Find the derivative of y with respect to x.

14) $y = \tan^{-1}\frac{4x}{5}$

$$y' = \frac{\frac{4}{5}}{1+\left(\frac{4x}{5}\right)^2}$$

$$y' = \frac{\frac{4}{5}}{1+\frac{16x^2}{25}}$$

$$y' = \frac{4}{5\left(\frac{25+16x^2}{25}\right)}$$

$$y' = \frac{20}{25+16x^2}$$

17) $\int \frac{dx}{x\sqrt{9x^2-7}} = \int \frac{du/3}{(u/3)\sqrt{u^2-(\sqrt{7})^2}} = \int \frac{du}{u\sqrt{u^2-(\sqrt{7})^2}} = \frac{1}{\sqrt{7}}\operatorname{arcsec}\frac{|u|}{\sqrt{7}} + C$

$u^2 = 9x^2$ so $u = 3x$, $x = \frac{u}{3}$

$a^2 = 7$ so $a = \sqrt{7}$

$u = 3x$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$$= \frac{1}{\sqrt{7}}\operatorname{arcsec}\frac{|3x|}{\sqrt{7}} + C$$

20) The charcoal from a tree killed in a volcanic eruption contained 61.3% of the carbon-14 found in living matter. How old is the tree, to the nearest year? Use 5700 years for the half-life of carbon-14.

Find k: $y = Ce^{kt}$

$\frac{1}{2}C = Ce^{k(5700)}$

$\frac{1}{2}C = Ce^{k\cdot 5700}$

$-\ln 2 = \ln e$

$k = \frac{-\ln 2}{5700}$

Find t: $y = Ce^{(-\ln 2/5700)t}$

$.613C = Ce^{(-\ln 2/5700)t}$

$\ln .613 = \ln e$

$t = \frac{5700 \ln .613}{-\ln 2}$

21) The amount of alcohol in the bloodstream, A, declines at a rate proportional to the amount, that is, $\frac{dA}{dt} = -kA$. If $k = 0.3$ for a particular person, how long will it take for his alcohol concentration to decrease from 0.10% to 0.05%? Give your answer to the nearest tenth of an hour.

$\frac{dA}{dt} = -kA$

$\int \frac{dA}{A} = \int -k$

$\ln|A| = -kt + C_1$

$A = e^{-kt+C_1}$

$A = e^{-kt} e^{C_1}$

$A = Ce^{-kt}$

$K = 0.3,\ A = Ce^{-0.3t}$

$.001C = Ce^{-0.3t}$

$t = \frac{\ln .001}{-0.3} \approx 23.0$

$.0005C = Ce^{-.3t}$

$t = \frac{\ln .0005}{-.3}$

≈ 25.3

$25.3 - 23 = 2.3$

It will take 2.3 hours

23) $2\frac{dy}{dx} - 4xy = 8x;\ y(0) = 12$

$2\frac{dy}{dx} = 4xy + 8x$

$\frac{dy}{dx} = 2xy + 4x$

$\frac{dy}{dx} = 2x(y+2)$

$\int \frac{dy}{y+2} = \int 2x\,dx$

$\ln|y+2| = x^2 + C$

$y + 2 = Ce^{x^2}$

$y = -2 + Ce^{x^2}$

$12 = -2 + Ce^{0^2}$

$12 = -2 + C$

$14 = C$

$y = -2 + 14e^{x^2}$

2) $y = \ln(\ln 8x)$

$u = \ln 8x$

$u' = \frac{8}{8x} = \frac{1}{x}$

$y' = \frac{u'}{u}$

$y' = \frac{1/x}{\ln 8x}$

$y' = \frac{1}{x \ln 8x}$

4) $y = \ln(\cos(\ln \theta))$

$u = \cos(\ln\theta)$

$u' = (-\sin\ln\theta)(1/\theta)$

$u' = -\frac{1}{\theta}\sin\ln\theta$

$y' = \frac{u'}{u}$

$y' = \frac{-1/\theta \sin\ln\theta}{\cos\ln\theta}$

$y' = -\frac{1}{\theta}\tan\ln\theta$

6) $\int \frac{dx}{x(2+5\ln x)} = \int \frac{\left(\frac{x}{5}du\right)}{x\,u}$

$u = 2+5\ln x$

$\frac{du}{dx} = \frac{5}{x}$

$dx = \frac{x}{5}du$

$= \frac{1}{5}\int \frac{du}{u}$

$= \frac{1}{5}\ln|u| + C$

$= \boxed{\frac{1}{5}\ln|2+5\ln x| + C}$