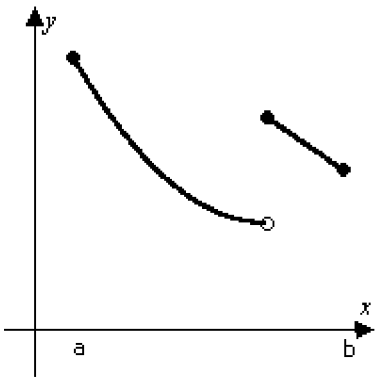
MATH 150/GRACEY
EXAM 3 PRACTICE Name________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

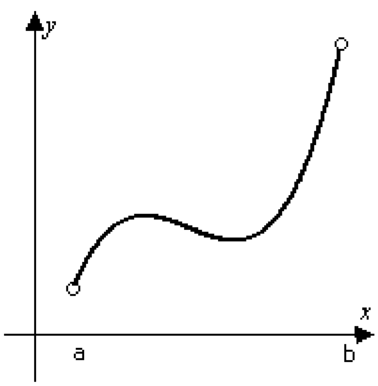
Determine from the graph whether the function has any absolute extreme values on the interval [a, b].

1) 1) ________



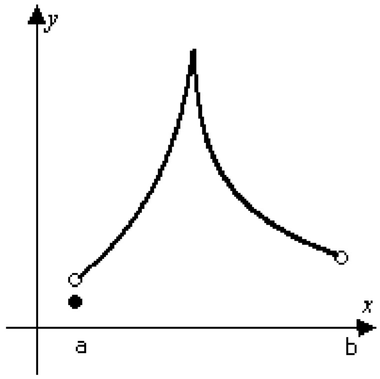
A) Absolute minimum only.
B) Absolute minimum and absolute maximum.
C) No absolute extrema.
D) Absolute maximum only.

2) 2) ________



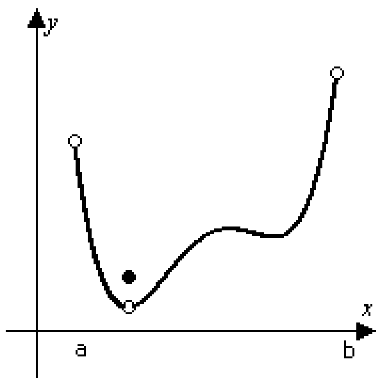
A) No absolute extrema.
B) Absolute minimum and absolute maximum.
C) Absolute maximum only.
D) Absolute minimum only.

3) 3) ________



A) Absolute minimum and absolute maximum.
B) No absolute extrema.
C) Absolute maximum only.
D) Absolute minimum only.

4) 4) ________



A) No absolute extrema.
B) Absolute minimum and absolute maximum.
C) Absolute maximum only.
D) Absolute minimum only.

Determine all critical points for the function.

5) $f(x) = x^2 + 18x + 81$ 5) ________

A) $x = -9$ B) $x = 9$ C) $x = 0$ D) $x = -18$

6) $f(x) = 80x^3 - 3x^5$ 6) ________

A) $x = 4$ B) $x = 0$, $x = -4$, and $x = 4$
C) $x = -4$ and $x = 4$ D) $x = -4$

7) $f(x) = \frac{3x}{x - 7}$ 7) ________

A) $x = -21$ and $x = 0$ B) $x = 0$ and $x = 7$
C) $x = 7$ D) $x = -7$

8) $y = 4x^2 - 128\sqrt{x}$ 8) ________

A) $x = 0$
B) $x = 4$
C) $x = 0$ and $x = 4$
D) $x = 0$, $x = 4$, and $x = -4$

Find the absolute extreme values of the function on the interval.

9) $g(x) = -x^2 + 11x - 30,\ 5 \le x \le 6$ 9) ________

A) absolute maximum is $\frac{5}{4}$ at $x = \frac{13}{2}$; absolute minimum is 0 at 6 and 0 at $x = 5$

B) absolute maximum is $\frac{241}{4}$ at $x = \frac{11}{2}$; absolute minimum is 0 at 6 and 0 at $x = 5$

C) absolute maximum is $\frac{1}{4}$ at $x = \frac{11}{2}$; absolute minimum is 0 at 6 and 0 at $x = 5$

D) absolute maximum is $\frac{1}{4}$ at $x = \frac{13}{2}$; absolute minimum is 0 at 6 and 0 at $x = 5$

10) $f(\theta) = \sin\left(\theta + \frac{\pi}{2}\right),\ 0 \le \theta \le \frac{5\pi}{4}$ 10) ________

A) absolute maximum is 1 at $\theta = \frac{5}{4}\pi$; absolute minimum is -1 at $\theta = \frac{3}{4}\pi$,

B) absolute maximum is 1 at $\theta = \frac{1}{4}\pi$; absolute minimum is -1 at $\theta = \frac{3}{4}\pi$,

C) absolute maximum is 1 at $\theta = 0$; absolute minimum is -1 at $\theta = \pi$

D) absolute maximum is 1 at $\theta = \frac{3}{4}\pi$; absolute minimum is -1 at $\theta = \frac{1}{4}\pi$

11) $F(x) = -\frac{1}{x^2},\ 0.5 \le x \le 5$ 11) ________

A) absolute maximum is $-\frac{1}{25}$ at $x = 5$; absolute minimum is -4 at $x = -\frac{1}{2}$

B) absolute maximum is $-\frac{1}{25}$ at $x = \frac{1}{2}$; absolute minimum is -4 at $x = -5$

C) absolute maximum is $-\frac{1}{25}$ at $x = 5$; absolute minimum is -4 at $x = \frac{1}{2}$

D) absolute maximum is $\frac{1}{25}$ at $x = \frac{1}{2}$; absolute minimum is -4 at $x = 5$

12) $g(x) = 10 - 6x^2,\ -2 \le x \le 5$ 12) ________

A) absolute maximum is 20 at $x = 0$; absolute minimum is -14 at $x = 5$
B) absolute maximum is 10 at $x = 0$; absolute minimum is -140 at $x = 5$
C) absolute maximum is 6 at $x = 0$; absolute minimum is -160 at $x = 5$
D) absolute maximum is 60 at $x = 0$; absolute minimum is -14 at $x = -2$

Find the absolute extreme values of the function on the interval.

