When you are done with your homework you should be able to…

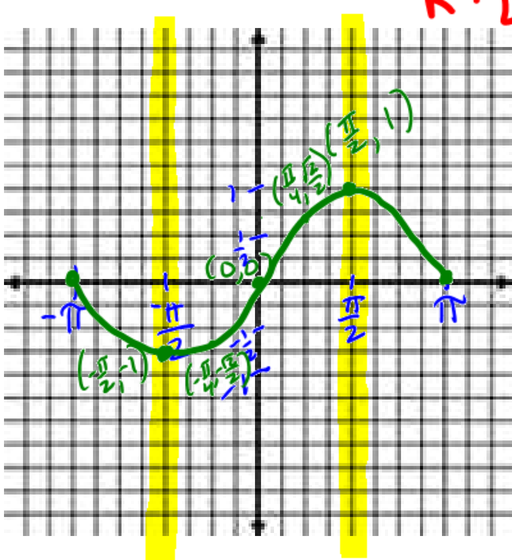
- π Develop properties of the six inverse trigonometric functions
- π Differentiate an inverse trigonometric function
- π Review the basic differentiation rules for elementary functions

Warm-up: Draw the following graphs by hand from $[-\pi,\pi]$. List the domain and range in interval notation.

1. Graph $f(x)=\sin x$.

restricted Dom: $[-\frac{\pi}{2},\frac{\pi}{2}]$

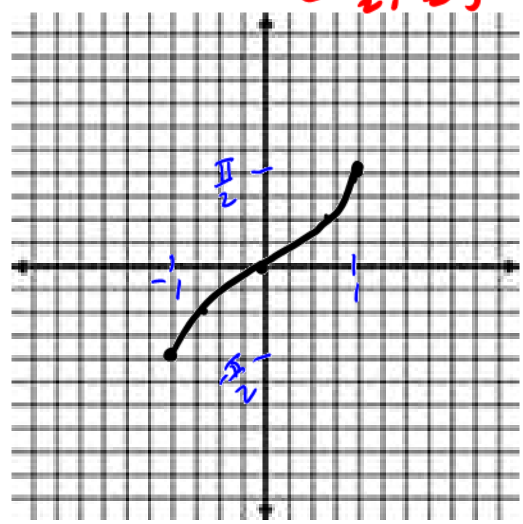
R: $[-1,1]$



2. Graph $g(x)=\arcsin x$.

D: $[-1,1]$

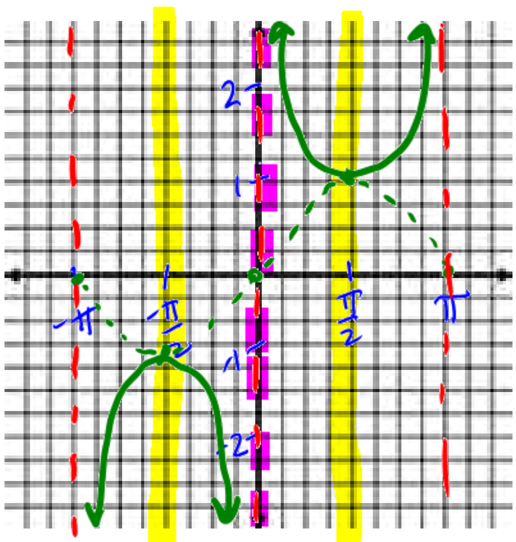
R: $[-\frac{\pi}{2},\frac{\pi}{2}]$



3. Graph $f(x)=\csc x$.

Rest. Dom: $[-\frac{\pi}{2},0)\cup(0,\frac{\pi}{2}]$

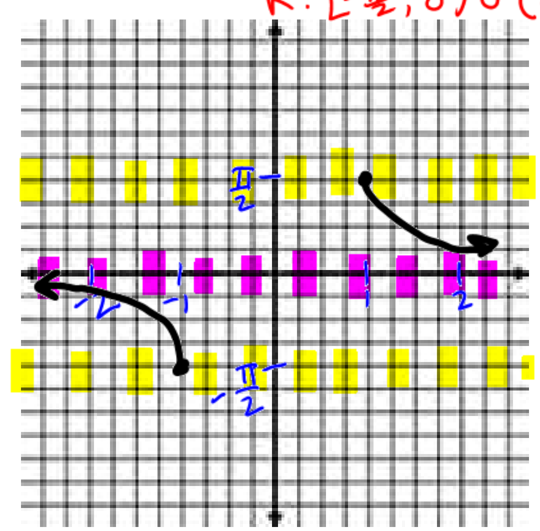
R: $(-\infty,-1]\cup[1,\infty)$



4. Graph $g(x)=\operatorname{arc}\csc x$.

D: $(-\infty,-1]\cup[1,\infty)$

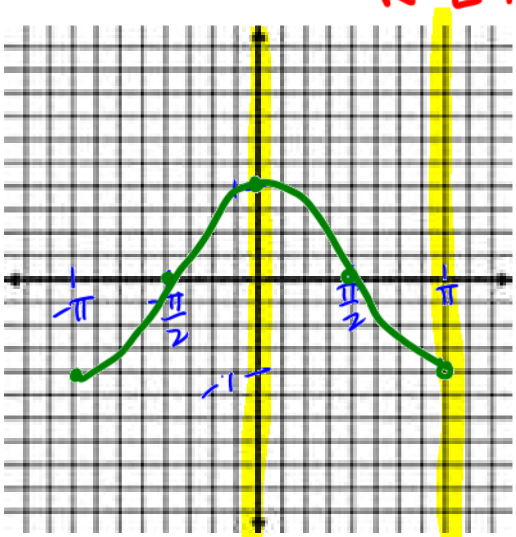
R: $[-\frac{\pi}{2},0)\cup(0,\frac{\pi}{2}]$



