MATH 64/GRACEY/INTERMEDIATE ALGEBRA NAME Key

EXAM 3/75 POINTS POSSIBLE

CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANISED, EASY TO READ FASHION.

1. (6 POINTS) Find the distance between the pair of points $(-1,4)$ and $(-3,-8)$. If necessary, express the answer in **simplified radical form**.

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(-3-(-1))^2+(-8-4)^2}$

$d=\sqrt{(-2)^2+(-12)^2}$

$d=\sqrt{4+144}$

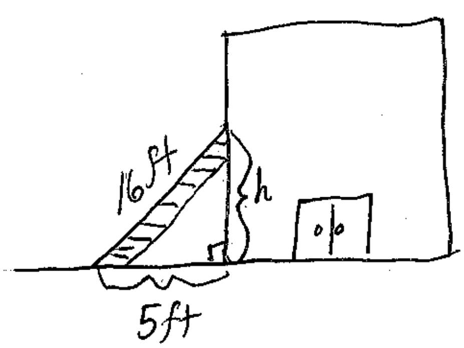
$d=\sqrt{148}$

$d=\sqrt{4\cdot 37}$

$d=2\sqrt{37}\approx 12.2$

Distance: $2\sqrt{37}$ units

2. (10 POINTS) No credit will be awarded for guessing. A 16 foot ladder is leaning against a building, with the base of the ladder 5 feet from the building. How high up on the building will the top of the ladder reach? You may round to the nearest tenth of a foot, if necessary.



$h^2+(5)^2=(16)^2$

$h^2+25=256$

$\quad -25 \quad -25$

$\sqrt{h^2}=\sqrt{231}$

$h=\pm\sqrt{231}$

$h\approx 15.2$

The ladder will reach approximately 15.2ft high up on the building.

3. (14 POINTS) You may round the intercepts to the nearest tenth, if necessary. (2 POINTS) Sketch the graph.

Use the vertex and intercepts to sketch the graph of $f(x)=(x+4)^2-9$.

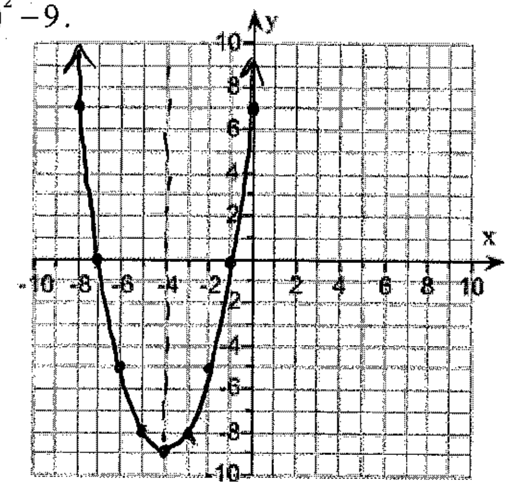
(4 POINTS) Vertex: $\boxed{(-4,-9)}$

~~f(x) ...~~

$f(x) = a(x-h)^2 + K$ a=1

$f(x) = 1(x-(-4))^2+(-9)$ h=-4

K=-9



$f(-3) = 1-9 = -8$

$f(-2) = 4-9 = -5$

(4 POINTS) *x*-intercepts: $\boxed{(-7,0) \text{ and } (-1,0)}$

$0 = (x+4)^2 - 9$

$+9 \qquad +9$

$\sqrt{9} = \sqrt{(x+4)^2}$

$\pm 3 = x+4$ → $x = -4 \pm 3$

$x = -7, -1$

(1 POINT) Axis of symmetry: $\boxed{x=-4}$

(1 POINT) *y*-intercept: $\boxed{(0,7)}$

$f(0) = (0+4)^2 - 9 \rightarrow f(0) = 16-9 \rightarrow f(0) = 7$

(2 POINTS) Domain in interval notation: $(-\infty, \infty)$

(2 POINTS) Range in interval notation: $[-9, \infty)$

4. (2 POINTS) Find the midpoint of the line segment with endpoints $(-2,4)$ and $(9,1)$.

midpoint $= \left(\frac{9+(-2)}{2}, \frac{1+4}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$

Midpoint: $\boxed{\left(\frac{7}{2}, \frac{5}{2}\right)}$

5. (8 POINTS) Solve the following equation using the method of your choice. Give exact answers, using radicals and i as needed.

