____________ or more ___________ have no special names. The ________________ of a __________________ is the ________________ degree of ____________ the _____________ of a __________________.

Example 2: Find the degree of the polynomial.

a. $5x^2 - x^8 + 16x^4$

b. -2

ADDING POLYNOMIALS

Recall that _________ ____________ are terms containing __________ the same ______________ to the __________ powers. ______________ are added by _______________ ____________ ____________.

Example 3: Add the polynomials.

a. $(8x-5)+(-13x+9)$

b. $(7y^3+5y-1)+(2y^2-6y+3)$

c. $\left(\frac{2}{5}x^4+\frac{2}{3}x^3+\frac{5}{8}x^2+7\right)+\left(-\frac{4}{5}x^4+\frac{1}{3}x^3-\frac{1}{4}x^2-7\right)$

d.

$$\begin{array}{r} 7x^2-5x-6 \\ \underline{-9x^2+4x+6} \end{array}$$

SUBTRACTING POLYNOMIALS

We ______________ real numbers by _____________ the _______________ of the number being ________________. Subtraction of polynomials also involves ____________________. If the sum of two polynomials is _________, the polynomials are __________________ of each other.

Example 4: Find the opposite of the polynomial.

a. $x+8$

b. $-12x^3-x+1$

SUBTRACTING POLYNOMIALS

To ______________ two polynomials, _______ the first polynomial and the _______________ of the second polynomial

Example 5: Subtract the polynomials.

a. $(x-2)-(7x+9)$

b. $(3x^2-2x)-(5x^2-6x)$

c. $\left(\frac{3}{8}x^2-\frac{1}{3}x-\frac{1}{4}\right)-\left(-\frac{1}{8}x^2+\frac{1}{2}x-\frac{1}{4}\right)$

d.

$$\begin{array}{r} 3x^5-5x^3+6 \\ \underline{-\left(7x^5+4x^3-2\right)} \end{array}$$

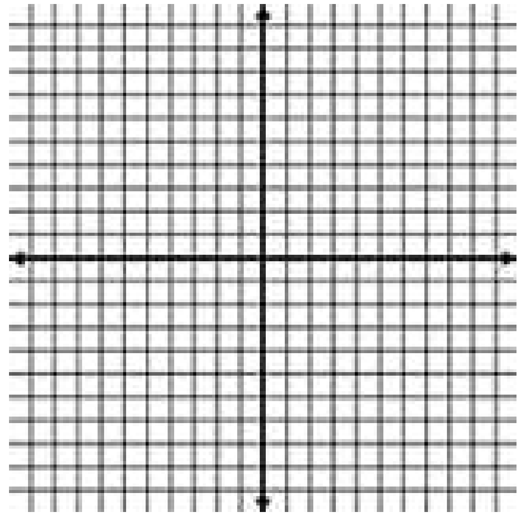
GRAPHING EQUATIONS DEFINED BY POLYNOMIALS

Graphs of equations defined by ___________________ of degree ______ have a ___________________ quality. We can obtain their graphs, shaped like _____________ or _______________ bowls, using the _____________-___________________ method for graphing an equation in two variables.

Example 6: Graph the following equations by plotting points.

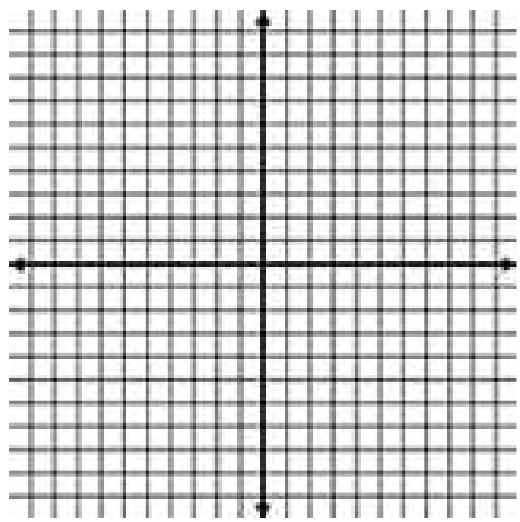
a. $y = x^2 - 1$

x	$y = x^2 - 1$	(x, y)



b. $y = 9 - x^2$

x	$y = 9 - x^2$	(x, y)



Section 5.2: MULTIPLYING POLYNOMIALS

When you are done with your homework you should be able to…

- π Use the product rule for exponents
- π Use the power rule for exponents
- π Use the products-to-power rule
- π Multiply monomials
- π Multiply a monomial and a polynomial
- π Multiply polynomials when neither is a monomial

WARM-UP:

Add or subtract the following polynomials:

a. $\left(-22r^7+6r^3-r^2\right)-\left(2r^7+r^2-1\right)$

b. $\left(8x^4-x^3-x^2\right)+\left(-8x^4+x^3\right)$

THE PRODUCT RULE FOR EXPONENTS

We have seen that __________________ are used to indicate ________________ multiplication. Recall that 3^4 = __________________. Now consider $3^4\cdot 3^2$:

THE PRODUCT RULE

When multiplying _________________ expressions with the ___________ base, ________ the _________________. Use this ________ as the _______________ of the _________________ base.

Example 1: Simplify each expression.

a. $2^5 \cdot 2^3$

b. $x^2 \cdot x \cdot x^4$

THE POWER RULE (POWERS TO POWERS)

When an ___________________ _________________ is ___________ to a _______________, _______________ the _________________. Place the _______________ of the __________________on the __________ and ________________ the ____________________.

Example 2: Simplify each expression.

a. $\left(4^2\right)^3$

b. $\left(x^{12}\right)^5$

THE PRODUCTS-TO-POWERS RULE FOR EXPONENTS

When a ________________ is _______________ to a __________, __________ each _____________ to the ________________.

