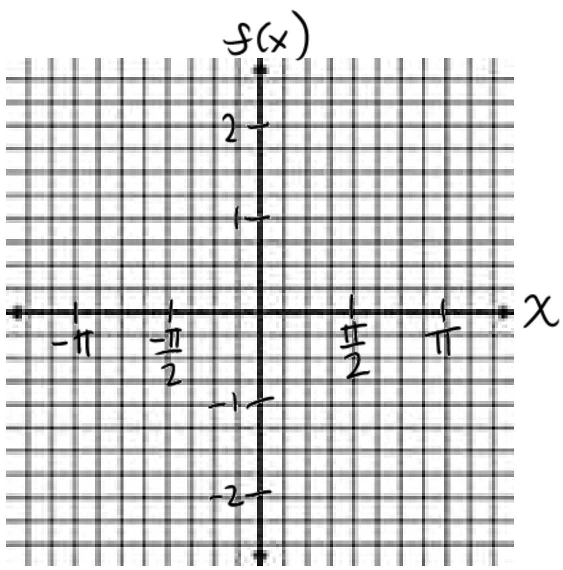
When you are done with your homework you should be able to…

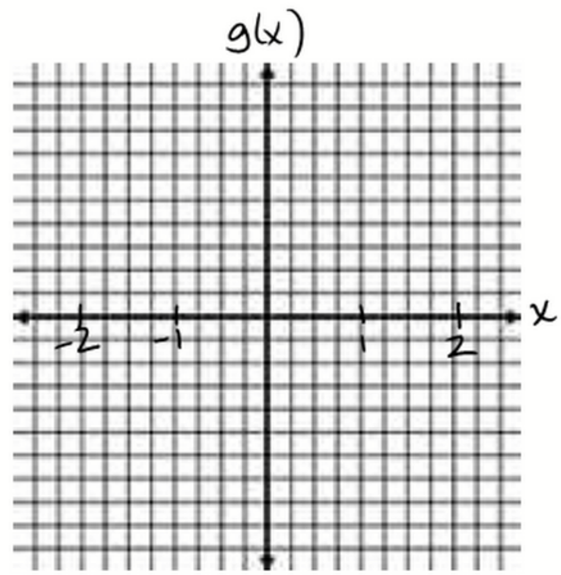
- π Develop properties of the six inverse trigonometric functions
- π Differentiate an inverse trigonometric function
- π Review the basic differentiation rules for elementary functions

Warm-up: Draw the following graphs by hand from $[-\pi, \pi]$. List the domain and range in interval notation.

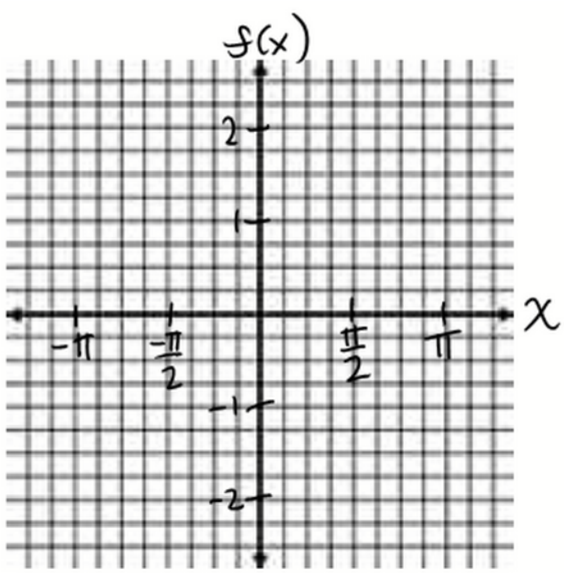
1. Graph $f(x) = \sin x$.



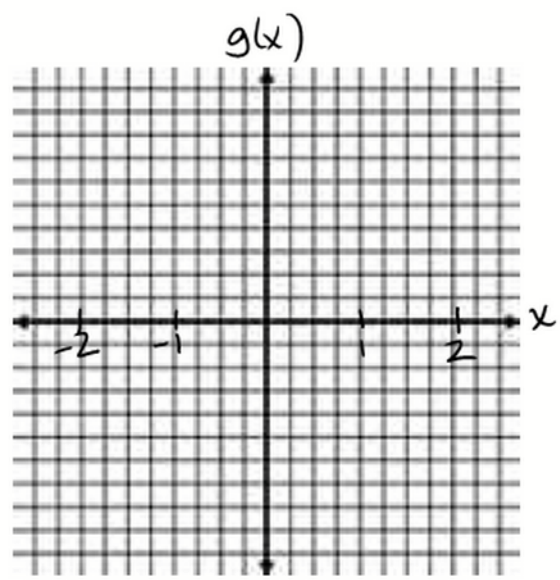
2. Graph $g(x) = \arcsin x$.



