

YOUR WORK MUST BE ORGANIZED AND CLEAR  
NO GRAPHING CALCULATOR IS PERMITTED

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1. (9 POINTS) Find the solution set of the system of equations represented in the augmented matrix. If applicable, please parameterize.

$$\begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 3 & 9 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. (15 POINTS) Please circle true or false.

- a. T    F    A homogeneous system of equations must have at least one solution.
- b. T    F    Multiplication of matrices is commutative.
- c. T    F    If  $A$  and  $B$  are  $n \times n$  matrices and  $A$  is invertible then  $(A^{-1}BA)^2 = AB^2A^{-1}$ .
- d. T    F    All  $n \times n$  matrices are invertible.
- e. T    F    If  $C$  is invertible, and  $AC = BC$  then  $A = B$ .

3. (9 POINTS) Please match the term/phrase with the correct definition. Each item is worth 3 points.

a. \_\_\_\_\_

b. \_\_\_\_\_

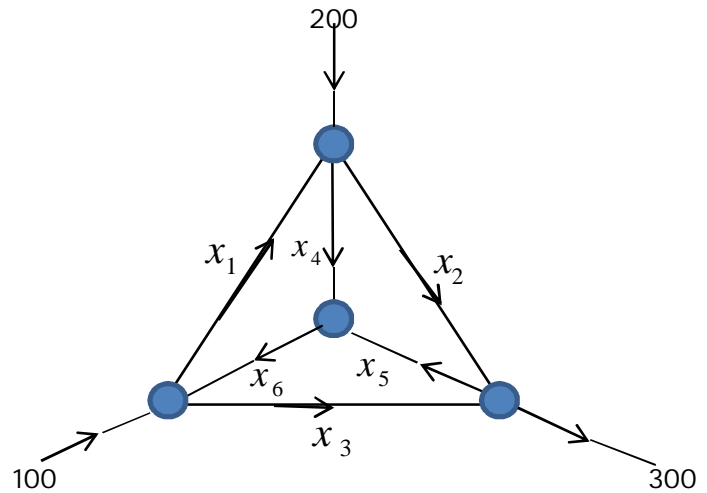
c. \_\_\_\_\_

Term/Phrase	Definition
a. SCALAR MULTIPLICATION	1. A matrix which does not have an inverse.
b. ELEMENTARY MATRIX	2. ...can be obtained from the identity matrix with a single row operation.
c. SINGULAR MATRIX	3. $c \begin{bmatrix} a_{ij} \end{bmatrix} = \begin{bmatrix} ca_{ij} \end{bmatrix}$

4. (10 POINTS) The figure shows the flow through a network.

Set up the system for

$$x_i, i = 1, 2, \dots, 6$$



5. (10 POINTS) Use an  $LU$ -factorization of the coefficient matrix to solve the linear system. Recall, you need to solve the lower triangular system,  $L\mathbf{y} = \mathbf{b}$ , and the upper triangular system,  $U\mathbf{x} = \mathbf{y}$ .

$$x + z = 3$$

$$2x + y + 2z = 7$$

$$3x + 2y + 6z = 8$$

6. (18 POINTS) Consider the following matrix  $A$ .

$$A = \begin{bmatrix} -2 & 5 \\ -1 & 2 \end{bmatrix}$$

a. (10 POINTS) Find  $A^{-1}$ .

b. (4 POINTS) Find  $(A^T)^{-1}$ .

c. (4 POINTS) Find  $(5A)^{-1}$ .

7. (9 POINTS) A country is divided into three regions. Each year, 10% of the residents of Region 1 move to Region 2 and 5% move to Region 3; 15% of the residents of Region 2 move to Region 1 and 5% move to Region 3; and 10% of the residents of Region 3 move to Region 1 and 10% move to Region 2. This year each region has a population of 100,000. Find the population of each region in (a) 1 year and (b) in 3 years.

8. (10 POINTS) A matrix is idempotent if  $A^2 = A$ . Prove that if  $A$  is an  $n \times n$  matrix that is idempotent and invertible, then  $A = I_n$ .
9. (10 POINTS) A square matrix is called skew-symmetric if  $A^T = -A$ . Prove that if  $A$  and  $B$  are  $n \times n$  skew-symmetric matrices, then  $A + B$  is skew-symmetric.