Example 3: Simplify each expression.

a. $(-2y)^5$

b. $(10x^3)^2$

MULTIPLYING MONOMIALS

To _________________ __________________ with the _____________ _________________ base, _______________ the ____________________ and then multiply the _________________. Use the _____________ rule for _______________ to multiply the __________________.

Example 4: Multiply.

d. $(8x)(-11x^4)$

e. $(7y^3)(2y^2)$

f. $\left(\frac{2}{5}x^4\right)\left(-\frac{5}{6}x^7\right)$

MULTIPLYING A MONOMIAL AND A POLYNOMIAL THAT IS NOT A MONOMIAL

To _________________ a __________________ and a _________________, use the _______________ property to ________________ each ____________ of the _________________ by the ______________________.

Example 5: Multiply.

a. $3x^2(2x-5)$

b. $-x(x^2+6x-5)$

MULTIPLYING POLYNOMIALS WHEN NEITHER IS A MONOMIAL

Multiply each _________________ of one _________________ by each __________ of the other polynomial. Then _____________ _____________ terms.

Example 6: Multiply.

a. $(x+2)(x+5)$

b. $(2x+5)(x+3)$

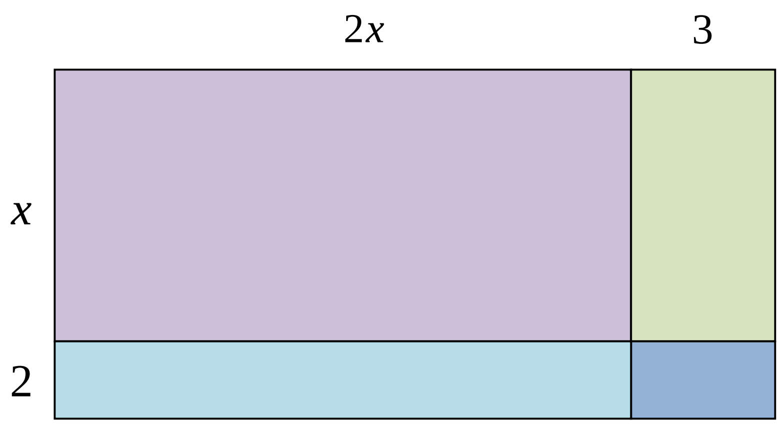
c. $\left(x^2-7x+9\right)\left(x+4\right)$

Example 7: Simplify.

a. $3x^2\left(6x^3+2x-3\right)-4x^3\left(x^2-5\right)$

b. $\left(y+6\right)^2-\left(y-2\right)^2$

APPLICATION



a. Express the area of the large rectangle as the product of two binomials.

b. Find the sum of the areas of the four smaller rectangles.

c. Use polynomial multiplication to show that your expressions for area in parts (a) and (b) are equal.

Section 5.3: SPECIAL PRODUCTS

When you are done with your homework you should be able to…

- π Use FOIL in polynomial multiplication
- π Multiply the sum and difference of two terms
- π Find the square of a binomial sum
- π Find the square of a binomial difference

WARM-UP:

Multiply the following polynomials:

a. $(x-1)^2$

b. $(x-5)(x+5)$

THE PRODUCT OF TWO BINOMIALS: FOIL

F represents the ______________ of the _____________ terms in each ______________, **O** represents the ________________ of the ___________ terms, **I** represents the _______________ of the _______________ terms, and **L** represents the _______________ of the _______________ terms.

USING THE FOIL METHOD TO MULTIPLY BINOMIALS

$(ax+b)(cx+d)=$ ____

Example 1: Multiply using FOIL.

a. $(5x+3)(3x+8)$

b. $(x-10)(x+9)$

THE PRODUCT OF THE SUM AND DIFFERENCE OF TWO TERMS

$(A+B)(A-B) =$ ____

The ________________ of the ________ and the __________________ of the ___________ two terms is the ____________ of the _____________ _______________ the _________________ of the second.

Example 2: Multiply.

a. $(x+4)(x-4)$

b. $(3x-7y)(3x+7y)$

THE SQUARE OF A BINOMIAL SUM

$(A+B)^2 =$ ____

The ________________ of a _______________ _________ is the ____________ term _____________ ________ ___ times the _____________ of the terms _____________ the last term _______________.

Example 3: Multiply.

a. $(x+6)^2$

b. $(x^2+9)^2$

THE SQUARE OF A BINOMIAL DIFFERENCE

$(A-B)^2 =$ ____

The _______________ of a ______________ _________ is the ____________ term _____________ ________ ___ times the _____________ of the terms _____________ the last term ______________.

Example 4: Multiply.

a. $(5x-y)^2$

b. $(x^3-11)^2$

Section 5.4: POLYNOMIALS IN SEVERAL VARIABLES

When you are done with your homework you should be able to…

- π Evaluate polynomials in several variables
- π Understand the vocabulary of polynomials in two variables
- π Add and subtract polynomials in several variables
- π Multiply polynomials in several variables

WARM-UP:

Evaluate the polynomial:

$x^3y + 2xy^2 + 5x - 2;\ x = -2 \text{ and } y = 3$

EVALUATING A POLYNOMIAL IN SEVERAL VARIABLES

1. ___________________ the given value for each ______________________.
2. Perform the resulting _______________________ using the ____________ of _______________________.

DESCRIBING POLYNOMIALS IN TWO VARIABLES

In general, a ____________________ in ___________ ___________________, ___ and ___, contains the ____________ of one or more ____________________ in the form ______________. The constant, ______, is the __________________. The ____________________, _____ and _____, represent _______________ numbers. The ___________________ of the ____________________ __________ is ______________.

