MATH 270/GRACEY
EXAM 3/SECTIONS 5.1-5.4, 6.1-6.4

NAME(S)______________________________

100 POINTS POSSIBLE

YOUR WORK MUST BE ORGANIZED AND CLEAR

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1. (10 POINTS, 2 POINTS EACH) Please circle true or false.

a. T F If A is an $m \times n$ matrix, then $R(A)$ and $N(A^T)$ are Orthogonal subspaces of R^n.

b. T F The vector spaces R^2 and P_1 are isomorphic to each other.

c. T F A set S of vectors is orthogonal when at least one pair of vectors in S is orthogonal.

d. T F The standard basis for R^n will always make the coordinate matrix for the linear transformation *T* the simplest matrix possible.

e. T F In general, the compositions $T_2 \circ T_1$ and $T_1 \circ T_2$ have the same standard matrix A.

2. (6 POINTS) Determine all vectors that are orthogonal to $(-1,-2,5)$.

3. (8 POINTS) Let $f(x) = x$, $g(x) = e^{-x}$, and use the inner product in $C[a,b]$ $\langle f, g \rangle = \int_a^b f(x) g(x) dx$. Find the orthogonal projection of f onto g.

4. (8 POINTS) Prove that if W is a subspace of R^n, the intersection of W and $W^{\perp}$ is $\{\mathbf{0}\}$, where $W^{\perp}$ is the subspace of R^n given by $W^{\perp} = \{\mathbf{v} : \mathbf{w} \cdot \mathbf{v} = 0 \text{ for every } \mathbf{w} \text{ in } W\}$.

5. (12 POINTS) Apply the Gram-Schmidt orthonormalization process to transform the basis $B=\{(7,24,0,0),(0,0,1,1),(0,0,1,-2)\}$ for a subspace of R^4 into an orthonormal basis for the subspace. Use the Euclidean inner product on R^4 and use the vectors in the order which they are given. Please show all work.

6. (8 POINTS) Find the least squares regression line for the set of data points $\{(-2,2),(-1,1),(0,1),(1,3)\}$ using the technique you learned in section 5.4 of your linear text. Please show all work except for matrix multiplication and rref (for these operations you can use the calculator). Is this an appropriate model? Please explain.

7. (8 POINTS) Prove that if A and B are similar and A is nonsingular, then B is also nonsingular and A^{-1} and B^{-1} are similar.

8. (12 POINTS) Define the linear transformation T by $T(\mathbf{v}) = A\mathbf{v}$. Let

$$A = \begin{bmatrix} -1 & 3 & 2 & 1 & 4 \\ 2 & 3 & 5 & 0 & 0 \\ 2 & 1 & 2 & 1 & 0 \end{bmatrix}.$$

Please find the following and show all work.

a. (5 POINTS) A basis for ker (T):

b. (1 POINT) nullity (T) = ________

c. (5 POINTS) A basis for the range (T):

d. (1 POINT) rank (T) = _________

9. (8 POINTS) Let T be a linear transformation from $M_{2,2}$ into $M_{2,2}$ such that

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right)=\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix},\ T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right)=\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix},$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right)=\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix},\ T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)=\begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}.$$

Find $T\left(\begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}\right)$. Please show all work.

10. (10 POINTS) Consider the linear transformation $T : R^3 \to R^3,\ T(x, y, z) = (x, x + 2y, x + y + 3z)$. Let $\mathrm{B}' = \{(1, -1, 0), (0, 0, 1), (0, 1, -1)\}$. Please find the matrix A' for *T* relative to the basis B'.