CHAPTER 2 ❑ MATLAB EXERCISES

1. Enter the matrices

$$A = \begin{bmatrix} 0 & -4 & 5 \\ 3 & 1 & -2 \\ 2 & 1 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -5 & 6 & 7 \\ 0 & -1 & 2 \\ 4 & 0 & -3 \end{bmatrix}.$$

Use MATLAB to find

(a) $A + B$ (b) $B - 3A$ (c) AB (d) BA.

2. Enter the three matrices

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix} \quad B = \begin{bmatrix} 1.0000 & 0.5000 & 0.3333 & 0.2500 \\ 0.5000 & 0.3333 & 0.2500 & 0.2000 \\ 0.3333 & 0.2500 & 0.2000 & 0.1667 \\ 0.2500 & 0.2000 & 0.1667 & 0.1429 \end{bmatrix}$$

$$C = \begin{bmatrix} 16 & -120 & 240 & -140 \\ -120 & 1200 & -2700 & 1680 \\ 240 & -2700 & 6480 & -4200 \\ -140 & 1680 & -4200 & 2800 \end{bmatrix}.$$

(a) Calculate $A - B$. Use the **format long** command to verify the result. Return to the standard short format with **format short.**

(b) Calculate AC and BC. Define what is meant by the inverse of a square matrix. What is the inverse of the matrix A? Of the matrix C?

3. Write the following system of linear equations in the form $AX = B$ and use the MATLAB command **A\ B** to solve the system.

$$\begin{aligned} 3x + 3y + 4z &= 2 \\ x + y + 4z &= -2 \\ 2x + 5y + 4z &= 3 \end{aligned}$$

Check your answer using **rref.**

4. Enter the matrices

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 2 & -1 & -5 \\ 7 & -5 & 0 & 6 \\ -4 & 0 & 7 & 12 \end{bmatrix} \quad \text{and} \quad B = \textbf{pascal(4)}.$$

(a) Use the MATLAB command **trace** to find the traces of A, B, and $A + B$. What do you observe?

(b) What is the relationship between the trace of AB and the trace of BA?

5. Use the MATLAB command **diag** to form the 5×5 diagonal matrix D with diagonal entries 0, -1, -2, -3, and -4. Find the product $D^4 = DDDD$. If D is an $n \times n$ diagonal matrix, describe how to find the product D^k for any positive integer k.

6. Enter the three matrices

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -6 & 3 & 5 \\ 0 & -3 & -6 & 8 \\ 3 & 5 & 0 & 7 \\ -1 & 0 & -7 & 9 \end{bmatrix} \quad C = \begin{bmatrix} 16 & -1 & 4 & -1 \\ -3 & 12 & -7 & 8 \\ 4 & -5 & 0 & 0 \\ -14 & 3 & 2 & 8 \end{bmatrix}$$

(a) Calculate $AB - AC$ and $A(B - C)$. What do you observe?
(b) Calculate $3(AC)$, $A(3C)$, and $(3A)C$. What do you observe?

7. Let

$$A = \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & \frac{1}{4} \end{bmatrix}.$$

Compute A^2, A^3, and A^8. Describe the matrix A^n for large n.

8. Enter the matrices

$$A11 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad A12 = \textbf{zeros(2, 2)} \quad A22 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

(a) Form the 4×4 matrix A using the following MATLAB construction:
A = [A11 A12; A12 A22]
(b) Find the smallest value of n such that $A^n = A$.

9. Use the MATLAB command **inv** to find the inverse of the following matrix A. Then adjoin the identity matrix $I =$ **eye(3)** to A to form the 3×6 matrix $B = [A \quad I]$. Row-reduce B to compute the inverse of A again. What do you observe?

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{bmatrix}$$

10. Let A and B be the following 3×3 matrices.

$$A = \begin{bmatrix} 2 & 4 & \frac{5}{2} \\ -\frac{3}{4} & 2 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{4} \\ \frac{3}{2} & \frac{1}{2} & -2 \\ \frac{1}{4} & 1 & \frac{1}{2} \end{bmatrix}$$

(a) Calculate $A^{-1}B^{-1}$, $(AB)^{-1}$, and $(BA)^{-1}$. What do you observe?
(b) Find $(A^{-1})^T$ and $(A^T)^{-1}$. What do you observe? Remember: The MATLAB command for the transpose of a matrix A is A'.

11. In this project you will use MATLAB to find the least squares regression line for the set of data $(1, 1)$, $(2, 2)$, $(3, 4)$, $(4, 4)$, and $(5, 6)$ from Example 10, Section 2.5.

(a) Form the following matrices.

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \quad Y = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 4 \\ 6 \end{bmatrix}$$

(b) Let $A = (X^TX)^{-1}X^TY$.
(c) Compare your answer with the MATLAB least squares command **polyfit.** (Hint: Let $X1 = X(:, 2)$ and type **polyfit(X1,Y,1).**)

(d) Plot the data using the MATLAB commands

t = (0: 0.1: 6);
plot(X1,Y, '+')

(e) Plot the least squares line for these data by using the MATLAB commands

t = (0: 0.1: 6);
p=polyfit(X1,Y,1);
f=polyval(p,t);
plot(t,f, '*')

(f) You can combine the previous plots as follows

plot(X1,Y, '+',t,f, '*')

12. Repeat Exercise 11 for the data $(0, 6)$, $(4, 3)$, $(5, 0)$, $(8, -4)$, $(10, -5)$. Plot the data and the least squares line on the interval $[0, 10]$. That is, use **t = (0: 0.1: 10);.**