Example 1: Determine the coefficient of each term, the degree of each term, and the degree of the polynomial.

$$8xy^4 - 17x^5y^3 + 4x^2y - 9y^3 + 7$$

ADDING AND SUBTRACTING POLYNOMIALS IN SEVERAL VARIABLES

________________________ in __________________ variables are added by

___________________ _______________ _________________.

Example 2: Add or subtract.

a. $\left(x^3 - y^3\right) - \left(-4x^3 - x^2y + xy^2 + 3y^3\right)$

b. $\left(7x^2y + 5xy + 13\right) + \left(-3x^2y + 6xy + 4\right)$

MULTIPLYING POLYNOMIALS IN SEVERAL VARIABLES

The _______________ of ______________ _________ the basis of ___________ ___________________. ______________________ can be done _____________ by ___________________ ___________________ and _______________ __________________ on ____________________ with the ________________ _______________.

Example 3: Multiply.

a. $\left(5xy^3\right)\left(-10x^2y^4\right)$

c. $\left(x-2y^4\right)\left(x+2y^4\right)$

b. $-x^7y^2\left(x^2+7xy-4\right)$

d. $\left(x^2-y\right)^2$

Section 5.5: DIVIDING POLYNOMIALS

When you are done with your homework you should be able to…

- π Use the quotient rule for exponents
- π Use the zero-exponent rule for exponents
- π Use the quotients-to-power rule
- π Divide monomials
- π Check polynomial division
- π Divide a polynomial by a monomial

WARM-UP:

1. Find the missing exponent, designated by the question mark, in the final step:

$$\frac{x^8}{x^3} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^?$$

2. Simplify:

$$\frac{\left(2a^3\right)^5}{\left(b^4\right)^5}$$

THE QUOTIENT RULE FOR EXPONENTS

When dividing _________________ expressions with the ___________ nonzero base, ________________ the exponent in the _________________ from the ____________________ in the __________________. Use this _______________________ as the _______________ of the _________________ base.

Example 1: Simplify each expression.

a. $\frac{2^5}{2^3}$

b. $\frac{x^{10}}{x^8}$

THE ZERO-EXPONENT RULE

If ______ is any _____________ number other than ______,

Example 2: Simplify each expression.

a. $\left(4^2\right)^0$

b. $-7x^0$

THE QUOTIENTS-TO-POWERS RULE FOR EXPONENTS

If _____ and _____ are real numbers and ____ is nonzero, then

When a ________________ is _______________ to a ___________, __________ the _____________ to the ________________ and _______________ by the ________________ raised to the _____________________.

Example 3: Simplify each expression.

a. $\left(\frac{x}{3}\right)^5$

b. $\left(\frac{4x^3}{5y}\right)^2$

DIVIDING MONOMIALS

To _________________ __________________, ____________________the

_______________________ and then divide the ______________________.

Use the _____________ rule for _______________ to divide the ____________.

Example 4: Divide.

a. $\frac{16x^4}{2x^4}$

b. $\frac{6x^2y^5}{21xy^3}$

c. $\frac{35r^8}{14r^7}$

DIVIDING A POLYNOMIAL THAT IS NOT A MONOMIAL BY A MONOMIAL

To __________________ by a ___________________, _________________ each

___________________ of the _____________________ by the _______________.

Example 5: Find the quotient.

a. $\left(24x^6 - 12x^4 + 8x^3\right) \div \left(4x^3\right)$

b. $\frac{459x^{10}y^9 + 18x^5y^3 - 9x^4y}{-9x^3y}$

Section 5.6: LONG DIVISION OF POLYNOMIALS AND SYNTHETIC DIVISION

When you are done with your homework you should be able to…

- π Use long division to divide by a polynomial containing more than one term
- π Divide polynomials using synthetic division

WARM-UP:

a. Divide using long division:

$$56\overline{)1234567}$$

b. Simplify:

$$\frac{5x^5 - 8x^3 + x^2}{2x^2}$$

STEPS FOR DIVIDING A POLYNOMIAL BY A BINOMIAL

1. _________________ the terms of ___________ the _________________ and the _________________ in ____________________ powers of the variable.
2. __________________ the _____________ term in the ________________ by the _______________ term in the __________________. The result is the ________________ term of the ___________________.
3. ________________ every term in the ___________________ by the _______________ term in the _________________. Write the resulting _________________ beneath the _________________ with __________ terms lined up.
4. ______________ the ________________ from the ____________________.
5. __________________ down the next term in the _________________ dividend and write it next to the ___________________ to form a new ___________________.
6. Use this new expression as the ____________________ and repeat the process until the ______________________ can no longer be ___________________. This will occur when the _______________ of the ___________________ is _______________ than the ________________ of the _________________.

Example 1: Divide.

a. $\dfrac{x^2+7x+10}{x+5}$

b. $\dfrac{2y^2-13y+21}{y-3}$

c. $\dfrac{x^3+2x^2-3}{x-2}$

d. $\left(8y^3+y^4+16+32y+24y^2\right)\div\left(y+2\right)$

DIVIDING POLYNOMIALS USING SYNTHETIC DIVISION

We can use ___________________ division to divide _________________ if the ________________ is of the form ________________. This method provides a ________________ more quickly than ______________ division.

STEPS FOR SYNTHETIC DIVISION

1. Arrange the ___________________ in __________________ powers, with a _______ coefficient for any _______________ term.
2. Write _____ for the ______________, ___________. To the __________, write the _________________ of the _________________.
3. Write the _______________ ___________________ of the _______________ on the _______________ row.
4. __________________ _____ times the ____________ just written on the _______________ row. Write the _______________ in the next ______________ in the ________________ row.
5. __________ the values in this new column, writing the __________ in the ______________ row.
6. Repeat this series of _______________________ and ________________ until all _________________ are filled in.