13) $f(x) = \tan x,\ -\frac{\pi}{6} \le x \le \frac{\pi}{4}$ 13) _______

A) absolute maximum is $-\frac{\sqrt{3}}{3}$ at $x = \frac{\pi}{4}$; absolute minimum is 1 at $x = -\frac{\pi}{6}$

B) absolute maximum is 1 at $x = \frac{\pi}{4}$ and $-\frac{\pi}{6}$; no minimum value

C) absolute maximum is 1 at $x = \frac{\pi}{4}$; absolute minimum is $-\frac{\sqrt{3}}{3}$ at $x = -\frac{\pi}{6}$

D) absolute maximum is 1 at $x = \frac{2\pi}{12}$; absolute minimum is $-\frac{\sqrt{3}}{3}$ at $x = -\frac{\pi}{12}$

14) $f(x) = |x - 8|,\ 6 \le x \le 11$ 14) _______

A) absolute maximum is 2 at x = 6; absolute minimum is 0 at x = 8
B) absolute maximum is -2 at x = 6; absolute minimum is 3 at x = 11
C) absolute maximum is 3 at x = 11; absolute minimum is 2 at x = 6
D) absolute maximum is 3 at x = 11; absolute minimum is 0 at x = 8

15) $f(x) = x^{2/3},\ -1 \le x \le 27$ 15) _______

A) absolute maximum is 8 at x = 27; absolute minimum is 0 at x = 01
B) absolute maximum is 9 at x = 27; absolute minimum does not exist
C) absolute maximum is 9 at x = 27; absolute minimum is 0 at x = 01
D) absolute maximum is 9 at x = 27; absolute minimum is 1 at x = -1

Find the extreme values of the function and where they occur.

16) $f(x) = x^2 + 2x - 3$ 16) _______

A) Absolute minimum is -1 at x = 4.
B) Absolute minimum is 1 at x = 4.
C) Absolute minimum is 1 at x = -4.
D) Absolute minimum is -4 at x = -1.

17) $f(x) = x^3 - 3x^2 + 1$ 17) _______

A) Local maximum at (0, 1), local minimum at (2, -3).
B) None
C) Local minimum at (2, -3).
D) Local maximum at (0, 1).

18) $f(x) = \frac{4x}{x^2 + 1}$ 18) _______

A) Absolute minimum value is 0 at x = 1. Absolute maximum value is 0 at x = -1.
B) Absolute minimum value is 0 at x = 0.
C) Absolute minimum value is - 2 at x = -1. Absolute maximum value is 2at x = 1.
D) Absolute maximum value is 0 at x = 0.

19) $f(x) = (x - 4)^{2/3}$ 19) _______

A) Absolute minimum value is 0 at x = -4.
B) There are no definable extrema.
C) Absolute maximum value is 0 at x = -4.
D) Absolute minimum value is 0 at x = 4.

20) $h(x) = \dfrac{x + 1}{x^2 + 3x + 3}$ 20) _______

A) Absolute maximum is 3 at x = 0; absolute minimum is $\frac{1}{3}$ at x = -2.

B) Absolute maximum is $-\frac{1}{3}$ at x = 0; absolute minimum is 1 at x = -2.

C) None

D) Absolute maximum is $\frac{1}{3}$ at x = 0; absolute minimum is - 1 at x = -2.

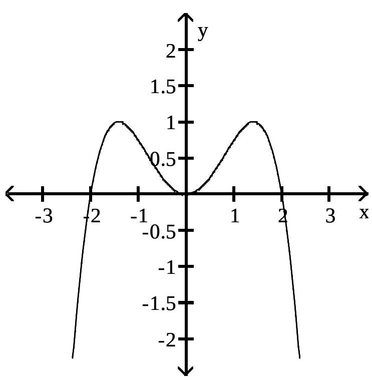
Solve the problem.

21) Select an appropriate graph of a twice-differentiable function y = f(x) that passes through the points $(-\sqrt{2},1)$, $\left(-\frac{\sqrt{6}}{3},\frac{5}{9}\right)$, (0,0), $\left(\frac{\sqrt{6}}{3},\frac{5}{9}\right)$ and $(\sqrt{2},1)$, and whose first two derivatives have the following sign patterns. 21) _______

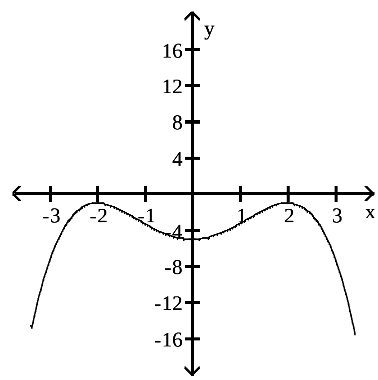
y′ : + $-\sqrt{2}$ - 0 + $\sqrt{2}$ -

y″ : + $-\frac{\sqrt{6}}{3}$ - $\frac{\sqrt{6}}{3}$ +

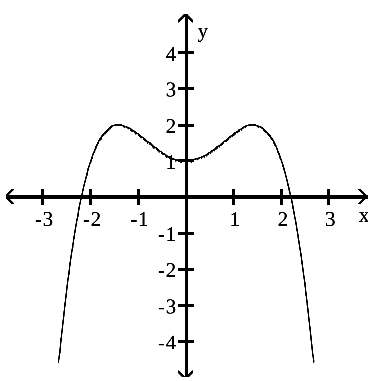
A)



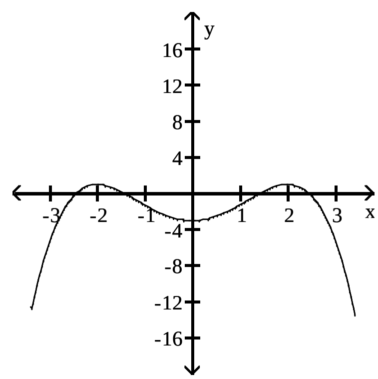
B)



C)



D)



SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

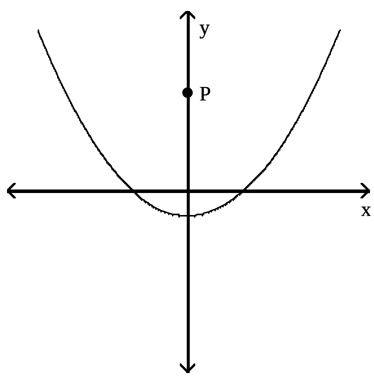
22) Sketch a continuous curve $y = f(x)$ with the following properties: 22) _______________

$f(2) = 3$; $f''(x) > 0$ for $x > 4$; and $f''(x) < 0$ for $x < 4$.

