MATH 270/GRACEY NAME______________________________

EXAM 2/Chapter 3-4.7

100 POINTS POSSIBLE

YOUR WORK MUST BE ORGANISED AND CLEAR

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1. (8 POINTS) Please circle true or false.

a. T F If A is an $n \times n$ nonsingular matrix, then the determinant of the matrix A is zero.

b. T F If A is a square matrix, then $\det(A) = \det(A^T)$.

c. T F The zero vector in R^n is defined as the additive inverse of a vector.

d. T F Every vector space V contains at least one subspace that is the zero subspace.

2. (8 POINTS) Please match the term/phrase with the correct term/phrase/definition. Please note that the blank can contain more than one word. Each item is worth 3 points.

a. ________ b. ________ c. ________

Term/Phrase	Definition
a. A nonempty subset W of a vector space V is called a ________________ of V when W is a vector space under the operations of addition and scalar multiplication defined in V.	1. subspace 2. subset 3. is zero 4. is not zero 5. linear combination
b. If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots \mathbf{v}_n\}$ is a set of vectors in a vector space V, then the span of S is the set of all ____________ of the vectors in S.	
c. If a set of vectors in a vector space is linearly dependent, the vector sum $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$ ______________.	

3. (5 POINTS) Show that the set, together with the indicated operations, is not a vector space identifying one of the ten vector space axioms that fail.

$W = \{(x, y) : x - y = 1\}, \ V = R^2$.

4. (5 POINTS) Use a determinant to find the equation of the line passing through $(3, -1), \text{ and } (4, 3)$. Do not use your graphing calculator. Show all work for full credit.

5. (6 POINTS) Proof: Let $S = \{\mathbf{u}, \mathbf{v}\}$, be a linearly independent set. Prove that the set $\{\mathbf{u}+\mathbf{v}, \mathbf{u}-\mathbf{v}\}$ is linearly independent.

6. (10 POINTS) Use Cramer's Rule to solve the linear system. **<u>No graphing calculator—show all steps for full credit</u>**.

$$\begin{aligned} 2x - \ \ y &= -10 \\ 3x + 2y &= \ \ -1 \end{aligned}$$

7. (20 POINTS) Consider the following matrices. **Do not use a graphing calculator—show all steps for full credit**. It is fine to use properties of determinants

$$A=\begin{bmatrix} -1 & 2 \\ -7 & 4 \end{bmatrix},\ B=\begin{bmatrix} -3 & 1 \\ 6 & 5 \end{bmatrix}$$

a. (4 POINTS) Find $\det(A)$.

b. (4 POINTS) Find $\det(B)$.

c. (4 POINTS) Find $\det(A^{-1})$.

d. (4 POINTS) Find $\det(A^{T})$.

e. (4 POINTS) Find $\det(AB)$

8. (8 POINTS) Determine whether the set is a basis for $M_{2,2}$.

$$S=\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}-1 & 0\\1 & 1\end{bmatrix},\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix},\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\right\}$$

9. (16 POINTS) Use the fact that the following matrices are row-equivalent.

$$A=\begin{bmatrix}-2 & -5 & 8 & 0 & -17\\1 & 3 & -5 & 1 & 5\\3 & 11 & -19 & 7 & 1\\1 & 7 & -13 & 5 & -3\end{bmatrix},\quad B=\begin{bmatrix}1 & 0 & 1 & 0 & 1\\0 & 1 & -2 & 0 & 3\\0 & 0 & 0 & 1 & -5\\0 & 0 & 0 & 0 & 0\end{bmatrix}$$

a. (2 POINTS) Find the rank of A.

b. (2 POINTS) Find the nullity of A.

c. (4 POINTS) Find a basis for the nullspace of A.

d. (4 POINTS) Find a basis for the row space of A.

e. (4 POINTS) Find a basis for the column space of A.

10. (5 POINTS) Find the coordinate matrix of $\mathbf{x} = (1, -3, 0)$ in R^3 relative to the standard basis.

11. (9 POINTS) Find the transition matrix from $B = \{(1,3,2),(2,-1,2),(5,6,1)\}$ to $B' = \{(1,0,0),(0,1,0),(0,0,1)\}$.