MATH 64/INTERMEDIATE ALGE BRA/GRACEY

NAME Key

EXAM 2/80 POINTS POSSIBLE

CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANISED, EASY TO READ FASHION.

1. (4 POINTS) Consider $f(x)=\sqrt{1-x}$

   a. (2 POINTS) Find $f(-3)$.

   $f(-3)=$ 2

   $f(-3)=\sqrt{1-(-3)}$
   $=\sqrt{4}$
   $=2$

   b. (2 POINTS) Find the domain of f. Give your result in interval notation.

   Domain: $(-\infty, 1]$

   $1-x \geq 0$
   $-x \geq -1$
   $x \leq 1$

2. (6 POINTS) No credit will be awarded for guessing. Solve the radical equation.

   $(3x+5)^{1/3}-4=6$
   $((3x+5)^{1/3})^3=(10)^3$
   $3x+5=1000$
   $3x=995$
   $x=\frac{995}{3}$

   $\left\{\frac{995}{3}\right\}$

3. (3 POINTS) Simplify. Assume variables can be any real number. Include absolute value bars where necessary.

   a. (3 POINTS)
   $\sqrt[4]{(x-1)^4}=|x-1|$

   b. (2 POINTS)
   $\sqrt[5]{-32}=\sqrt[5]{(-2)^5}$
   $=-2$

4. (6 POINTS) Multiply and simplify. Assume that all variables in a radicand represent positive real numbers.

   $\sqrt[3]{4x^8y^5}\cdot\sqrt[3]{4x^2y^3}=\sqrt[3]{(4x^8y^5)(4x^2y^3)}$
   $=\sqrt[3]{16x^{10}y^8}$
   $=\sqrt[3]{8\cdot 2x^9\cdot x\,y^6\cdot y^2}$
   $=\sqrt[3]{(2)^3\cdot 2(x^3)^3\cdot x\,(y^2)^3\,y^2}$
   $=2x^3y^2\sqrt[3]{2xy^2}$

5. (8 POINTS) No credit will be awarded for guessing. Solve the radical equation.

$\sqrt{x+2}+\sqrt{3x+7}=1$

$-\sqrt{3x+7} \quad -\sqrt{3x+7}$

$(\sqrt{x+2})^2 = (1-\sqrt{3x+7})^2$

$x+2 = (1-\sqrt{3x+7})(1-\sqrt{3x+7})$

$x+2 = 1-\sqrt{3x+7}-\sqrt{3x+7}+3x+7$

$x+2 = 8-2\sqrt{3x+7}+3x$

$-3x-8 \quad -8 \quad -3x$

$\frac{-2x-6}{-2} = \frac{-2\sqrt{3x+7}}{-2}$

$(x+3)^2 = (\sqrt{3x+7})^2$

$x^2+6x+9 = 3x+7$

$-3x-7 \quad -3x-7$

$x^2+3x+2=0$

$(x+2)(x+1)=0$

$x+2=0$ or $x+1=0$

$x=-2$ ~~$x=-1$~~

$\{-2\}$

Check

$x=-1$

$\sqrt{-1+2}+\sqrt{3(-1)+7} \stackrel{?}{=} 1$

$\sqrt{1}+\sqrt{4} \stackrel{?}{=} 1$

$1+2 \neq 1$

$x=-2$

$\sqrt{-2+2}+\sqrt{3(-2)+7} \stackrel{?}{=} 1$

$\sqrt{0}+\sqrt{1} \stackrel{?}{=} 1$

$1=1\checkmark$

6. (5 POINTS) Use rational exponents to simplify. If rational exponents appear after simplifying, write the answer in radical notation. Assume that all variable represent positive numbers.

$\sqrt{\sqrt{xy^2}} = \sqrt{(xy^2)^{1/2}}$

$= [(xy^2)^{1/2}]^{1/2}$

$= (xy^2)^{1/4}$

$= \sqrt[4]{xy^2}$

7. (5 POINTS) Simplify. Assume that all variables represent positive real numbers.

$\sqrt{\frac{128x^6y}{2xy^{-3}}} = \sqrt{64x^5y^4}$

$= \sqrt{(8)^2(x^2)^2 x (y^2)^2}$

$= 8x^2y^2\sqrt{x}$

8. (4 POINTS) Subtract. Write the result in the form $a+bi$.

$(8+5i)-(6+2i) = 8+5i-6-2i$

$= \boxed{2+3i}$

9. (6 POINTS) Divide and simplify to the form $a+bi$.

$\frac{(5-i)}{(2+i)}\frac{(2-i)}{(2-i)} = \frac{10-5i-2i+i^2}{(2)^2-(i)^2}$

$= \frac{10-7i+(-1)}{4-(-1)}$

$= \frac{9-7i}{5}$

$= \boxed{\frac{9}{5}-\frac{7}{5}i}$

10. (5 POINTS) Rationalize the denominator. Simplify, if possible.

$\frac{2}{(\sqrt{x}-3)}\frac{(\sqrt{x}+3)}{(\sqrt{x}+3)} = \frac{2(\sqrt{x}+3)}{(\sqrt{x})^2-(3)^2}$

$= \frac{2(\sqrt{x}+3)}{x-9}$

$= \boxed{\frac{2\sqrt{x}+6}{x-9}}$

11. (4 POINTS) Rationalize the numerator. Simplify, if possible.

$\frac{\sqrt[3]{25x}}{4} = \frac{\sqrt[3]{(5)^2x}}{4}\cdot\frac{\sqrt[3]{5x^2}}{\sqrt[3]{5x^2}}$

$= \frac{\sqrt[3]{(5)^3(x)^3}}{4\sqrt[3]{5x^2}}$

$= \boxed{\frac{5x}{4\sqrt[3]{5x^2}}}$

12. (4 POINTS) Simplify.

$i^{24} = (i^2)^{12} \rightarrow = (-1)^{12}$

$= \boxed{1}$

13. (6 POINTS) Multiply. Write imaginary results in the form $a+bi$.

$$(2+\sqrt{-16})(4-\sqrt{-9}) = (2+4i)(4-3i)$$
$$= 8-6i+16i-12i^2$$
$$= 8+10i-12(-1)$$
$$= \boxed{20+10i}$$

14. (8 POINTS) $\sqrt{x}+2=x$

$$(-\sqrt{x})^2 = (x-2)^2$$
$$x = (x-2)(x-2)$$
$$x = x^2-2x-2x+4$$
$$x = x^2-4x+4$$
$$0 = x^2-5x+4$$
$$0 = (x-4)(x-1)$$
$$x-4=0 \text{ or } x-1=0$$

extraneous

$\rightarrow x=4$ or $x=1$

$\{4\}$

Check:

$x=4$:
$\sqrt{4}+2 \stackrel{?}{=} 4$
$2+2=4$
$4=4$ ✓

$x=1$:
$\sqrt{1}+2 \stackrel{?}{=} 1$
$1+2 \stackrel{?}{=} 1$
$3 \neq 1$

15. (5 POINTS) Use rational exponents to simplify the expression. If rational exponents appear after simplifying, write the answer in radical notation. Assume that all variables represent positive numbers.

$$\frac{\sqrt[10]{x^9}}{\sqrt[5]{x^3}} = \frac{x^{9/10}}{x^{3/5}}$$
$$= x^{\frac{9}{10}-\frac{3}{5}\cdot\frac{2}{2}}$$
$$= x^{\frac{9}{10}-\frac{6}{10}}$$
$$= x^{\frac{3}{10}}$$
$$= \boxed{\sqrt[10]{x^3}}$$