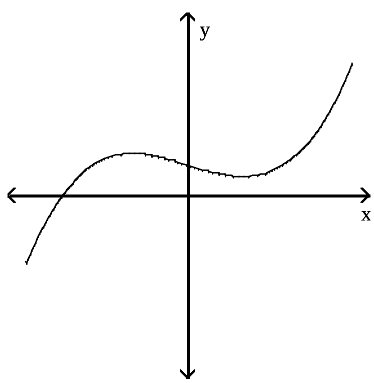
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

23) The graph below shows the first derivative of a function $y = f(x)$. Select a possible graph of f that passes through the point P. 23) _______

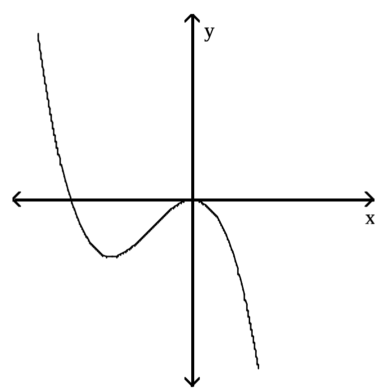
f′



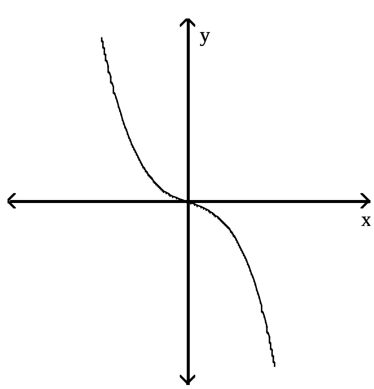
A)



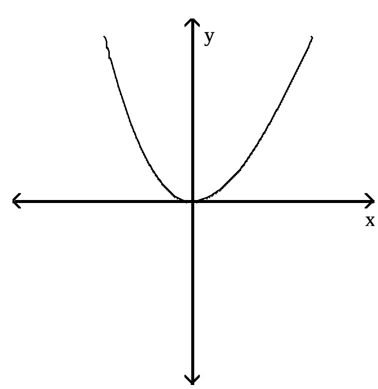
B)



C)

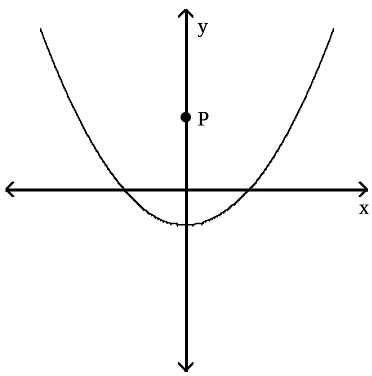


D)

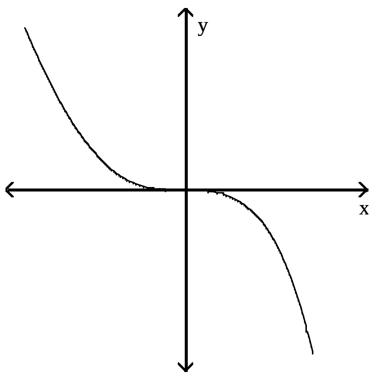


24) The graph below shows the first derivative of a function y = f(x). Select a possible graph f that passes through the point P. 24) _______

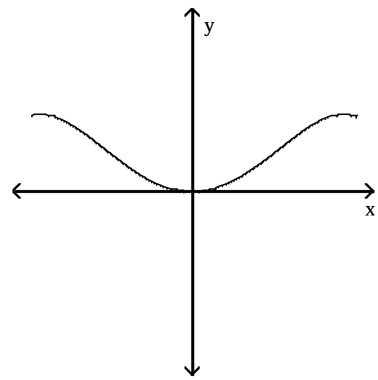
f′



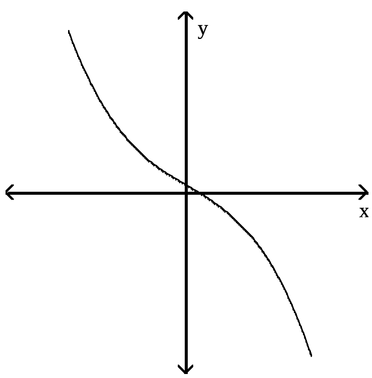
A)



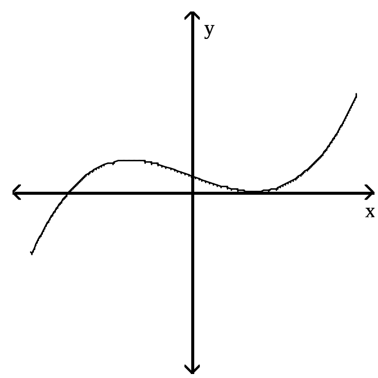
B)



C)



D)



Find the largest open interval where the function is changing as requested.

25) Increasing $y = 7x - 5$ 25) _______

A) $(-5, \infty)$ B) $(-\infty, 7)$ C) $(-\infty, \infty)$ D) $(-5, 7)$

26) Increasing $f(x) = \frac{1}{x^2 + 1}$ 26) _______

A) $(1, \infty)$ B) $(-\infty, 1)$ C) $(-\infty, 0)$ D) $(0, \infty)$

27) Decreasing $f(x) = \sqrt{4 - x}$ 27) _______

A) $(4, \infty)$ B) $(-\infty, 4)$ C) $(-4, \infty)$ D) $(-\infty, -4)$

28) Decreasing $f(x) = |x - 8|$ 28) _______

A) $(-\infty, -8)$ B) $(8, \infty)$ C) $(-\infty, 8)$ D) $(-8, \infty)$

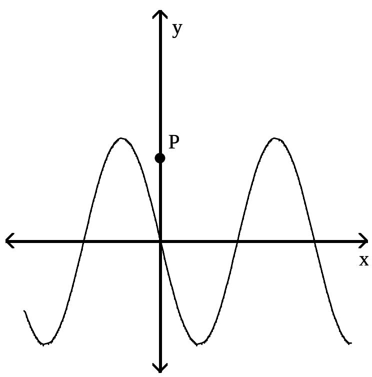
29) Decreasing $f(x) = x^3 - 4x$ 29) _______

A) $\left(-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$ B) $(-\infty, \infty)$ C) $\left(\frac{2\sqrt{3}}{3}, \infty\right)$ D) $\left(-\infty, -\frac{2\sqrt{3}}{3}\right)$

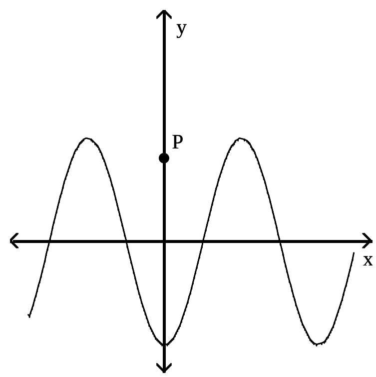
Solve the problem.