7. Use the numbers in the last row to write the _______________ plus the

_________________ ________________ the ________________. The

________________ of the _______________ term of the quotient will be

___________ less than the _________________ of the first term of the

___________________. The final value in this row is the _____________.

Example 2: Divide using synthetic division.

a. $\left(x^2 + x - 2\right) \div \left(x - 1\right)$

b. $\left(x^2 - 6x - 6x^3 + x^4\right) \div \left(6 + x\right)$

c. $\dfrac{x^7 - 128}{x - 2}$

d. $\left(y^5 - 2y^4 - y^3 + 3y^2 - y + 1\right) \div \left(y - 2\right)$

APPLICATION

You just signed a contract for a new job. The salary for the first year is $30,000 and there is to be a percent increase in your salary each year. The algebraic expression

$$\frac{30000x^n - 30000}{x - 1}$$

describes your total salary over *n* years, where *x* is the sum of 1 and the yearly percent increase, expressed as a decimal.

a. Use the given expression and write a quotient of polynomials that describes your total salary over four years.

b. Simplify the expression in part (a) by performing the division.

c. Suppose you are to receive an increase of 8% per year. Thus, *x* is the sum of 1 and 0.08, or 1.08. Substitute 1.08 for *x* in the expression in part (a) as well as the simplified expression in part (b). Evaluate each expression. What is your total salary over the four-year period?

Section 5.7: NEGATIVE EXPONENTS AND SCIENTIFIC NOTATION

When you are done with your homework you should be able to…

- π Use the negative exponent rule
- π Simplify exponential expressions
- π Convert from scientific notation to decimal notation
- π Convert from decimal notation to scientific notation
- π Compute with scientific notation
- π Solve applied problems using scientific notation

WARM-UP:

1. Divide:

$$\left(7x^4-8x\right)\div\left(x+3\right)$$

2. Simplify:

$$\frac{1}{\left(6x^3\right)^2}$$

NEGATIVE INTEGERS AS EXPONENTS

A nonzero base can be raised to a _________________ power. The _______________ rule can be used to help determine what a ______________ __________________ as an __________________ should mean.

THE NEGATIVE EXPONENT RULE

If _____ is any real number other than ____ and ____ is a natural number, then

NEGATIVE EXPONENTS IN NUMERATORS AND DENOMINATORS

If _____ is any real number other than ____ and ____ is a natural number, then

When a _______________ number appears as an ________________, ______________ the position of the ___________ (from ________________ to _________________ or from _________________ to ____________________) and make the ___________________ _________________. The sign of the _____________ does ______________ change.

Example 1: Write each expression with positive exponents only. Then simplify, if possible.

a. -7^{-2}

c. $3^{-1}-6^{-1}$

b. $(-7)^{-2}$

d. $\frac{x^{-12}}{y^{-1}}$

SIMPLIFYING EXPONENTIAL EXPRESSIONS

Properties of ____________________ are used to ___________________ exponential expressions. An exponential ____________________ is ___________________ when

- Each ________________ occurs only ___________________
- No __________________________ appear
- No _____________________ are raised to _____________________
- No __________________ or ________________ exponents appear

STEPS FOR SIMPLIFYING EXPONENTIAL EXPRESSIONS

1. If necessary, be sure that each _____________ appears only __________, using ___________________ or __________________.
2. If necessary, ________________ parentheses using _________________ or _______________________.
3. If necessary, simplify ________________ to _________________ using _____________________.
4. If necessary, _________________ exponential expressions with _______ powers as _____ (_____________). Furthermore, write the answer with _________________ exponents using ____________________________.

Example 2: Simplify. Assume that variables represent nonzero real numbers.

a. $\dfrac{45z^4}{15z^{12}}$

c. $\dfrac{\left(5x^3\right)^2}{x^7}$

b. $\dfrac{\left(3y^4\right)^3 y^{-7}}{y^7}$

d. $\left(\dfrac{x^3}{y^2}\right)^{-4}$

SCIENTIFIC NOTATION

A _________________ number is written in _________________ notation when it is expressed in the form

where ____ is a number ______________ than or equal to ____ and __________ than _____ (_____________________) and ____ is an _____________________.

It is customary to use the _____________________ symbol, ____, rather than a dot, when writing a number in ___________________ _________________. We can use ____, the exponent on the ______ in _______________, to change a number in scientific notation to ________________ notation. If _____ is _________________, move the decimal point in _____ to the _____________ ____ places. If _____ is __________________, move the decimal point in ____ to the ______________ _________ places.

Example 3: Write each number in decimal notation.

a. 7.85×10^8

c. 1.001×10^2

b. 9×10^{-5}

d. 9.999×10^{-1}

CONVERTING FROM DECIMAL TO SCIENTIFIC NOTATION

Write the number in the form _________________.

- Determine _____, the numerical _______________. Move the ____________ point in the _____________ number to obtain a number _______________ than or equal to _____ and __________ than ______.
- Determine _____, the _______________ on _______. The ____________ ________________ of _____ is the ________________ of places the decimal point was _____________. The exponent _____ is _____________ if the given number is ________________ than ______ and ____________ if the given number is _________________ _____ and _____.

Example 4: Write each number in scientific notation.

a. 0.00000006589

c. 0.234

b. $6,789,000,000,000$

d. $1,000,234,000$

COMPUTATIONS WITH NUMBERS IN SCIENTIFIC NOTATION

MULTIPLICATION

DIVISION

EXPONENTIATION

After the computation is ____________________, the ________________ may require an additional __________________ before it is expressed in ______________________ notation.