5. Graph $f(x) = \cos x$.

Rest. Dom: $[0, \pi]$

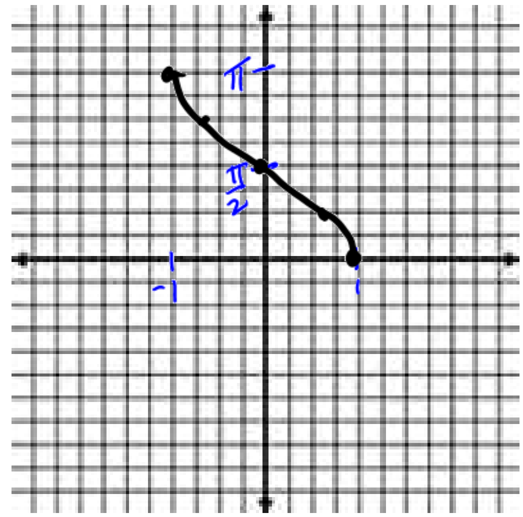
R: $[-1, 1]$



6. Graph $g(x) = \arccos x$.

D: $[-1, 1]$

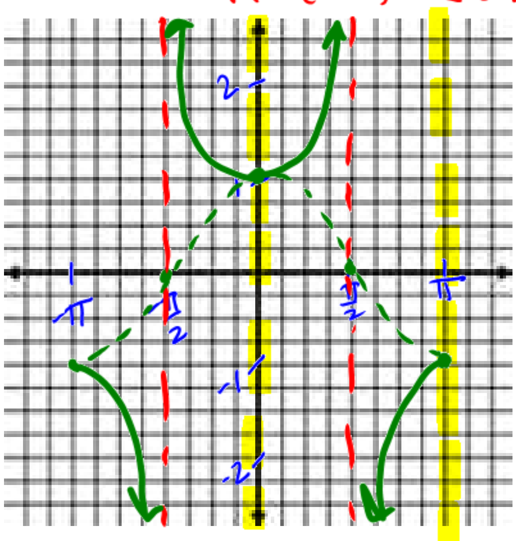
R: $[0, \pi]$



7. Graph $f(x) = \sec x$.

Rest Dom: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

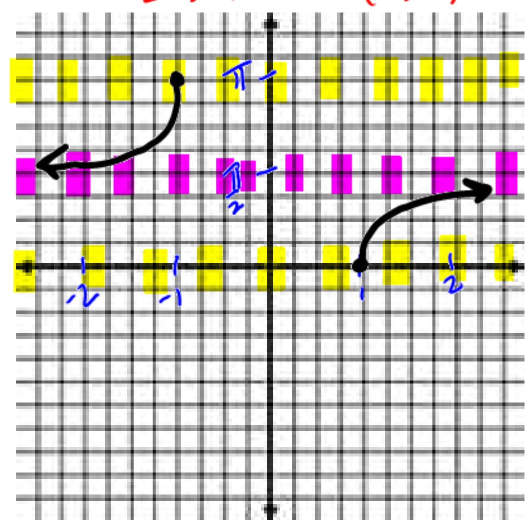
R: $(-\infty, -1] \cup [1, \infty)$



8. Graph $g(x) = \operatorname{arcsec} x$.

D: $(-\infty, -1] \cup [1, \infty)$

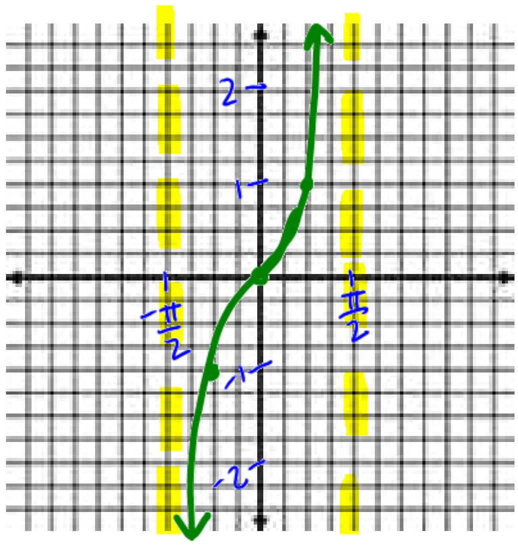
R: $[0, \pi/2) \cup (\pi/2, \pi]$



9. Graph $f(x) = \tan x$.

Rest. Dom: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

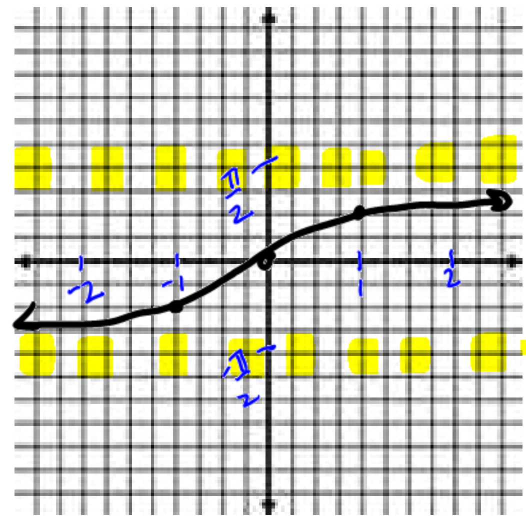
R: $(-\infty, \infty)$



10. Graph $g(x) = \arctan x$.

D: $(-\infty, \infty)$