30) The graphs below show the first and second derivatives of a function $y = f(x)$. Select a possible graph f that passes through the point P. 30) _______

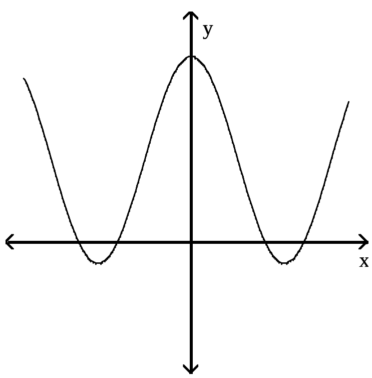
f'



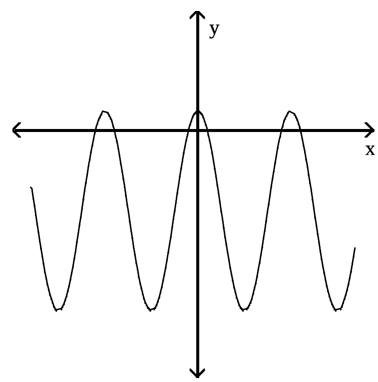
f''



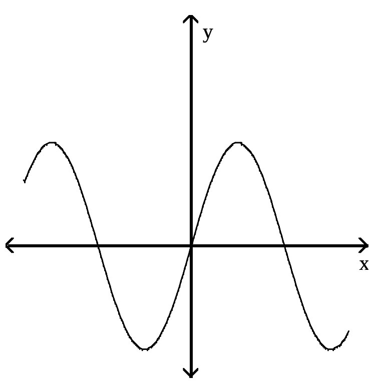
A)



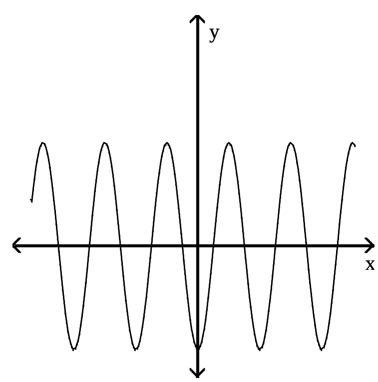
B)



C)

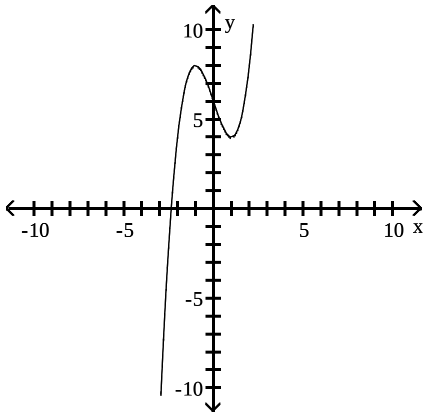


D)



Use the graph of the function f(x) to locate the local extrema and identify the intervals where the function is concave up and concave down.

31) 31) _______



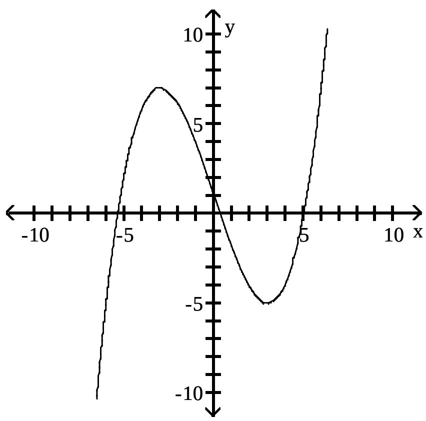
A) Local minimum at x = 1; local maximum at x = -1; concave down on $(0, \infty)$; concave up on $(-\infty, 0)$

B) Local minimum at x = 1; local maximum at x = -1; concave down on $(-\infty, \infty)$

C) Local minimum at x = 1; local maximum at x = -1; concave up on $(0, \infty)$; concave down on $(-\infty, 0)$

D) Local minimum at x = 1; local maximum at x = -1; concave up on $(-\infty, \infty)$

32) 32) _______



A) Local minimum at x = 3 ; local maximum at x = -3 ; concave down on $(-\infty, -3)$ and $(3, \infty)$; concave up on $(-3, 3)$

B) Local minimum at x = 3 ; local maximum at x = -3 ; concave up on $(-\infty, -3)$ and $(3, \infty)$; concave down on $(-3, 3)$

C) Local minimum at x = 3 ; local maximum at x = -3 ; concave down on $(0, \infty)$; concave up on $(-\infty, 0)$

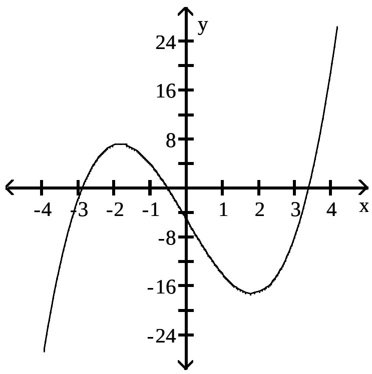
D) Local minimum at x = 3 ; local maximum at x = -3 ; concave up on $(0, \infty)$; concave down on $(-\infty, 0)$

Solve the problem.

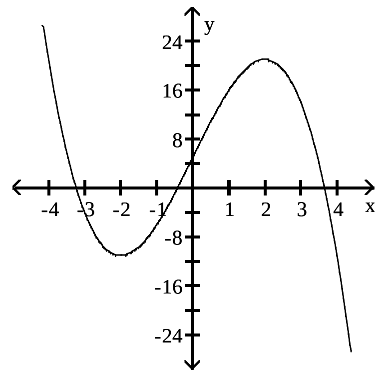
33) Using the following properties of a twice-differentiable function $y = f(x)$, select a possible graph of f. 33) _______

x	y	Derivatives
$x < 2$		$y' > 0, y'' < 0$
-2	11	$y' = 0, y'' < 0$
$-2 < x < 0$		$y' < 0, y'' < 0$
0	-5	$y' < 0, y'' = 0$
$0 < x < 2$		$y' < 0, y'' > 0$
2	-21	$y' = 0, y'' > 0$
$x > 2$		$y' > 0, y'' > 0$

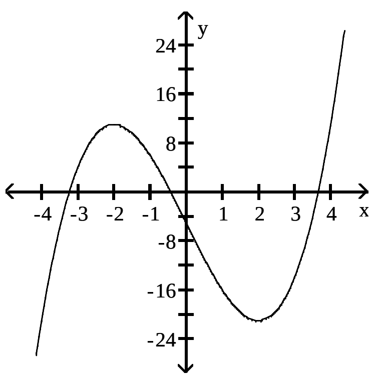
A)