Example 5: Perform the indicated operations, writing the answers in scientific notation.

a. $\left(3\times10^{4}\right)\left(4\times10^{2}\right)$

b. $\left(2\times10^{-3}\right)^{5}$

c. $\frac{180\times10^{8}}{2\times10^{4}}$

d. $\left(5\times10^{4}\right)^{-1}$

APPLICATIONS

1. A human brain contains 3×10^{10} neurons and a gorilla brain contains 7.5×10^{9} neurons. How many times as many neurons are in the brain of a human as in the brain of a gorilla?

2. If the sun is approximately 9.14×10^{7} miles from the earth, how many seconds, to the nearest tenth of a second does it take sunlight to reach Earth? Use the motion formula, $d=rt$, and the fact that light travels at the rate of 1.86×10^{5} miles per second.

Section 6.1: THE GREATEST COMMON FACTOR AND FACTORING BY GROUPING

When you are done with your homework you should be able to…

- π Find the greatest common factor (GCF)
- π Factor out the GCF of a polynomial
- π Factor by grouping

WARM-UP:

1. Multiply:

$x^2\left(7x^4 - 8\right)$

2. Divide:

$$\frac{16x^4 - 8x^2}{4x^2}$$

FACTORING A ______________________ CONTAINING THE SUM OF _________________ MEANS FINDING AN ________________ EXPRESSION THAT IS A ____________________.

FACTORING OUT THE GREATEST COMMON FACTOR (GCF)

We use the ____________________ property to ________________ a monomial and a ____________________ of ____________ or more _____________.

When we ________________, we _________________ this process, expressing the __________________ as a ____________________.

MULTIPLICATION **FACTORING**

In any ________________________ problem, the first step is to look for the ____________________ ___________________ __________________. The ____________ is an __________________ of the _________________ degree that _________________ each _____________ of the __________________.

The ________________ part of the ______________ always contains the ________________ _________________ of a ________________ that appears in __________ terms of the ____________________.

Example 1: Find the greatest common factor of each list of monomials:

a. 5 and $15x$

b. $-3x^4$ and $6x^3$

c. $x^2y,\ 7x^3\text{y}$ and $14x^2$

STEPS FOR FACTORING A MONOMIAL FROM A POLYNOMIAL

1. Determine the _______________ ______________ factor of ________ terms in the _________________________.
2. Express each ________________ as the _________________ of the _______ and its other __________________.
3. Use the ________________ ________________ to factor out the ______.

Example 2: Factor each polynomial using the GCF:

a. $9x+9$

b. $32x-24$

c. $18x^3y^2-12x^3y-24x^2y$

d. $7(x+1)+21x(x+1)$

FACTORING BY GROUPING

1. _________________ terms that have a _________________ ______________ factor. There will usually be ________ groups. Sometimes the terms must be _____________________.
2. ________________ out the _________________ monomial ____________ from each ________________.
3. _________________ out the remaining common _________________ factor (if one exists).

Example 3: Factor by grouping:

a. $x^2+3x+5x+15$

c. $xy-6x+2y-12$

b. $x^3-3x^2+4x-12$

d. $10x^2-12xy+35xy-42y^2$

Example 4: Factor each polynomial:

a. $x^3 - 5 + 2x^3y - 10y$

c. $8x^5(x+2) - 10x^3(x+2) - 2x^2(x+2)$

b. $7x^5 - 7x^4 + x^3 - x^2 + 3x - 3$

d. $12x^2 - 25$

APPLICATION

An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. The polynomial $72x - 16x^2$ describes the height of the debris above the ground, in feet, after x seconds.

a. Find the height of the debris after 4 seconds.

b. Factor the polynomial.

c. Use the factored form of the polynomial in part (b) to find the height of the debris after 4 seconds. Do you get the same answer as you did in part (a)? If so, does this prove that your factorization is correct?

Section 6.2: FACTORING TRINOMIALS WHOSE LEADING COEFFICIENT IS 1

When you are done with your homework you should be able to…

- π Factor trinomials of the form x^2+bx+c

WARM-UP:

Multiply:

a. $(x+1)(x+8)$

c. $(x+1)(x-8)$

b. $(x-1)(x-8)$

d. $(x-1)(x+8)$

A STRATEGY FOR FACTORING ax^2+bx+c: USING GROUPING

1. Multiply the leading coefficient (in this case 1) and the constant, ______.
2. Find the _________________ of ________ whose ________ is ______.
3. Rewrite the ____________ term, _____, as a ________ or a

   _______________ using the factors from step 2.
4. ______________ by ________________.

Example 1: Factor each trinomial

a. $x^2 + 9x + 8$

b. $x^2 + 7x + 10$

c. $x^2 - 13x + 40$

d. $x^2 + 3x - 28$

e. $x^2 - 4x - 5$

f. $w^2 + 12w - 64$

g. $y^2 - 15y + 5$

h. $x^2 - 9xy + 14y^2$

Some ___________________ can be _________________ using more than one ____________________. **Always begin by looking for the _______________ __________________ _________________ and, if there is one, ________ it out!** A polynomial is ______________ ________________ when it is written as the __________________ of ______________ ______________________.

Example 4: Factor completely

a. $3x^2 + 21x + 36$

b. $20x^2y - 5xy - 120y$

c. $y^4 - 12y^3 + 35y^2$

d. $(a+b)x^2 - 13(a+b)x + 36(a+b)$

APPLICATION

You dive directly upward from a board that is 48 feet high. After *t* seconds, your height above the water is described by the polynomial $-16t^2 + 32t + 48$.

a. Factor the polynomial completely.

b. Evaluate both the original polynomial and its factored form for *t* = 3.

c. Do you get the same answer? Describe what this answer means?

