MATH 270/GRACEY
DEFINITIONS AND THEOREMS TO MEMORIZE FOR EXAM 1

DEFINITIONS

SOLUTIONS OF SYSTEMS OF LINEAR EQUATIONS

A solution of a system of linear equations is a sequence of numbers $s_1, s_2, s_3, \ldots, s_n$ that is a solution of each of the linear equations in the system.

ELEMENTARY MATRIX

An $n \times n$ matrix is called an elementary matrix when it can be obtained from the identity matrix I_n by a single elementary row operation.

LINEAR COMBINATIONS

The matrix product $A\mathbf{x}$ is a linear combination of the column vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots, \mathbf{a}_n$ that form the coefficient matrix A.

$$x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + x_3 \begin{bmatrix} a_{13} \\ a_{23} \\ \vdots \\ a_{m3} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

The system $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b}$ can be expressed as such a linear combination, where the coefficients of the linear combination are a solution of the system.

THEOREMS

THEOREM 1.1: THE NUMBER OF SOLUTIONS OF A HOMOGENEOUS SYSTEM

Every homogeneous system of linear equations is consistent. If the system has fewer equations than variables, then it must have infinitely many solutions.

THEOREM 2.12: REPRESENTING ELEMENTARY ROW OPERATIONS

Let E be the elementary matrix obtained by performing an elementary row operation on I_n. If that same elementary row operation is performed on an $m \times n$ matrix A, then the resulting matrix is given by the product EA.