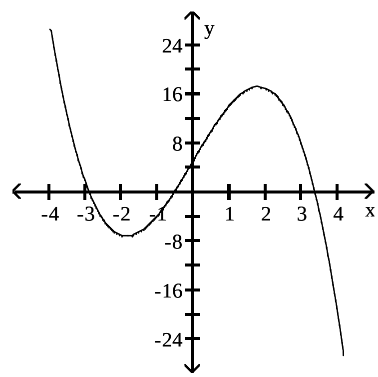
B)



C)

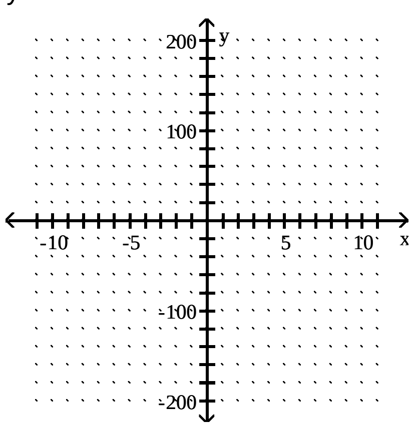


D)

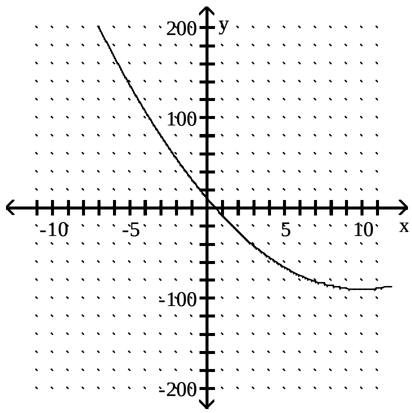


Graph the equation. Include the coordinates of any local extreme points and inflection points.

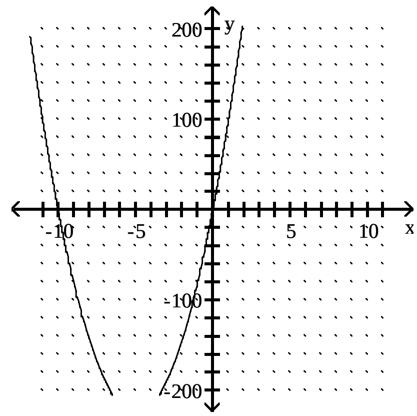
34) $y = 9x^2 + 90x$ 34) _______



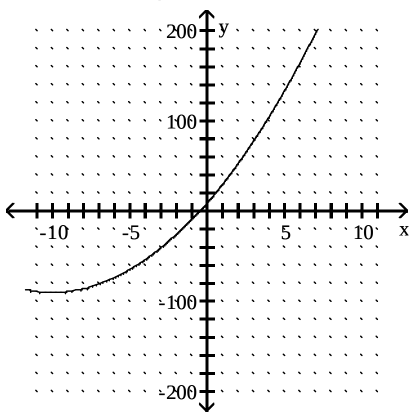
A) local minimum: (10, -90)
no inflection points



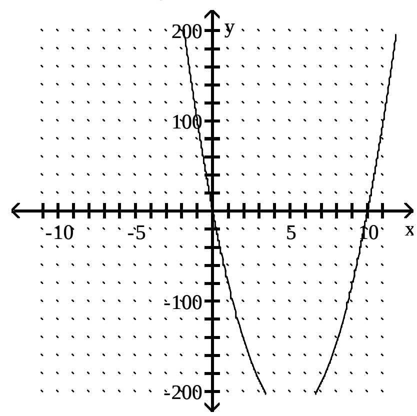
B) local minimum: (-5, -225)
no inflection points



C) local minimum: (-10, -90)
no inflection points

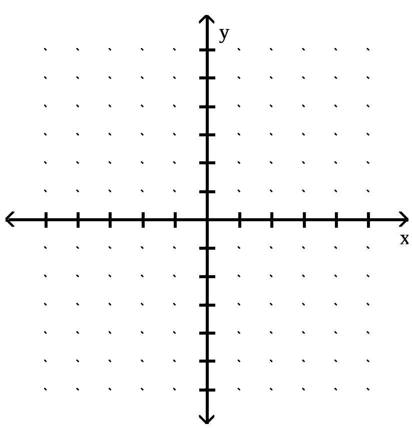


D) local minimum: (5, -225)
no inflection points

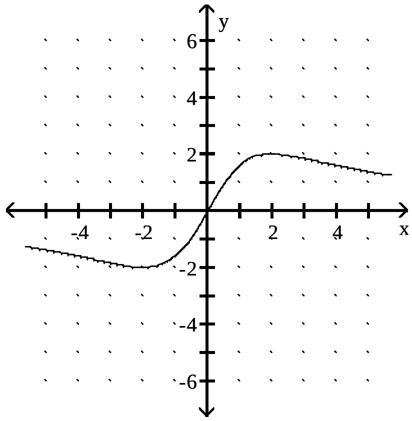


35) $y = \dfrac{8x}{x^2 + 4}$

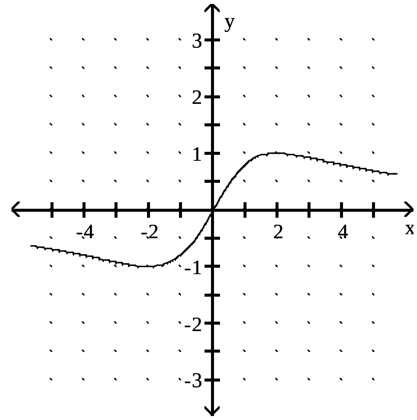
35) ______



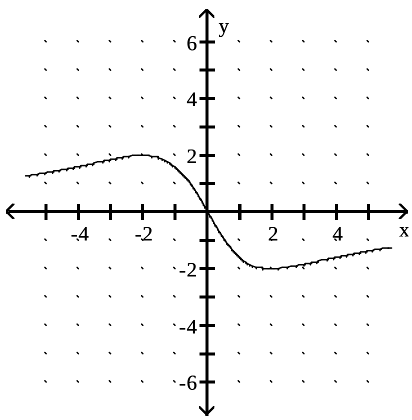
A) local minimum: (-2, -2)
local maximum: (2, 2)
inflection points: (0, 0), $(-2\sqrt{3}, -2\sqrt{3})$, $(2\sqrt{3}, 2\sqrt{3})$



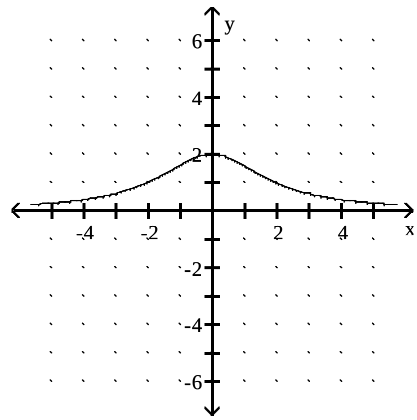
B) local minimum: $\left(-2, -1\right)$
local maximum: $\left(2, 1\right)$
inflection point: (0,0)