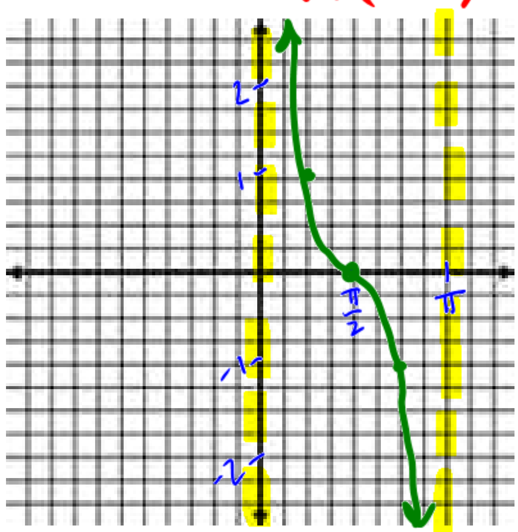
R: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



11. Graph $f(x) = \cot x$.

Rest. Dom: $(0, \pi)$

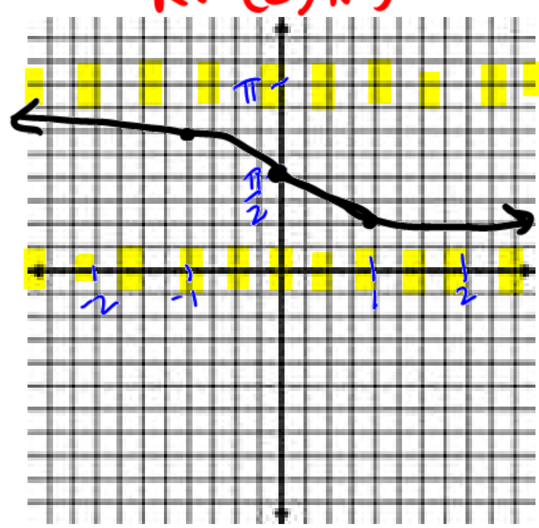
R: $(-\infty, \infty)$



12. Graph $g(x) = \operatorname{arccot} x$.

D: $(-\infty, \infty)$

R: $(0, \pi)$



PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

If $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, then

$$\sin(\arcsin x) = x \quad \text{and} \quad \arcsin(\sin y) = y.$$

If $-\frac{\pi}{2} < y < \frac{\pi}{2}$, then

$$\tan(\arctan x) = x \quad \text{and} \quad \arctan(\tan y) = y.$$

If $|x| \ge 1$ and $0 \le y < \frac{\pi}{2}$ or $\frac{\pi}{2} < y \le \pi$, then

$$\sec(\operatorname{arcsec} x) = x \quad \text{and} \quad \operatorname{arcsec}(\sec y) = y.$$