Section 6.3: FACTORING TRINOMIALS WHOSE LEADING COEFFICIENT IS NOT 1

When you are done with your homework you should be able to…

- π Factor trinomials by trial and error
- π Factor trinomials by grouping

WARM-UP:

Factor:

a. $x^2y - xy^2$

c. $2x^3 - 6x^2 + 4x$

b. $x^2 - 14x - 51$

d. $z^2 + z - 72$

A STRATEGY FOR FACTORING ax^2+bx+c: USING TRIAL AND ERROR

Assume, for the moment, that there is no _______________ ______________ factor other than _____.

1. ____________ two First ______________ whose ___________ is ________.

2. ____________ two Last ______________ whose ___________ is _____.

3. By _____________ and _______________, perform steps 1 and 2 until the __________ of the Outside ________________ and the Inside ______________ is ___________.

If ___________ such __________________ exist, the polynomial is _________.

Example 1: Factor using trial and error.

a. $5x^2-14x+8$

b. $6x^2+19x-7$

c. $3x^2 - 13xy + 4y^2$

d. $9z^2 + 3z + 2$

A STRATEGY FOR FACTORING $ax^2 + bx + c$: USING GROUPING

1. Multiply the leading coefficient and the constant, _______.
2. Find the ___________________ of _________ whose _________ is _______.
3. Rewrite the _____________ term, ______, as a _________ or a

   ________________ using the factors from step 2.
4. _______________ by _________________.

Example 1: Factor using grouping.

a. $3x^2 - x - 10$

b. $8x^2 - 10x + 3$

c. $9y^2+5y-4$

d. $12x^2+7xy-12y^2$

Example 4: Factor completely

a. $4x^2-18x-10$

c. $24y^4+10y^3-4y^2$

b. $3x^3+14x^2+8x$

d. $6(y+1)x^2+33(y+1)x+15(y+1)$

Section 6.4: FACTORING SPECIAL FORMS

When you are done with your homework you should be able to…

- π Factor the difference of two squares
- π Factor perfect square trinomials
- π Factor the sum of two cubes
- π Factor the difference of two cubes

WARM-UP:

Factor:

a. $3a^2 - ab - 14b^2$

c. $80z^3 + 80z^2 - 60z$

b. $12x^2 - 33x + 21$

d. $-10x^2y^4 + 14xy^4 + 12y^4$

THE DIFFERENCE OF TWO SQUARES

If _____ and _____ are real numbers, or ________________ expressions, then

The __________________ of the _____________ of __________ __________ factors as the _______________ of a ____________ and a _________________ of those terms.

16 PERFECT SQUARES

$1 =$ ________	$25 =$ ________	$81 =$ ________	$169 =$ ________
$4 =$ ________	$36 =$ ________	$100 =$ ________	$196 =$ ________
$9 =$ ________	$49 =$ ________	$121 =$ ________	$225 =$ ________
$16 =$ _______	$64 =$ ________	$144 =$ ________	$256 =$ ________

Example 1: Factor.

a. $x^2 - 144$

c. $25 - 4x^{10}$

b. $16x^2 - 196y^2$

d. $18x^3 - 2x$

FACTORING PERFECT SQUARE TRINOMIALS

Let ______ and ______ be real numbers, _____________________, or _________________ expressions.

1. $A^2 + 2AB + B^2 =$ ____________________________

2. $A^2 - 2AB + B^2 =$ ____________________________

- The ________________ and ________________ terms are _______________ of__________________ or ___________________.
- The _____________________ term is __________________ the _______________ of the _____________________ being ________________ in the ___________ and ________________ terms.

Example 2: Factor.

a. $9x^2 + 6x + 1$

c. $x^2 - 18xy + 81y^2$

b. $x^2 + 4x + 4$

d. $2y^2 - 40y + 200$

FACTORING THE SUM OR DIFFERENCE OF TWO CUBES

Let _____ and _____ be real numbers, _________________, or _______________ expressions.

1. $A^3 + B^3 =$ ____________________________

2. $A^3 - B^3 =$ ____________________________

Example 3: Factor.

a. $x^3 + 64$

c. $128 - 250y^3$

b. $8y^3 - 1$

d. $125x^3 + y^3$

Example 4: Factor completely

a. $25x^2 - \frac{4}{49}$

b. $20x^3 - 5x$

c. $(y+6)^2 - (y-2)^2$

d. $0.064 - x^3$

Section 6.5: A GENERAL FACTORING STRATEGY

When you are done with your homework you should be able to…

- π Recognize the appropriate method for factoring a polynomial
- π Use a general strategy for factoring polynomials

WARM-UP:

Multiply:

a. $(x+1)(x^2-x+1)$

b. $(2x-3y)(4x^2+6xy+9y^2)$

A STRATEGY FOR FACTORING A POLYNOMIAL

1. If there is a ___________________ factor other than _____, factor the _____________.

2. Determine the _______________ of _____________ in the polynomial and try factoring as follows:

   a. If there are __________ terms, can the _____________ be factored by one of the following special forms?

      _____________________ of ______ ______________:

______________ of ________ ______________:

________________ of __________ _____________:

b. If there are ______________ terms, is the ________________ a

_________________ _________________ ________________? If so,

factor by one of the following special forms:

_________________________ = ____________________________

_________________________ = ____________________________

If the trinomial is ___________ a _________________

______________, try _________________ by _____________ and

__________________ or _____________________.

c. If there are ________________ or _______________ terms, try

__________________ by ____________________.

3. Check to see if any ___________________ with more than one term in the

_________________ ___________________ can be factored

______________. If so, __________________ completely.

4. __________________ by _______________________.