C) local minimum: (2, -2)
local maximum: (-2, 2)
inflection point: (0, 0)

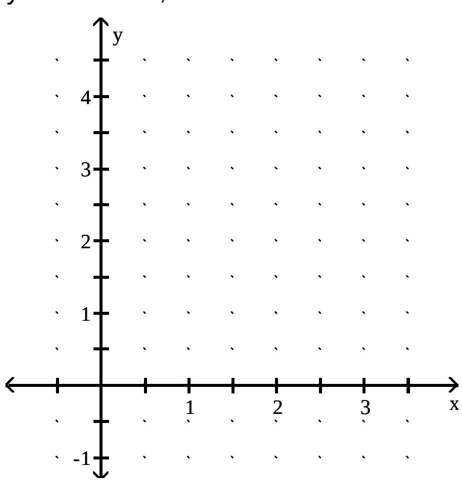


D) absolute maximum: $\left(0, 2\right)$
no inflection point

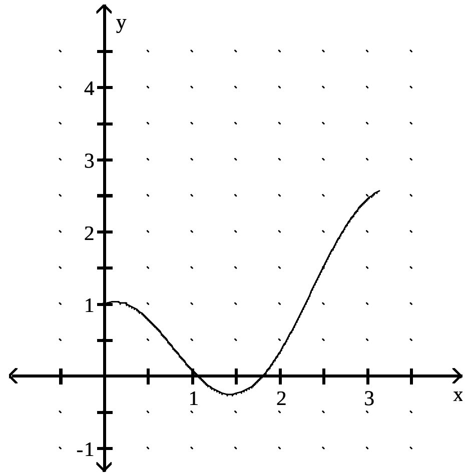


36) $y = x + \cos 2x$, $0 \le x \le \pi$

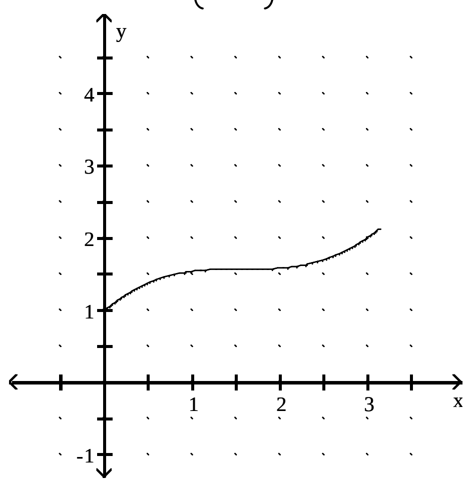
36) _______



A) local minimum: (1.444, -0.246)
local maximum: (0.126, 1.031)
inflection points: (0.785, 0.393), (2.356, 1.178)



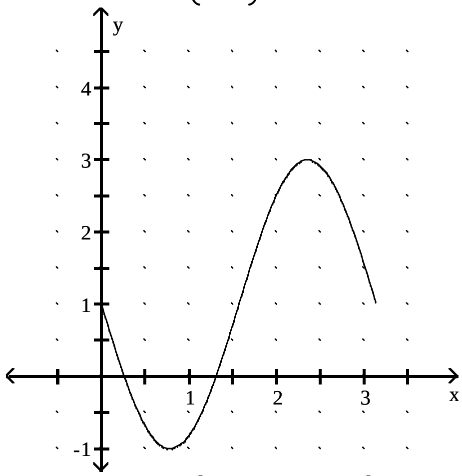
B) no local extrema
inflection point: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$



C) local minimum: $\left(\frac{\pi}{4}, -1\right)$

local maximum: $\left(\frac{3\pi}{4}, 3\right)$

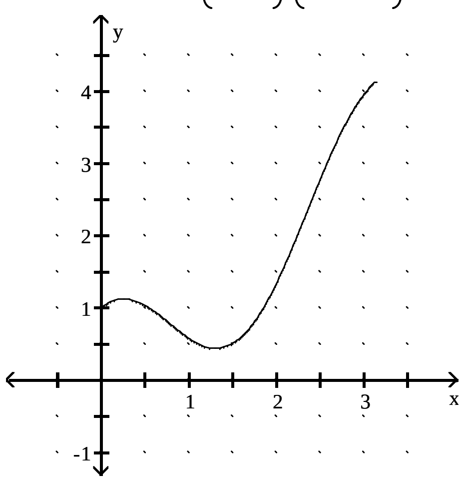
inflection point: $\left(\frac{\pi}{2}, 1\right)$



D) local minimum: $\left(\frac{5\pi}{12}, \frac{5\pi - 6\sqrt{3}}{12}\right)$

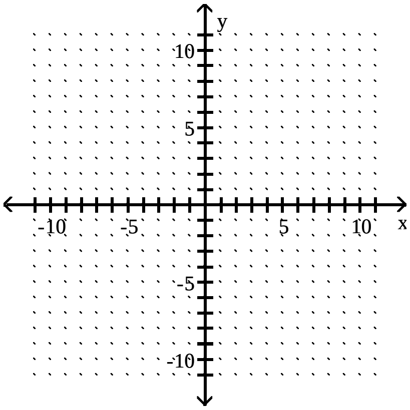
local maximum: $\left(\frac{\pi}{12}, \frac{\pi + 6\sqrt{3}}{12}\right)$

inflection points: $\left(\frac{\pi}{4}, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \frac{3\pi}{4}\right)$

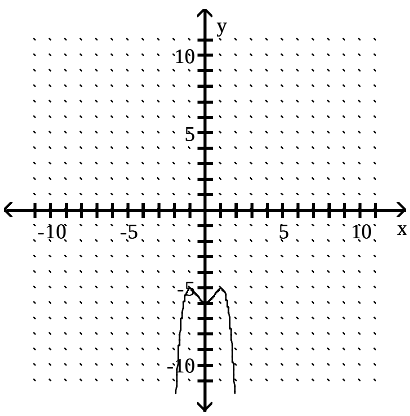


Sketch the graph and show all local extrema and inflection points.

