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Part I: (30 Points) Problems 1-4: Complete the following problems.

1. (10 POINTS)
   a. (8 POINTS) Find the 2nd degree Taylor polynomial for $f(x) = x^2 \sin x$, centered at $\frac{\pi}{2}$.

   b. (2 POINTS) Use your result from part a to approximate $f\left(\frac{3\pi}{8}\right)$

2. (10 POINTS)

a. (8 POINTS) Find the 4^{th} degree Maclaurin polynomial for $f(x) = \dfrac{1}{x+1}$.

b. (2 POINTS) Use your result from part a to approximate $f\left(\dfrac{1}{4}\right)$

3. (6 POINTS) Find the radius of convergence and the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n n!(x-5)^n}{3^n}$.

4. (4 POINTS) Write a series which is equivalent to $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ with the index of summation starting at 1.

Part II: (25 points) Problems 5-6. Find a geometric power series for the function, centered at *c*, and determine the interval of convergence.

5. (10 POINTS) $f\left(x\right)=\dfrac{4}{8-x},\ c=2$

6. (15 POINTS) $f(x) = \dfrac{4x}{x^2 + 2x - 3}, \; c = 0$

Part III: (15 points) Problem 7. **Use the definition of Taylor series** to find the Taylor series for the function, centered at *c*. Be sure to find the interval of convergence and test the endpoints. You may not use a series known from a list. Hint: you may need to integrate or differentiate.

7. $f\left(x\right)=\dfrac{1}{1-x},\ c=2$

Part IV: (30 points/15 points each) Problems 8-10. Solve the following problems as indicated. You do not need to find the interval of convergence.

8. Use the binomial series

$$(1+x)^k = 1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \frac{k(k-1)(k-2)(k-3)x^4}{4!} + \cdots$$

to find the Maclaurin series for the function $f(x) = \dfrac{1}{(1+x)^4}$.

9. Use the series $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$ to find the Maclaurin series for the function $f(x) = x\sin x$.