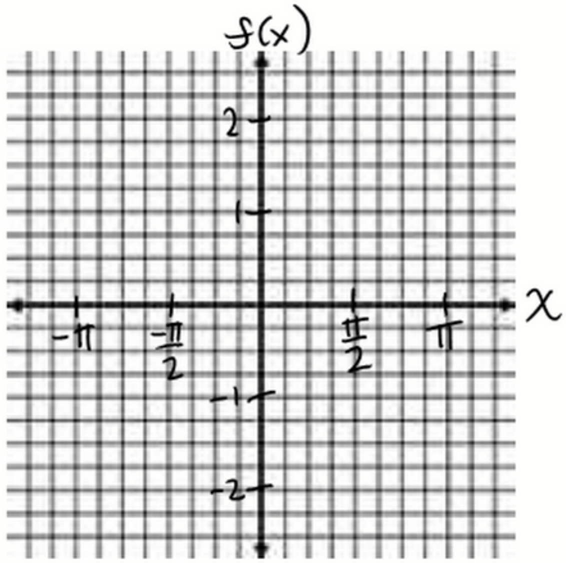
3. Graph $f(x) = \csc x$.



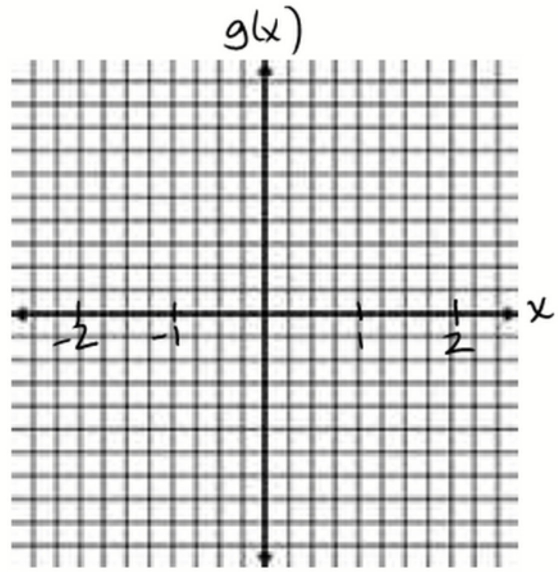
4. Graph $g(x) = \operatorname{arc}\csc x$.



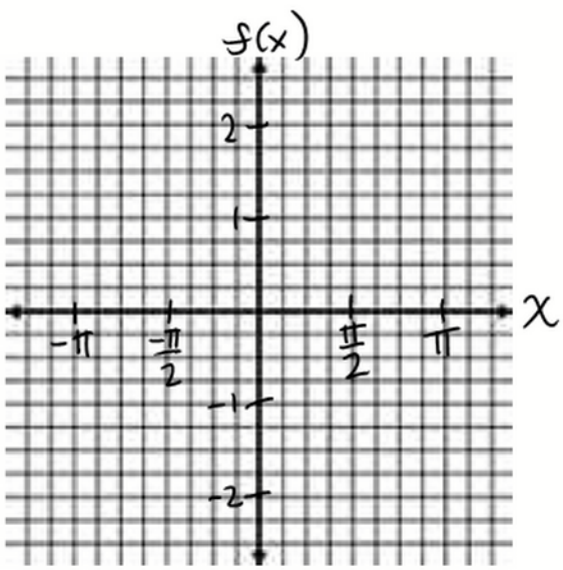
5. Graph $f(x)=\cos x$.



