MATH 270/MYERS NAME___________________________________

EXAM 2/CHAPTER 3 GUIDED NOTEBOOK

YOU MAY USE A TI-83/84/85/86 CALCULATOR

100 POINTS POSSIBLE

YOUR WORK MUST BE ORGANIZED AND CLEAR

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1. (10 POINTS, 2 POINTS EACH) Please circle true or false.

a. T F If A is a 15 by 6 matrix of rank 2, the number of independent vectors satisfying $A\mathbf{x} = \mathbf{0}$ is 13.

b. T F If the determinant of an $n \times n$ matrix is nonzero, then $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

c. T F The columns of a matrix are the basis for the column space.

d. T F The function $f(x) = \cos x$ is a linear transformation from R into R.

e. T F The range of a linear transformation is a subspace of the codomain.

2. (12 POINTS) Consider the determinants of two 3×3 matrices.

$|A| = -1$, $|B| = 12$

a. (3 POINTS) Find $\det\left(A^{-1}\right)$.

b. (3 POINTS) Find $\det\left(A^{T}\right)$.

c. (3 POINTS) Find $\det(AB)$.

d. (3 POINTS) Find $\det(5A)$.

3. (8 POINTS) Find the standard matrix A for $T = T_2 \circ T_1$ where

$T_1 : R^2 \to R^2,\ T_1(x, y) = (x - 2y, 2x + 3y)$

$T_2 : R^2 \to R^2,\ T_2(x, y) = (y, 0)$

4. (6 POINTS) Find the inverse of the linear transformation defined by

$T : R^2 \to R^2,\ T(x, y) = (-2x, 2y)$.

5. (6 POINTS) Determine whether the set of all 3×3 symmetric matrices is a subspace of $M_{3,3}$ with the standard operations. Be sure to justify your answer.

6. (6 POINTS) Use a determinant to find the area of a triangle with vertices $(3,8),\ (2,-1)$ and $(-3,2)$. Do not use your graphing calculator. Show all work for full credit.

7. (16 POINTS) Define the linear transformation *T* by $T(\mathbf{x}) = A\mathbf{x}$. Let

$$A = \begin{bmatrix} 1 & 3 \\ -1 & -3 \\ 2 & 2 \end{bmatrix}.$$

Please find the following and show all work.

a. (4 POINTS) A basis for ker (*T*):

b. (1 POINTS) nullity (*T*) = ________

c. (4 POINTS) A basis for the range (*T*):

d. (1 POINTS) rank (*T*) = _________

e. (2 POINTS) Is *T* one-to-one? YES NO

f. (2 POINTS) Is *T* onto? YES NO

g. (2 POINTS) Is *T* an isomorphism? YES NO

8. (6 POINTS) Let *T* be a linear transformation from $M_{2,2}$ into $M_{2,2}$ such that

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix},\ T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix},$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix},\ T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}.$$

Find $T\left(\begin{bmatrix} 5 & 3 \\ -1 & 4 \end{bmatrix}\right)$. Please show all work.

9. (6 POINTS) Prove that if *A* and *B* are similar, then A^T and B^T are similar.

10. (6 POINTS) Determine whether the function is a linear transformation.

$T : M_{2,2} \rightarrow R,\ T(A) = b^2$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

11. (8 POINTS) Determine whether the set $S = \{2, x+1, x^2+2\}$ is a basis for P_2.

12. (10 POINTS) Find the coordinate matrix of $\mathbf{x} = (3, 19, 2)$ in R^4 relative to the basis $B' = \{(8, 11, 0), (7, 0, 10), (1, 4, 6)\}$.