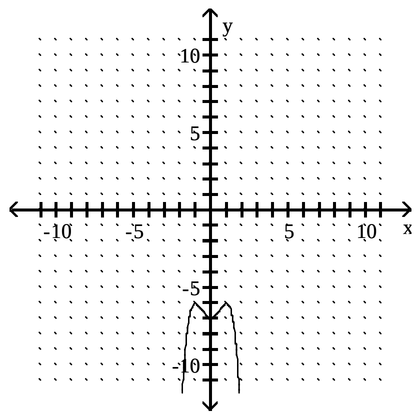
37) $f(x) = -x^4 + 2x^2 - 7$ 37) ______



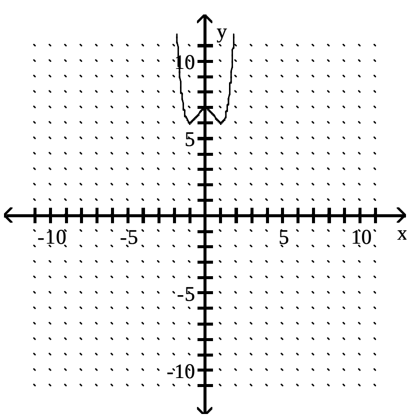
A) Local maxima: (-1, -6), (1, -6)
Inflection points: $\left(-\sqrt{\frac{1}{3}}, \frac{2}{3}\right), \left(\sqrt{\frac{1}{3}}, \frac{2}{3}\right)$



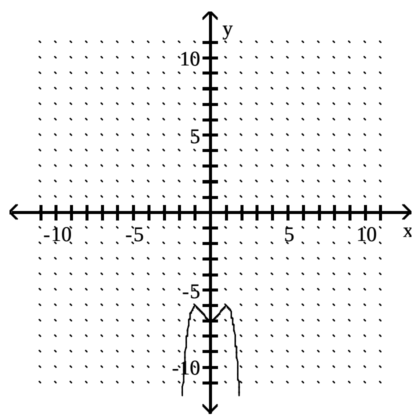
B) Local maxima: (-1, -6), (1, -6)
Local minimum: (0, -7)
Inflection points: $\left(-\sqrt{\frac{1}{3}}, \frac{2}{3}\right), \left(\sqrt{\frac{1}{3}}, \frac{2}{3}\right)$



C) Local minima: (-1, 6), (1, 6)
Local maximum: (0, 7)
Inflection point: $\left(-\sqrt{\frac{1}{3}}, \frac{58}{9}\right), \left(\sqrt{\frac{1}{3}}, \frac{58}{9}\right)$

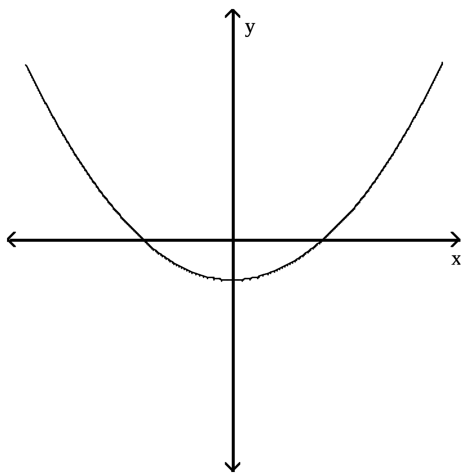


D) Local maxima: (-1, -6), (1, -6)
Local minimum: (0, -7)
No inflection points

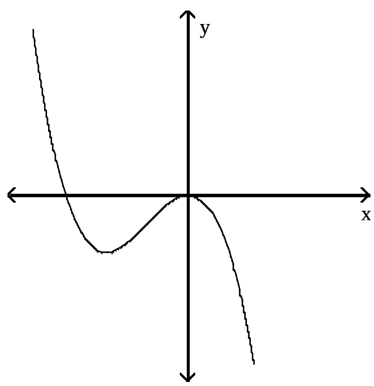


For the given expression y′, find y" and sketch the general shape of the graph of y = f(x).

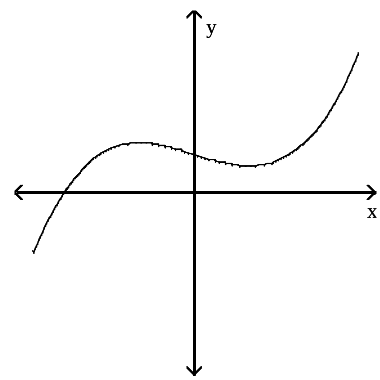
38) $y' = \frac{x^2}{5} - 1$ 38) _______



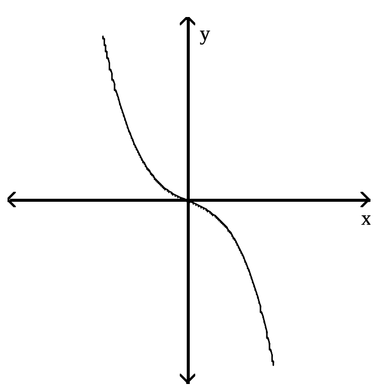
A)



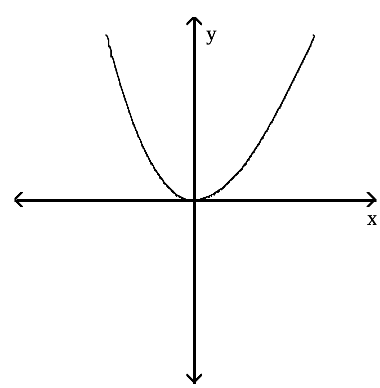
B)



C)



D)



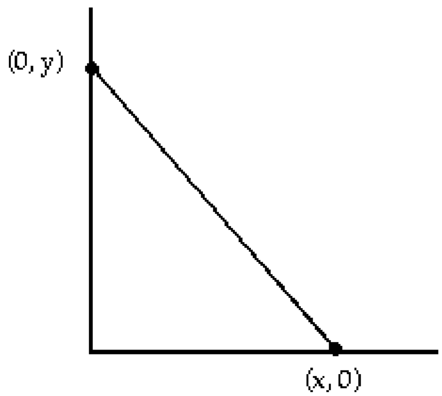
Solve the problem.