$7x^2 = -4x - 2$

$a=7$

$b=4$

$c=2$

$7x^2+4x+2=0$

$x = \frac{-(4) \pm \sqrt{(4)^2-4(7)(2)}}{2(7)}$

$x = \frac{-4 \pm \sqrt{16-56}}{14}$

$x = \frac{-4 \pm \sqrt{-40}}{14}$

$x = \frac{-4 \pm \sqrt{(-4)(10)}}{14}$

$x = \frac{-4 \pm 2\sqrt{10}\, i}{14}$

$x = -\frac{4}{14} \pm \frac{2\sqrt{10}}{14} i$

$x = -\frac{2}{7} \pm \frac{\sqrt{10}}{7} i$

$\boxed{\left\{ -\frac{2}{7} \pm \frac{\sqrt{10}}{7} i \right\}}$

6. (10 POINTS) Solve. Give exact answers, using radicals and i as needed.

$x^4 - x^2 - 6 = 0$

Let $u = x^2$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u-3=0$ or $u+2=0$

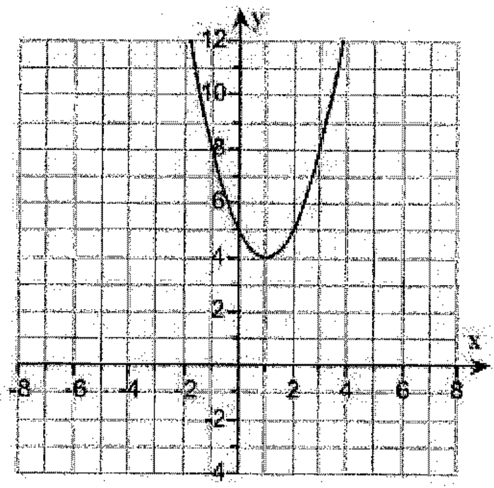
$u=3$ $\quad$ $u=-2$

$u=3$	$u=-2$
$\sqrt{3} = \sqrt{x^2}$	$\sqrt{-2} = \sqrt{x^2}$
$\pm\sqrt{3} = x$	$\pm\sqrt{2}\, i = x$

$\boxed{\left\{ \pm\sqrt{2}\, i, \pm\sqrt{3} \right\}}$

7. (5 POINTS)

The graph of a quadratic function is given. Select the function's equation from the choices given.



Choose the correct equation below.

- ○ A. $f(x) = (x+1)^2 + 4$
- ● B. $f(x) = (x-1)^2 + 4$
- ○ C. $f(x) = (x-1)^2 - 4$
- ○ D. $f(x) = (x+1)^2 - 4$

8. (10 POINTS) **<u>Solve by completing the square</u>**. Give exact answers, using radicals and *i* as needed.

$x^2 - 8x - 4 = 0$

$x^2 - 8x + (4)^2 = 4 + (-4)^2$

$(x-4)^2 = 4 + 16$

$\sqrt{(x-4)^2} = \sqrt{20}$

$x - 4 = \pm\sqrt{20}$

$x = 4 \pm \sqrt{4 \cdot 5}$

$x = 4 \pm 2\sqrt{5}$

$\boxed{\{4 \pm 2\sqrt{5}\}}$

9. (10 POINTS) The profit, $P(x)$, generated after producing and selling x units of a product is given by the function $P(x)=R(x)-C(x)$, where R and C are the revenue and cost functions, respectively. A local sandwich store has a fixed weekly cost of $545.00, and variable costs for making a roast beef sandwich are $0.60.

a. (2 POINTS) Let x represent the number of roast beef sandwiches made and sold each week. Write the weekly cost function, C, for the local sandwich store.

$C(x)=$ $0.60x + 545$

b. (3 POINTS) The function $R(x)=-0.001x^2+3x$ describes the money that the local sandwich store takes in each week from the sale of x roast beef sandwiches. What is the weekly profit function?

$P(x)=R(x)-C(x)$

$P(x)=-0.001x^2+3x$

$\underline{\quad -0.60x-545}$

$-0.001x^2+2.40x-545$

$P(x)=$ $-0.001x^2+2.40x-545$

c. (5 POINTS) Use the store's profit function to determine the number of roast beef sandwiches it should make and sell each week to maximize profit, and find the maximum weekly profit.

$a=-0.001$

$b=2.4$

$x=-\frac{b}{2a}=-\frac{2.4}{2(-.001)}=1200$

$P(1200)=-0.001(1200)^2+2.40(1200)-545$

$P(1200)=-1440+2880-545$

$P(1200)=895$

The maximum weekly profit is $895.00 when 1,200 sandwiches are sold.