When you are done with your homework you should be able to…

- Integrate functions whose antiderivatives involve inverse trigonometric functions
- Use the method of completing the square to integrate a function
- Review the basic integration rules involving elementary functions

Warm-up:

1. Differentiate the following functions with respect to x.

a. $y = \arctan\frac{x}{2} - \frac{1}{2\left(x^2+4\right)}$

b. $\arctan\left(xy\right) = \arcsin\left(x+y\right)$.

2. Complete the square.

a. $3+4x-x^2$

b. $2x^2-6x+9$

What did you notice about the derivatives of the inverse trigonometric functions?

THEOREM: INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x, and let $a>0$.

1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a}+C$

2. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a}+C$

3. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a}+C$

Example 1: Find or evaluate the integral.

a. $\int \frac{dx}{x\sqrt{x^2-1}}$

b. $\int \frac{xdx}{\sqrt{x^2-1}}$

c. $\int \frac{dx}{\sqrt{1-x^2}}$

d. $\int \frac{dx}{x \ln x}$

e. $\int \frac{(\ln x)^2 \, dx}{x}$

f. $\int \ln x dx$

Example 2: Find the integral by completing the square.

a. $\int \frac{dx}{x^2+4x+13}$

b. $\int \frac{dx}{x\sqrt{x^4-4}}$

c. $\int \frac{2dx}{\sqrt{-x^2+4x}}$

d. $\int \frac{2x-5}{x^2+2x+2}dx$

e. $\int \frac{x}{\sqrt{9+8x^2-x^4}}dx$

f. $\int_1^3 \frac{1}{\sqrt{x}(1+x)}dx$

g. $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x} dx$

Example 3: Find the area of the region bound by the graphs of $y = \frac{4e^x}{1+e^{2x}}$, $x = 0$, $y = 0$ and $x = \ln\sqrt{3}$.