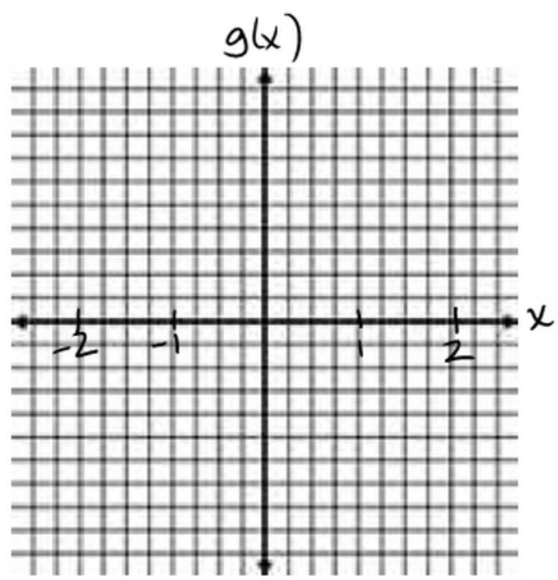
6. Graph $g(x)=\arccos x$.



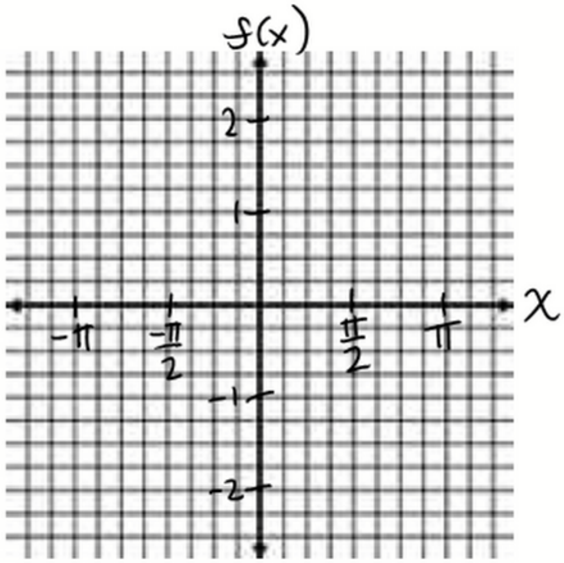
7. Graph $f(x)=\sec x$.



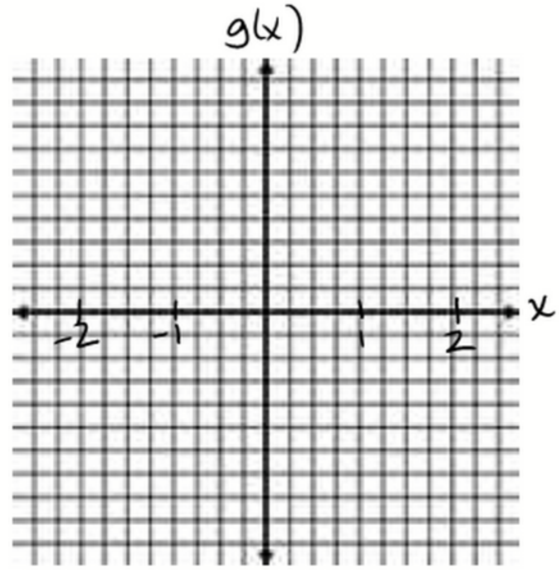
8. Graph $g(x)=\text{arc}\sec x$.



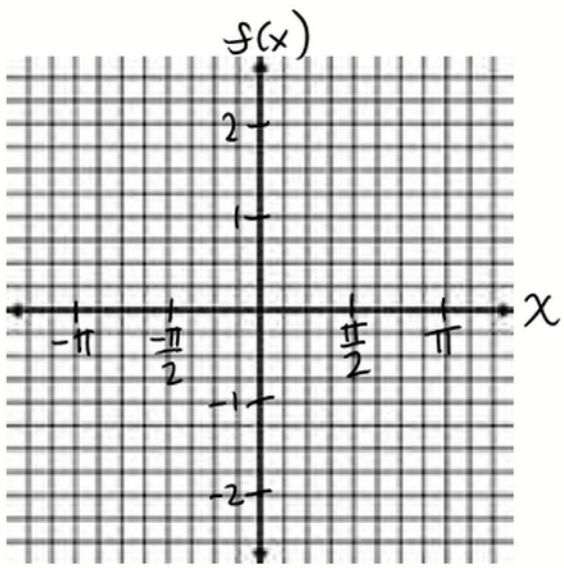
9. Graph $f(x) = \tan x$.