Example 1: Factor

a. $5x^4 - 45x^2$

b. $4x^2 - 16x - 48$

c. $4x^5 - 64x$

d. $x^3 - 4x^2 - 9x + 36$

e. $3x^3 - 30x^2 + 75x$

f. $2w^5 + 54w^2$

g. $3x^4y - 48y^5$

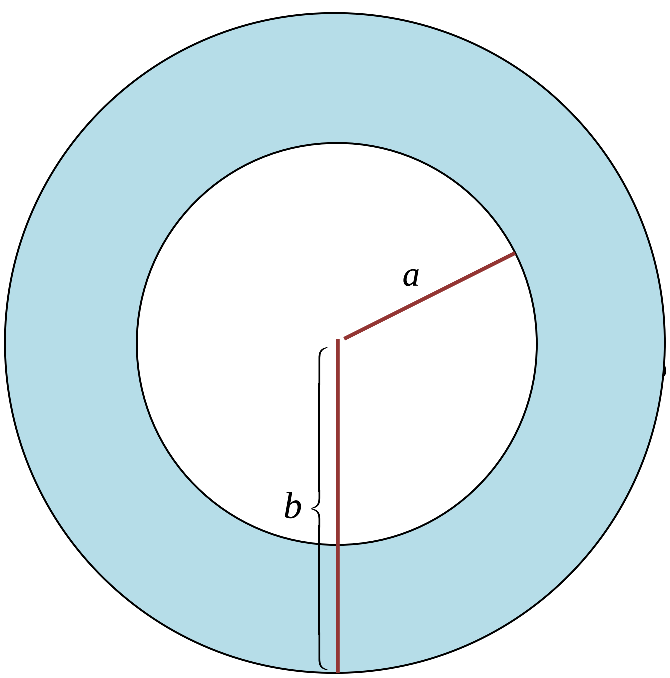
h. $12x^3 + 36x^2y + 27xy^2$

i. $12x^2(x-1)-4x(x-1)-5(x-1)$

j. $x^2+14x+49-16a^2$

APPLICATION

Express the area of the shaded ring shown in the figure in terms of π. Then factor this expression completely.



Section 6.6: SOLVING QUADRATIC EQUATIONS BY FACTORING

When you are done with your homework you should be able to…

- π Use the zero-product principle
- π Solve quadratic equations by factoring
- π Solve problems using quadratic equations

WARM-UP:

a. Factor:

$x^2 - 8x + 7$

b. Solve:

$x - 7 = 0$

DEFINITION OF A QUADRATIC EQUATION

A ________________ __________________ in ____ is an equation that can

be written in the ________________ ______________

where _____, _____, and _____ are real numbers, with ____________. A

________________ ________________ in _____ is also called a ___________-

_______________ ________________ equation in ______.

SOLVING QUADRATIC EQUATIONS BY FACTORING

Consider the quadratic equation $x^2 - 8x + 7 = 0$. How is this different from the first warm-up?

We can _______________ the ____________ side of the __________________

equation __________________ to get _____________________. If a quadratic

equation has a zero on one side and a __________________ ________________

on the other side, it can be _________________ using the ________________-

__________________ principle.

THE ZERO-PRODUCT PRINCIPLE

If the __________________ of two or more ______________ expressions is

________________, then _____ _______________ one of them is __________

to ________________.

Example 1: Solve the following equations:

a. $2x-11=0$

b. $x+1=0$

c. $(2x-11)(x+1)=0$

STEPS FOR SOLVING A QUADRATIC EQUATION BY FACTORING

1. If necessary, ________________ the equation in ________________ form _______________________, moving all _____________ to one side, thereby obtaining ______________ on the other side.
2. ____________________.
3. Apply the _____________-_________________ principle, setting each _______________ equal to ________________.
4. ________________ the equations formed in step 3.
5. ________________ the _________________ in the _________________ equation.

Example 2: Solve:

a. $x(x+9)=0$

b. $8(x-5)(3x+11)=0$

c. $x^2+x-42=0$

d. $x^2=8x$

e. $4x^2 = 12x - 9$

f. $(x+3)(3x+5) = 7$

g. $x^3 - 4x = 0$

h. $(x-3)^2+2(x-3)-8=0$

APPLICATION

An explosion causes debris to rise vertically with an initial velocity of 72 feet per second. The formula $h=-16t^2+72t$ describes the height of the debris above the ground, *h*, in feet, *t* seconds after the explosion.

a. How long will it take for the debris to hit the ground?

b. When will the debris be 32 feet above the ground?

Section 7.1: RATIONAL EXPRESSIONS AND THEIR SIMPLIFICATION

When you are done with your homework you should be able to…

- π Find numbers for which a rational expression is undefined
- π Simplify rational expressions
- π Solve applied problems involving rational expressions

WARM-UP:

a. Factor:

$x^3 - 8x^2 + 2x - 16$

b. Solve:

$2x^2 - x - 10 = 0$

EXCLUDING NUMBERS FROM RATIONAL EXPRESSIONS

A ________________ expression is the _________________ of two _____________________. Rational expressions indicate ___________________ and division by ________________ is ____________________. This means that we _______________ ____________________ any value or values of the ___________ that make a ____________________ _______________!

Example 1: Find all numbers for which the rational expression is undefined:

a. $\frac{5}{x}$

b. $\frac{x+1}{x-4}$

c. $\frac{8x-40}{x^2+3x-28}$

d. $\frac{x-12}{x^2+4}$

SIMPLIFYING RATIONAL EXPRESSIONS

A ________________ ________________ is ________________ if its

__________________ and _________________ have ______ common

_______________ other than ______ or ______.

FUNDAMENTAL PRINCIPLE OF RATIONAL EXPRESSIONS

If _____, _____, and _____ are __________________ and ______ and _____

are __________ ______,

STEPS FOR SIMPLIFYING RATIONAL EXPRESSIONS

1. _______________ the _______________ and the __________________ completely.

2. ________________ both the _________________ and the ________________ by any _______________ _______________.

