THEOREM: THE CONSTANT RULE

The derivative of a constant function is zero. That is, if c is a real number, then

$$\frac{d}{dx}[c]=0$$

Example 1: Find the derivative of the function $g(x)=-5$.

THEOREM: THE POWER RULE

If n is a rational number, then the function $f(x)=x^n$ is differentiable and

$$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$$

For f to be differentiable at $x=0$, n must be a number such that x^{n-1} is defined on an interval containing zero.

Example 2: Find the following derivatives.

a. $f(x)=x^{-5}$

b. $f(x)=x^{1/2}$

THEOREM: THE CONSTANT MULTIPLE RULE

If f is a differentiable function and c is a real number, then cf is also differentiable and

$$\frac{d}{dx}\left[cf(x)\right]=cf'(x)$$

Example 3: Find the slope of the graph of $f(x)=4x^{2/3}$ at

a. $x=x$

b. $x=125$

c. $x=-64$

THEOREM: THE SUM AND DIFFERENCE RULES

The sum (or difference) of two differentiable functions f and g is itself differentiable. Moreover, the derivative of $f+g$ (or $f-g$) is the sum (or difference) of the derivatives of f and g.

$$\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x)$$

$$\frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$$

Example 4: Find the equation of the line tangent to the graph of $f(x)=x-\sqrt{x}$ at x = 4.

1. Find ________ at $x=x$.

2. Find slope at ________.

3. Find the _____________ of the line _____________ to the graph at ______ using the _________-__________ form of the equation of the line.

THEOREM: DERIVATIVES OF THE TRIGONOMETRIC FUNCTIONS

$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\cos x] = -\sin x$
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\frac{d}{dx}[\sec x] = \sec x \tan x$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\frac{d}{dx}[\cot x] = -\csc^2 x$

Example 5: Find the derivative of the following functions:

a. $f(x) = \dfrac{\sin x}{6}$

b. $r(\theta) = 5\theta - 3\cos\theta$

c. $h(t) = \dfrac{\cos t}{\cot t}$

d. $f(x) = 12 + 7\sec x$

e. $f(x) = -\tan x + x$

THEOREM: THE PRODUCT RULE

The product of two differentiable functions f and g is itself differentiable. Moreover, the derivative of fg is the derivative of the first function times the second function, plus the first function times the derivative of the second function.

$$\frac{d}{dx}\left[f(x)g(x)\right]=f'(x)g(x)+f(x)g'(x)$$

This rule extends to cover products of more than two factors. For example the derivative of the product of functions $fghk$ is

$$\frac{d}{dx}[fghk]=f'(x)g(x)h(x)k(x)+f(x)g'(x)h(x)k(x)+f(x)g(x)h'(x)k(x)+f(x)g(x)h(x)k'(x)$$

Example 6: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

a. $g(x)=x\cos x$

b. $h(t)=\left(3-\sqrt{t}\right)^2$

THEOREM: THE QUOTIENT RULE

The quotient of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$. Moreover, the derivative of f/g is the derivative of the numerator times the denominator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Example 7: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

a. $g(x) = \dfrac{2x}{5x^2 + 3}$

b. $h(t) = \dfrac{t}{\sqrt{t} - 1}$

c. $h(t) = \dfrac{\cot t}{t}$

Example 8: Find the given higher-order derivative.

a. $f(x) = 2 - \frac{2}{x}, \quad f'''(x)$

b. $f^{(4)}(x) = 2x + 1, \quad f^{(6)}(x)$

Theorem: The Chain Rule

If $y = f(u)$ is a differentiable function of *u* and $u = g(x)$ is a differentiable function of *x*, then $y = f(g(x))$ is a differentiable function of *x* and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \text{ or } \frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x).$$

Example 9: Find the derivative using the Chain Rule.

a. $y = \left(\sqrt{x} - x^2\right)^{10}$

b. $f(x) = \frac{1}{\sqrt{2-x}}$

Example 10: Find the derivative of the following functions.

a. $y = \sin x$

b. $y = \sin 2x$

c. $y = \sin^2 x$

d. $y = \sin x^2$

e. $y = \sqrt{\cos x}$

f. $f(x) = x^2 (2 - x)^{2/3}$

g. $h(x) = x\sin^2 4x$

Theorem: Derivative of the Natural Logarithmic Function

Let *u* be a differentiable function of *x*.

1. $\frac{d}{dx}[\ln x] = \frac{1}{x}$

2. $\frac{d}{dx}[\ln u] = \frac{u'}{u}, \quad u > 0$

Example 11: Find the derivative.

a. $y = (\ln x)^3$

b. $f(x) = \ln|\cos x|$

c. $h(t) = \ln x^x$

Theorem: Derivative of the Natural Exponential Function

Let u be a differentiable function of x.

1. $\frac{d}{dx}\left[e^x\right] = e^x$

2. $\frac{d}{dx}\left[e^u\right] = e^u u'$

Example 12: Find the derivative.

a. $y = xe^{-x}$

b. $f(x) = e^{\sin 2x}$

c. $h(t) = \frac{e^t}{\ln e^{\sqrt{t}}}$

Theorem: Derivatives for Bases other than *e*

Let *a* be a positive real number $(a \neq 1)$ and let *u* be a differentiable function of *x*.

1. $\frac{d}{dx}\left[a^x\right] = (\ln a)a^x$
2. $\frac{d}{dx}\left[a^u\right] = (\ln a)a^u u'$
3. $\frac{d}{dx}\left[\log_a x\right] = \frac{1}{(\ln a)x}$
4. $\frac{d}{dx}\left[\log_a u\right] = \frac{u'}{(\ln a)u}$

Example 13: Find the derivative.

a. $y = 2^{3x}$

b. $f(x) = \log 5x$

THEOREM: DERIVATIVES OF THE INVERSE TRIGONOMETRIC FUNCTIONS (*u* is a function of *x*)

$$\frac{d}{dx}\left[\arcsin u\right] = \frac{du/dx}{\sqrt{1-u^2}} \qquad \frac{d}{dx}\left[\arccos u\right] = -\frac{du/dx}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}\left[\operatorname{arccsc} u\right] = -\frac{du/dx}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}\left[\operatorname{arcsec} u\right] = \frac{du/dx}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}\left[\arctan u\right] = \frac{du/dx}{1+u^2} \qquad \frac{d}{dx}\left[\operatorname{arccot} u\right] = -\frac{du/dx}{1+u^2}$$

Example 14: Find the derivative.

a. $y = \arctan 3x - \ln\left(1 + 9x^2\right)$

b. $f(x) = x \arcsin \sqrt{x}$