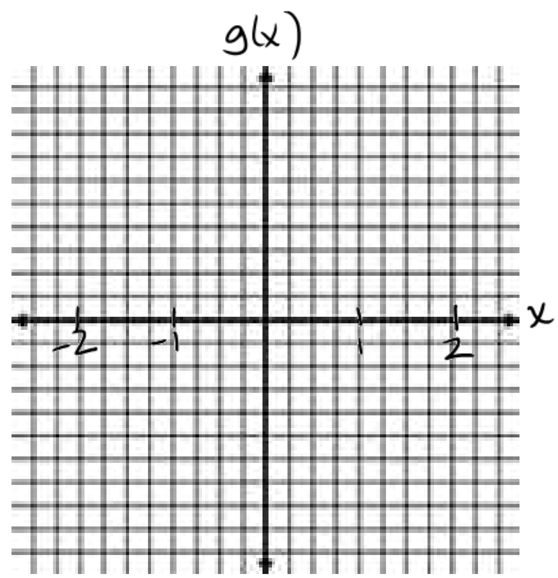
10. Graph $g(x) = \arctan x$.



11. Graph $f(x) = \cot x$.



12. Graph $g(x) = \operatorname{arccot} x$.



PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

If $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, then $$\sin(\arcsin x) = x \quad \text{and} \quad \arcsin(\sin y) = y.$$
If $-\frac{\pi}{2} < y < \frac{\pi}{2}$, then $$\tan(\arctan x) = x \quad \text{and} \quad \arctan(\tan y) = y.$$
If $\lvert x \rvert \ge 1$ and $0 \le y < \frac{\pi}{2}$ or $\frac{\pi}{2} < y \le \pi$, then $$\sec(\operatorname{arc\,sec} x) = x \quad \text{and} \quad \operatorname{arc\,sec}(\sec y) = y.$$
If $\_\_\_ \le x \le \_\_\_$ and $\_\_\_\_ \le y \le \_\_\_\_$, then $$\cos(\arccos x) = x \quad \text{and} \quad \arccos(\cos y) = y.$$
If $-\frac{\pi}{2} < y < \frac{\pi}{2}$, then $$\_\_\_(\text{arc}\_\_\_\_\; x) = x \quad \text{and} \quad \text{arc}\_\_\_\_(\_\_\_\_\; y) = y.$$
If $\lvert x \rvert \ge 1$ and $\_\_\_\_ \le y < \_\_\_\_$ or $\_\_\_\_ < y \le \_\_\_\_$, then $$\csc(\operatorname{arc\,csc} x) = x \quad \text{and} \quad \operatorname{arc\,csc}(\csc y) = y.$$

Example 1: Evaluate each function.

a. $\operatorname{arc\,cot}(1)$

b. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

c. $\operatorname{arc\,sec}\left(\frac{2\sqrt{3}}{3}\right)$

e. $\operatorname{arc\,cos}\left(-\frac{1}{2}\right)$

d. $\arctan\left(\sqrt{3}\right)$

f. $\operatorname{arc\,csc}\left(-\sqrt{2}\right)$

Example 2: Solve the equation for *x*.

$$\arctan\left(2x-5\right)=-1$$

Example 3: Write the expression in algebraic form. (HINT: Sketch a right triangle)

a. $\sec\left(\arctan 4x\right)$

b. $\cos\left(\arcsin x\right)$

Example 4: Differentiate with respect to *x*.

a. $y = \arcsin x$

b. $y = \arccos x$

c. $y = \arctan x$

d. $y = \operatorname{arc}\csc x$

e. $y = \operatorname{arc}\sec x$

f. $y = \operatorname{arc}\cot x$

What have we found out?!

DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
2. $\frac{d}{dx}[\arccos u] = -\frac{u'}{\sqrt{1-u^2}}$
3. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
4. $\frac{d}{dx}[\operatorname{arc}\cot u] = -\frac{u'}{1+u^2}$
5. $\frac{d}{dx}[\operatorname{arc}\sec u] = \frac{u'}{|u|\sqrt{u^2-1}}$
6. $\frac{d}{dx}[\operatorname{arc}\csc u] = -\frac{u'}{|u|\sqrt{u^2-1}}$

Example 5: Find the derivative of the function. Simplify your result to a single ration expression with positive exponents.

a. $f(t) = \arcsin t^3$

b. $g(x) = \arcsin x + \arccos x$

c. $y = x \arctan 2x - \frac{1}{4}\ln\left(1+4x^2\right)$

d. $y = 25\arcsin\frac{x}{5} - x\sqrt{25-x^2}$

Example 6: Find an equation of the tangent line to the graph of the function $y = \frac{1}{2}\arccos x$ at the point $\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{8}\right)$.