39) From a thin piece of cardboard 10 in. by 10 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary. 39) _______

A) 5 in. × 5 in. × 2.5 in.; 62.5 in^3
B) 6.7 in. × 6.7 in. × 3.3 in.; 148.1 in^3
C) 6.7 in. × 6.7 in. × 1.7 in.; 74.1 in^3
D) 3.3 in. × 3.3 in. × 3.3 in.; 37 in^3

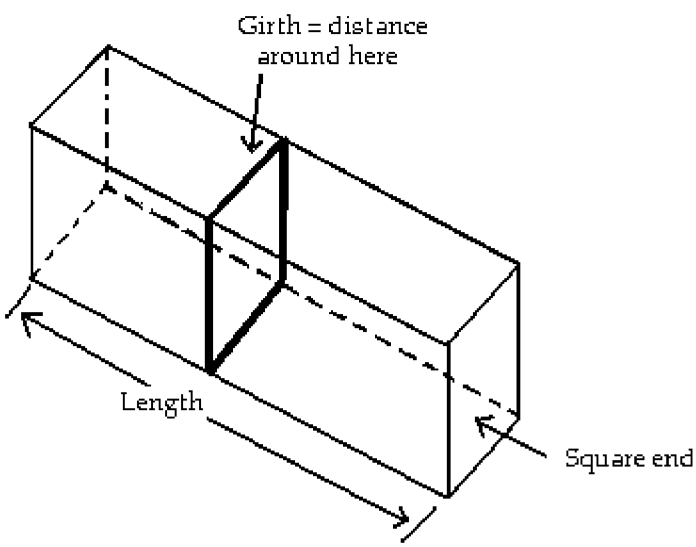
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

40) You are planning to close off a corner of the first quadrant with a line segment 19 units long running from (x, 0) to (0, y). Show that the area of the triangle enclosed by the segment is largest when x = y. 40) ____________



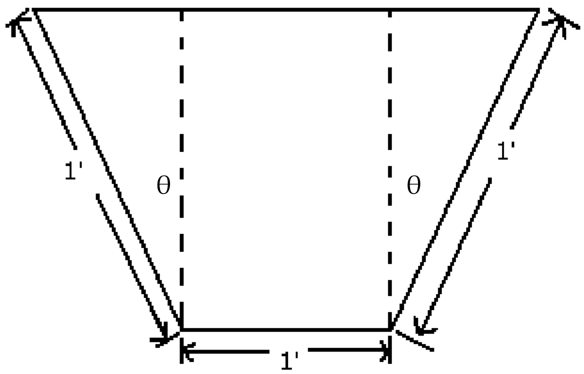
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

41) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 120 in. What dimensions will give a box with a square end the largest possible volume? 41) ______



A) 40 in. × 40 in. × 40 in.
B) 20 in. × 20 in. × 100 in.
C) 20 in. × 20 in. × 40 in.
D) 20 in. × 40 in. × 40 in.

42) A trough is to be made with an end of the dimensions shown. The length of the trough is to be 18 feet long. Only the angle θ can be varied. What value of θ will maximize the trough's volume? 42) _______



A) 30° B) 12° C) 48° D) 32°

43) If the price charged for a candy bar is p(x) cents, then x thousand candy bars will be sold in a certain city, where $p(x) = 124 - \frac{x}{16}$. How many candy bars must be sold to maximize revenue? 43) _______

A) 1984 candy bars
B) 992 candy bars
C) 992 thousand candy bars
D) 1984 thousand candy bars

44) Suppose $c(x) = x^3 - 22x^2 + 30{,}000x$ is the cost of manufacturing x items. Find a production level that will minimize the average cost of making x items. 44) _______

A) 12 items B) 13 items C) 10 items D) 11 items

45) The diameter of a tree was 9 in. During the following year, the circumference increased 2 in. About how much did the tree's diameter increase? (Leave your answer in terms of π.) 45) _______

A) $\frac{2}{\pi}$ in. B) $\frac{\pi}{2}$ in. C) $\frac{11}{\pi}$ in. D) $\frac{9}{\pi}$ in.

Express the relationship between a small change in x and the corresponding change in y in the form dy = f′(x) dx.

46) $f(x) = 9x^2 + 6x + 6$ 46) _______

A) $dy = 18x + 6\ dx$
B) $dy = (18x + 6)\ dx$
C) $dy = 18x + 12\ dx$
D) $dy = 18x\ dx$

47) $f(x) = x\sqrt{9x - 5}$ 47) _______

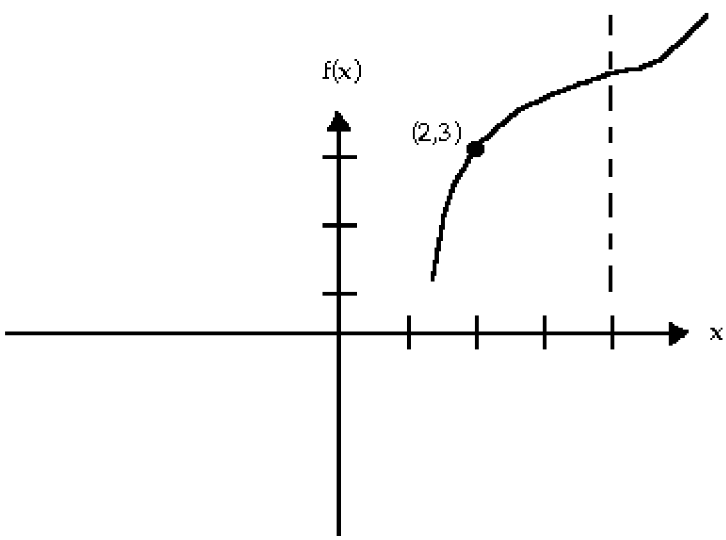
A) $dy = \frac{27x - 10}{2\sqrt{9x - 5}}\ dx$
B) $dy = \frac{27x + 10}{\sqrt{9x - 5}}\ dx$
C) $dy = \frac{27x - 10}{\sqrt{9x - 5}}\ dx$
D) $dy = \frac{27x + 10}{2\sqrt{9x - 5}}\ dx$

Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for the function and interval.

48) $f(x) = x^2 + 3x + 2$, [1, 2] 48) _______

A) $0, \frac{3}{2}$ B) 1, 2 C) $-\frac{3}{2}, \frac{3}{2}$ D) $\frac{3}{2}$

1) D
2) A
3) A
4) A
5) A
6) B
7) C
8) C
9) C
10) C
11) C
12) B
13) C
14) D
15) C
16) D
17) A
18) C
19) D
20) D
21) A
22) Answers will vary. A general shape is indicated below:



23) A
24) D
25) C
26) C
27) B
28) C
29) A
30) A
31) C
32) D

33) C
34) B
35) A
36) D
37) B
38) B
39) C
40) If x , y represent the legs of the triangle, then $x^2 + y^2 = 19^2$.

Solving for y, $y = \sqrt{361 - x^2}$

$A(x) = xy = x\sqrt{361 - x^2}$

$A'(x) = -\dfrac{x^2}{2\sqrt{361 - x^2}} + \dfrac{\sqrt{361 - x^2}}{2}$

Solving $A'(x) = 0$, $x = \pm\dfrac{19\sqrt{2}}{2}$

Substitute and solve for y: $\left(\dfrac{19\sqrt{2}}{2}\right)^2 + y^2 = 361$; $y = \dfrac{19\sqrt{2}}{2}$ $\therefore$ x = y.

41) C
42) A
43) C
44) D
45) A
46) B
47) A
48) D