Example 2: Simplify:

a. $\dfrac{4x-64}{16x}$

b. $\dfrac{6y+18}{11y+33}$

c. $\dfrac{x^2-12x+36}{4x-24}$

d. $\dfrac{x^3+4x^2-3x-12}{x+4}$

e. $\dfrac{x+5}{x-5}$

f. $\dfrac{x^3-1}{x^2-1}$

SIMPLIFYING RATIONAL EXPRESIONS WITH OPPOSITE FACTORS IN THE NUMERATOR AND DENOMINATOR

The _________________ of two _________________ that have ____________ signs and are _________________ ___________________ is _______.

Example 3: Simplify:

a. $\frac{x-3}{3-x}$

b. $\frac{9x-15}{5-3x}$

c. $\frac{x^2-4}{2-x}$

APPLICATION

A company that manufactures small canoes has costs given by the equation

$$C = \frac{20x + 20000}{x}$$

in which x is the number of canoes manufactured and C is the cost to manufacture each canoe.

a. Find the cost per canoe when manufacturing 100 canoes.

b. Find the cost per canoe when manufacturing 10000 canoes.

c. Does the cost per canoe increase or decrease as more canoes are manufactured?

Section 7.2: MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

When you are done with your homework you should be able to...

- π Multiply rational expressions
- π Divide rational expressions

WARM-UP:

Simplify:

a. $\frac{a^2-2ab+b^2}{a^2-b^2}$

b. $\frac{x^2-3x+2}{x-1}$

MULTIPLYING RATIONAL EXPRESSIONS

If _____, ______, ______, and _____ are polynomials, where ______ and _______, then

The _________________ of two _______________ _________________ is the _______________ of their _______________, divided by the ________________ of their ________________.

STEPS FOR MULTIPLYING RATIONAL EXPRESSIONS

1. ________________ all ___________________ and _____________________.
2. ________________ ___________________ and ___________________ by common ___________________.
3. ___________________ the remaining factors in the _________________ and ___________________ the remaining factors in the ________________.

Example 1: Multiply.

a. $\frac{x-5}{3}\cdot\frac{18}{x-8}$

c. $\frac{9y+21}{y^2-2y}\cdot\frac{y-2}{3y+7}$

b. $\frac{x}{5}\cdot\frac{30}{x-4}$

d. $\frac{x^2+5x+6}{x^2+x-6}\cdot\frac{x^2-9}{x^2-x-6}$

DIVIDING RATIONAL EXPRESSIONS

If ______, _______, _______, and ______ are polynomials, where _______, _______,

and________, then

The ____________________ of two _________________ ____________________ is the _________________ of the ____________ expression and the ____________ of the ________________.

Example 2: Divide.

a. $\frac{x}{3} \div \frac{3}{8}$

b. $\frac{x+5}{7} \div \frac{4x+20}{9}$

c. $\frac{y^2-2y}{15} \div \frac{y-2}{5}$

d. $\frac{x^2-4y^2}{x^2+3xy+2y^2} \div \frac{x^2-4xy+4y^2}{x+y}$

Example 3: Perform the indicated operation or operations.

e. $$\frac{5x^2-x}{3x+2} \div \left(\frac{6x^2+x-2}{10x^2+3x-1} \cdot \frac{2x^2-x-1}{2x^2-x} \right)$$

f. $$\frac{5xy-ay-5xb+ab}{25x^2-a^2} \div \frac{y^3-b^3}{15x+3a}$$

Section 7.3: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH THE SAME DENOMINATOR

When you are done with your homework you should be able to…

- π Add rational expressions with the same denominator
- π Subtract rational expressions with the same denominator
- π Add and subtract rational expressions with opposite denominators

WARM-UP:

Simplify:

a. $\frac{b^2-a^2}{a^2-b^2}$

b. $\frac{x^2-2x+1}{1-x}$

ADDING RATIONAL EXPRESSIONS WITH COMMON DENOMINATORS

If _____ and ______ are ___________________ expressions, then

To __________ rational expressions with the _________ ________________,

add ________________ and place the __________ over the _____________

_____________________. If possible, ___________________ the result.

SUBTRACTING RATIONAL EXPRESSIONS WITH COMMON DENOMINATORS

If _____ and ______ are ___________________ expressions, then

To ______________ rational expressions with the _________ ____________,

subtract ________________ and place the ____________________ over the

_________________ ____________________. If possible, ______________ the result.

Example 1: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{x}{15}+\frac{4x}{15}$

b. $\frac{x+4}{9}+\frac{2x-25}{9}$

c. $\frac{x}{x-1}-\frac{1}{x-1}$

d. $\frac{3x+2}{3x+4}+\frac{3x+6}{3x+4}$

e. $\dfrac{x^3-3}{2x^4}-\dfrac{7x^3-3}{2x^4}$

f. $\dfrac{x^2+9x}{4x^2-11x-3}+\dfrac{3x-5x^2}{4x^2-11x-3}$

g. $\dfrac{3y^2-2}{3y^2+10y-8}-\dfrac{y+10}{3y^2+10y-8}-\dfrac{y^2-6y}{3y^2+10y-8}$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS WITH OPPOSITE DENOMINATORS

When one denominator is the ____________________, or ________________ _________________, of the other, first __________________ either rational expression by ___________ to obtain a _______________ _________________.

Example 2: Add or subtract as indicated. Simplify the result, if possible.

a. $\frac{6x+7}{x-6}+\frac{3x}{6-x}$

c. $\frac{4-x}{x-9}-\frac{3x-8}{9-x}$

b. $\frac{x^2}{x-3}+\frac{9}{3-x}$

d. $\frac{2x+3}{x^2-x-30}+\frac{x-2}{30+x-x^2}$