THEOREM: THE CONSTANT RULE

Let k be a real number.

$$\int k\,dx = x + C$$

Example 1: Find the indefinite integral.

$\int -3dx$

THEOREM: THE POWER RULE

Let n be a rational number.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Example 2: Find the following indefinite integrals.

a. $\int x^{-5} dx$

b. $\int x^{1/2}\,dx$

THEOREM: THE CONSTANT MULTIPLE RULE

If f is an integrable function and c is a real number, then cf is also integrable and

$$\int cf(x)\,dx = c\int f(x)\,dx$$

Example 3: Find the area of the region bounded by $f(x) = 2x^3$, $x = 1$, $x = 3$, and $y = 0$.

THEOREM: THE SUM AND DIFFERENCE RULES

The sum (or difference) of two integrable functions f and g is itself integrable. Moreover, the antiderivative of $f+g$ (or $f-g$) is the sum (or difference) of the antiderivatives of f and g.

$$\int \left[f(x) + g(x) \right] dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int \left[f(x) - g(x) \right] dx = \int f(x)\,dx - \int g(x)\,dx$$

Example 4: Find the indefinite integral.

a. $\int \left(\frac{\sqrt{x} - 5x^2}{\sqrt{x}} \right) dx$

b. $\int \left(x^3 + 1 \right)^2 dx$

c. $\int_3^5 \frac{5+6x+x^2}{5+x}\,dx$

THEOREM: ANTIDERIVATIVES OF THE TRIGONOMETRIC FUNCTIONS

$\int \sin x dx = -\cos x + C$	$\int \cos x dx = \sin x + C$
$\int \csc x \cot x dx = -\csc x + C$	$\int \sec x \tan x dx = \sec x + C$
$\int \sec^2 x dx = \tan x + C$	$\int \csc^2 x dx = -\cot x + C$
$\int \tan x dx = -\ln\lvert\cos x\rvert + C$	$\int \cot x dx = \ln\lvert\sin x\rvert + C$
$\int \sec x dx = \ln\lvert\sec x + \tan x\rvert + C$	$\int \csc x dx = -\ln\lvert\csc x + \cot x\rvert + C$

Example 5: Integrate.

a. $\int \sec^2 x dx$

b. $\int\left(-\csc\theta+\csc\theta\cot\theta\right)d\theta$

c. $\int 3\tan x dx$

d. $\int\frac{1}{1+\cos\theta}d\theta$

THEOREM: ANTIDIFFERENTIATION OF A COMPOSITE FUNCTION

Let g be a function whose range is an interval I and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f\left(g\left(x\right)\right)g'\left(x\right)dx = F\left(g\left(x\right)\right)+C$$

Letting $u = g\left(x\right)$ gives $du = g'\left(x\right)dx$ and

$$\int f\left(u\right)du = F\left(u\right)+C$$

Example 6: Find the following definite and indefinite integrals.

a. $\int\left(x\sqrt{1-x}\right)dx$

b. $\int x\left(5-2x^{2}\right)^{5} dx$

c. $\int \cos^{2} 3x dx$

d. $\int_{\pi/4}^{\pi/3} \tan^3 x \sec^2 x dx$

Theorem: LOG RULE FOR INTEGRATION

Let *u* be a differentiable function of *x*.

1. $\int \frac{1}{x} dx = \ln|x| + C$

2. $\int \frac{1}{u} du = \ln|u| + C$

Theorem: INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let *u* be a differentiable function of *x*.

1. $\int e^x dx = e^x + C$

2. $\int e^u du = e^u + C$

3. $\int a^x dx = \left(\frac{1}{\ln a}\right) a^x + C,\ a$ is a positive real number, $a \neq 1$

Example 7: Find the following indefinite integrals and evaluate the definite integrals.

a. $\int \frac{5t^2 - t - 1}{2 - t}\,dx$

b. $\int \frac{5}{\left(\sqrt{x}\ln x\right)^2}\,dx$

c. $\int \frac{1}{x^{2/3}\left(1+x^{1/3}\right)}dx$

d. $\int_1^2 \frac{e^x+e^{-x}}{e^x-e^{-x}}dx$

e. $\int_{\pi/6}^{\pi/4} \sec^2 x dx$

f. $\int_{-\pi/2}^{\pi/2} \sin x \cos^2 x dx$

Theorem: INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x, and let $a>0$.

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

Example 7: Find the following indefinite integrals and evaluate the definite integrals.

a. $\displaystyle\int \frac{t}{t^4+2}\,dt$

b. $\displaystyle\int \frac{dx}{\sqrt{5-4x-x^2}}$

c. $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-25}}$