CHAPTER 7 ❑ MATLAB EXERCISES

1. The MATLAB command **poly(A)** produces the coefficients of the characteristic polynomial of the square matrix A, beginning with the highest degree term. Find the characteristic polynomial of the following matrices.

   (a) $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & -7 & 8 \\ -9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

2. If we set **p = poly(A)**, then the command **roots(p)** calculates the roots of the characteristic polynomial of the matrix A. Use this sequence of commands to find the eigenvalues of the matrices in Exercise 1.

3. The MATLAB command **[V, D] = eig(A)** produces a diagonal matrix D containing the eigenvalues of A on the diagonal, and a matrix V whose columns are the corresponding eigenvectors. Use this command to find the eigenvalues and corresponding eigenvectors of the three matrices in Exercise 1.

4. Let

   $$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

   Use MATLAB to find the eigenvalues and corresponding eigenvectors of A, A^T, and A^{-1}. What do you observe?

5. Let

   $$A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}.$$

   We can use MATLAB to diagonalize A as follows. First compute the eigenvalues and eigenvectors of A, using the command **[P, D] = eig(A).** The diagonal matrix D contains the eigenvalues of A, and the corresponding eigenvectors form the columns of P. Verify that P diagonalizes A by showing that $P^{-1}AP = D$.

6. Follow the procedure outlined in Exercise 5 to show that the matrix

   $$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

   is *not* diagonalizable.

7. Follow the procedure outlined in Exercise 5 to diagonalize (if possible) the following matrices.

(a) $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & -7 & 8 \\ -9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

8. For a symmetric matrix A, the MATLAB command **[P, D] = eig(A)** will produce a diagonal matrix D containing the eigenvalues of A, and an *orthogonal* matrix P containing the corresponding eigenvectors. For instance, if

$$A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

is the matrix from Section 7.3, Example 8, then the command **[P, D] = eig(A)** yields

$$P = \begin{bmatrix} -0.8944 & -0.4472 \\ 0.4472 & -0.8944 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix},$$

which is equivalent to the solution given in the text.

Use this procedure to orthogonally diagonalize the following symmetric matrices.

(a) $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & -3 \end{bmatrix}$