MATH 270/GRACEY NAME___________________________________

EXAM 3/SECTIONS 5.1-5.4, 6.1-6.2 HW______% Q______% O______% E______%

100 POINTS POSSIBLE

YOUR WORK MUST BE ORGANIZED AND CLEAR

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1. (10 POINTS, 2 POINTS EACH) Please circle true or false.

   a. T F The Cauchy-Schwarz Inequality states that if If $\mathbf{u}$ and $\mathbf{v}$ are vectors in R^n, then $|\mathbf{u}\cdot\mathbf{v}| \geq \|\mathbf{u}\|\|\mathbf{v}\|$.

   b. T F $c\langle \mathbf{u},\mathbf{v}\rangle = \langle c\mathbf{u}, c\mathbf{v}\rangle$.

   c. T F A set S of vectors is orthogonal when every pair of vectors in S is orthogonal.

   d. T F An orthonormal basis derived by the Gram-Schmidt orthonormalization process does not depend on the order of the vectors in the basis.

   e. T F A linear transformation *T* from V into W is one-to-one when the preimage of every $\mathbf{w}$ in the range consists of a single vector $\mathbf{v}$.

2. (10 POINTS, 2 POINTS PER BLANK) Please fill in the blank.

   a. If $S=\{\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_n\}$ is an orthogonal set of ____________________ vectors in an inner product space V, then S is ____________________ __________________________.

   b. Let $T:V\to W$ be a linear transformation. The dimension of the kernel of *T* is called the _________________ of *T*.

   c. Let $T:R^n\to R^m$ be a linear transformation given by $T(A)=A\mathbf{x}$. Then the _______________________ space is equal to the range of *T*.

3. (5 POINTS) Determine all vectors that are orthogonal to $(1,-1,2)$.

4. (6 POINTS) Let $f(x)=x$, $g(x)=\frac{1}{x^2+1}$, and $\langle f,g\rangle=\int_{-1}^{1} f(x)g(x)\,dx$.

a. (5 POINTS) Find the inner product.

b. (1 POINTS) Are the vectors are orthogonal?

Circle One: YES NO

5. (10 POINTS) Let $B = \{(-1, 2, 2), (1, 0, 0)\}$ be a basis for a subspace of R^3, and let $\mathbf{x} = \{(-3, 4, 4)\}$ be a vector in the subspace.

a. (3 POINTS) Find the coordinates of $\mathbf{x}$ relative to B.

b. (6 POINTS) Apply the Gram-Schmidt orthonormalization process to transform B into an orthonormal set B'.

c. (3 POINTS) Find the coordinates of $\mathbf{x}$ relative to B'.

6. (5 POINTS) Prove that if $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in R^2, then $(\mathbf{u}+\mathbf{v})\cdot\mathbf{w}=\mathbf{u}\cdot\mathbf{w}+\mathbf{v}\cdot\mathbf{w}$.

7. (10 POINTS) We are collecting data on the number of machine failures per day in some factory. We collected three data points in the form (day, number of failures). The three data points are (1, 1), (2, 2), and (3, 2).

a. Plot the data. Is there a linear trend?
Circle One: YES NO

b. Use your linear algebra techniques to find the least squares regression line.

8. (18 POINTS) Define the linear transformation T by $T(\mathbf{x}) = A\mathbf{x}$. Let

$$A = \begin{bmatrix} 5 & -3 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

Find the following.

a. (8 POINTS) ker (T):

b. (1 POINT) nullity (T) = _______

c. (8 POINTS) range (T):

d. (1 POINT) rank (T) = _________

9. (10 POINTS) Let $T(1,1,1) = (2, 0, -1)$, $T(0, -1, 2) = (-3, 2, -1)$, and $T(1, 0, 1) = (1, 1, 0)$. Find the image of $T(0,2,-1)$.

10. (6 POINTS) Define the linear transformation $T : R^n \to R^m$ by $T(\mathbf{v}) = A\mathbf{v}$.

$$A = \begin{bmatrix} -1 & 2 & 1 & 3 & 4 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix}.$$

a. $\dim(R^m) =$ _________

b. $\dim(R^n) =$ _________

11. (10 POINTS) Consider the linear transformation $T : R^2 \to R^2,\ T(x, y) = (2x + y, x - y)$.

a. (8 POINTS) Find ker(T).

b. (2 POINTS) T is (Circle One): one-to-one onto neither