MATH 270/GRACEY
DEFINITIONS AND THEOREMS TO MEMORIZE WORD FOR WORD FOR EXAM 2
PLEASE NOTE THAT YOU MUST KNOW ALL OTHER DEFINITIONS AND THEOREMS WELL ENOUGH TO USE THEM IN PROBLEMS AND PROOFS

DEFINITIONS

DEFINITION OF SUBSPACE OF A VECTOR SPACE

A nonempty set W of a vector space V is called a **subspace** of V when W is a vector space under the operations of addition and scalar multiplication defined in V.

DEFINITION OF SPANNING SET OF A VECTOR SPACE

Let $S=\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\ldots,\mathbf{v}_k\}$ be a subset of a vector space V. The set S is called a **spanning set** of V when *every* vector in V can be written as a linear combination of vectors in S.

DEFINITION OF LINEAR DEPENDENCE AND LINEAR INDEPENDENCE

A set of vectors $S=\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\ldots,\mathbf{v}_k\}$ in a vector space V is called **linearly independent** when the vector equation

$c_1\mathbf{v}_1+c_2\mathbf{v}_2+c_3\mathbf{v}_3+\cdots+c_k\mathbf{v}_k=\mathbf{0}$

has only the trivial solution

$c_1=0,\ c_2=0,\ c_3=0,\ldots,\ c_k=0$.

If there are also nontrivial solutions, then S is called **linearly dependent**.

THEOREMS

THEOREM 4.11: NUMBER OF VECTORS IN A BASIS

If a vector space V has one basis with n vectors, then every basis for V has n vectors.

THEOREM 4.21: TRANSITION MATRIX FROM B TO B'

Let $B=\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\ldots,\mathbf{v}_n\}$ and $B'=\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\ldots,\mathbf{u}_n\}$ be two bases for R^n. Then the transition matrix P^{-1} from B to B' can be found by using Gauss-Jordan elimination on the $n\times 2n$ matrix $[B' \quad B]$, as follows.

$$[B' \quad B] \longrightarrow [I_n \quad P^{-1}]$$