When you finish your homework you should be able to…

- π Write the general solution of a differential equation
- π Use indefinite integral notation for antiderivatives
- π Use basic integration rules to find antiderivatives
- π Find a particular solution of a differential equation

Warm-up: For each derivative, describe the original function F.

1. $F'(x) = 2x$
2. $F'(x) = x^3$
3. $F'(x) = \dfrac{1}{x^2}$
4. $F'(x) = \sec^2 x$
5. $F'(x) = \sin x$
6. $F'(x) = 6$

DEFINITION OF ANTIDERIVATIVE

A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Why is F called **an** antiderivative of f, rather than **the** antiderivative of f?

THEOREM: REPRESENTATION OF ANTIDERIVATIVES

If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form $G(x) = F(x) + C$, for all x in I where C is a constant.

NOTATION:

Example 1: Verify the statement by showing that the derivative of the right side equals the integrand of the left side.

$$\int \left(8x^3 + \frac{1}{2x^2}\right)dx = 2x^4 - \frac{1}{2x} + C$$

Example 2: Find the general solution of the differential equation.

a. $\frac{dy}{dx} = 2x^{-3}$

b. $\frac{dr}{d\theta} = \pi$

BASIC INTEGRATION RULES

Differentiation Formula	Integration Formula
$\frac{d}{dx}[C]=$	$\int 0dx=$
$\frac{d}{dx}[kx]=$	$\int kdx=$
$\frac{d}{dx}\left[kf(x)\right]=$	$\int kf(x)dx=$
$\frac{d}{dx}\left[f(x)\pm g(x)\right]=$	$\int\left[f(x)\pm g(x)\right]dx=$
$\frac{d}{dx}\left[x^n\right]=$	$\int x^n dx=$
$\frac{d}{dx}[\sin x]=$	$\int \cos xdx=$
$\frac{d}{dx}[\cos x]=$	$\int \sin xdx=$
$\frac{d}{dx}[\tan x]=$	$\int \sec^2 xdx=$
$\frac{d}{dx}[\sec x]=$	$\int \sec x\tan xdx=$
$\frac{d}{dx}[\cot x]=$	$\int \csc^2 xdx=$
$\frac{d}{dx}[\csc x]=$	$\int \csc x\cot xdx=$

Example 3: Find the indefinite integral and check the result by differentiation.

a. $\int (16 - x)dx$

b. $\int \frac{\sqrt[5]{x^3} - 2x}{\sqrt{x}}dx$

c. $\int (3x-4)^3 dx$

d. $\int (1-u)\sqrt{u}du$

e. $\int \sec t(\tan t - \sec t)dt$

f. $\int \left(4\theta - \csc^2 \theta\right)d\theta$

g. $\int \frac{\sin x}{1-\sin^2 x}dx$

INITIAL CONDITIONS AND PARTICULAR SOLUTIONS

You have already seen that the equation $y = \int f(x)dx$ has _________________

solutions, each differing from each other by a _________________________.

This means that the graphs of any two __________________________ of f are

__________________________ translations of each other.

In many applications of integration, you are given enough information to determine

a ____________________________ solution. To do this, you need only know the

value of $y = F(x)$ for one value of x. This information is called an

_____________________________ condition.

Example 4: Solve the differential equation.

a. $g'(x) = 6x^2, \quad g(0) = -1$

b. $f''(x) = \sin x, \quad f'(0) = 1, \quad f(0) = 6$