If $\underline{-1} \le x \le \underline{1}$ and $\underline{0} \le y \le \underline{\pi}$, then

$$\cos(\arccos x) = x \quad \text{and} \quad \arccos(\cos y) = y.$$

If $\cancel{-\frac{\pi}{2}}\,0 < y < \cancel{\frac{\pi}{2}}\,\pi$, then

$$\underline{\cot}(\operatorname{arc}\underline{\cot}\, x) = x \quad \text{and} \quad \operatorname{arc}\underline{\cot}(\underline{\cot}\, y) = y.$$

If $|x| \ge 1$ and $\underline{-\frac{\pi}{2}} \le y < \underline{0}$ or $\underline{0} < y \le \underline{\pi/2}$, then

$$\csc(\operatorname{arccsc} x) = x \quad \text{and} \quad \operatorname{arccsc}(\csc y) = y.$$

Example 1: Evaluate each function.

a. $\operatorname{arccot}(1) = \boxed{\frac{\pi}{4}}$

(since $\cot\frac{\pi}{4} = 1$)

b. $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\pi}{4}}$

c. $\operatorname{arcsec}\left(\frac{2\sqrt{3}}{3}\right) = \boxed{\frac{\pi}{6}}$

$\frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$

e. $\arccos\left(-\frac{1}{2}\right)$

d. $\arctan\left(\sqrt{3}\right)$

f. $\operatorname{arccsc}\left(-\sqrt{2}\right)$

Example 2: Solve the equation for *x*.

$\tan(\arctan(2x-5)) = \tan(-1)$

$2x - 5 = \tan(-1)$

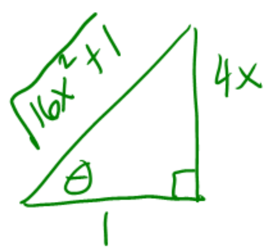
$\boxed{\begin{aligned} x &= \frac{5+\tan(-1)}{2} \quad \text{exact} \\ x &\approx 1.72 \quad \text{approximate} \end{aligned}}$

Example 3: Write the expression in algebraic form. (HINT: Sketch a right triangle)

a. $\sec(\arctan 4x)$

Let $\theta = \arctan 4x$

$\tan\theta = \frac{4x}{1}$



So...

$\sec\theta = \frac{\sqrt{16x^2+1}}{1}$

$\boxed{\sec\theta = \sqrt{16x^2+1}}$

b. $\cos(\arcsin x)$

Example 4: Differentiate with respect to *x*.

a. $y = \arcsin x$

b. $y = \arccos x$

c. $y = \arctan x$

d. $y = \operatorname{arc}\csc x$

e. $y = \operatorname{arc}\sec x$

f. $y = \operatorname{arc}\cot x$

What have we found out?!

DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
2. $\frac{d}{dx}[\arccos u] = -\frac{u'}{\sqrt{1-u^2}}$
3. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
4. $\frac{d}{dx}[\operatorname{arc}\cot u] = -\frac{u'}{1+u^2}$
5. $\frac{d}{dx}[\operatorname{arc}\sec u] = \frac{u'}{|u|\sqrt{u^2-1}}$
6. $\frac{d}{dx}[\operatorname{arc}\csc u] = -\frac{u'}{|u|\sqrt{u^2-1}}$

Example 5: Find the derivative of the function. Simplify your result to a single ration expression with positive exponents.

a. $f(t) = \arcsin t^3$

b. $g(x) = \arcsin x + \arccos x$

c. $y = x \arctan 2x - \frac{1}{4}\ln\left(1+4x^2\right)$

d. $y = 25 \arcsin \frac{x}{5} - x\sqrt{25-x^2}$

Example 6: Find an equation of the tangent line to the graph of the function $y=\frac{1}{2}\arccos x$ at the point $\left(-\frac{\sqrt{2}}{2},\frac{3\pi}{8}\right)$.