When you finish your homework you should be able to…

- π Determine finite limits at infinity
- π Determine the horizontal asymptotes, if any, of the graph of a function
- π Determine infinite limits at infinity

Warm-up: Evaluate the following limits analytically

1. $\lim_{x \to 1^+} \frac{3}{1-x}$

2. $\lim_{t \to 0} \frac{\sin 3t}{t}$

LIMITS AT INFINITY

This section discusses the ____________ behavior of a function on an

_____________ interval.

DEFINITION OF LIMITS AT INFINITY

Let L be a real number.

1. The statement $\lim_{x\to\infty} f(x) = L$ means that for each $\varepsilon > 0$ there exists an $M > 0$, such that $|f(x) - L| < \varepsilon$ whenever $x > M$.
2. The statement $\lim_{x\to-\infty} f(x) = L$ means that for each $\varepsilon > 0$ there exists an $N < 0$, such that $|f(x) - L| < \varepsilon$ whenever $x < N$.

DEFINITION OF A HORIZONTAL ASYMPTOTE

The line $y = L$ is a **horizontal asymptote** of the graph of f if $\lim_{x\to-\infty} f(x) = L$ or $\lim_{x\to\infty} f(x) = L$.

THEOREM: LIMITS AT INFINITY

If r is a positive rational number, then

$$\lim_{x\to\infty}\frac{c}{x^r}=0.$$

Furthermore, if x^r is defined when $x<0$, then

$$\lim_{x\to-\infty}\frac{c}{x^r}=0.$$

DEFINITION OF INFINITE LIMITS AT INFINITY

Let f be a function defined on the interval (a,∞).

1. The statement $\lim_{x\to\infty} f(x)=\infty$ means that for each $M>0$ there is a corresponding number $N>0$, such that $f(x)>M$ whenever $x>N$.
2. The statement $\lim_{x\to\infty} f(x)=-\infty$ means that for each $M<0$ there is a corresponding number $N>0$, such that $f(x)<M$ whenever $x>N$.

GUIDELINES FOR FINDING LIMITS AT +/- INFINITY

1. If the degree of the numerator is **<u>less than</u>** the degree of the denominator, then the limit of the rational function is ________.
2. If the degree of the numerator is **<u>equal to</u>** the degree of the denominator, then the limit of the rational function is the __________ of the leading ______________________.
3. If the degree of the denominator is **<u>greater than</u>** the degree of the denominator, then the limit of the rational function is plus or minus infinity, hence it does not ________________.

Example 1: Find the limit.

a. $\lim_{x\to-\infty}\left(\frac{5}{x}-\frac{x}{3}\right)$

b. $\lim_{x\to\infty}\left(\frac{x^2+3}{2x^2-1}\right)$

c. $\lim_{x\to-\infty}\sqrt{\frac{x^4-1}{x^3-1}}$

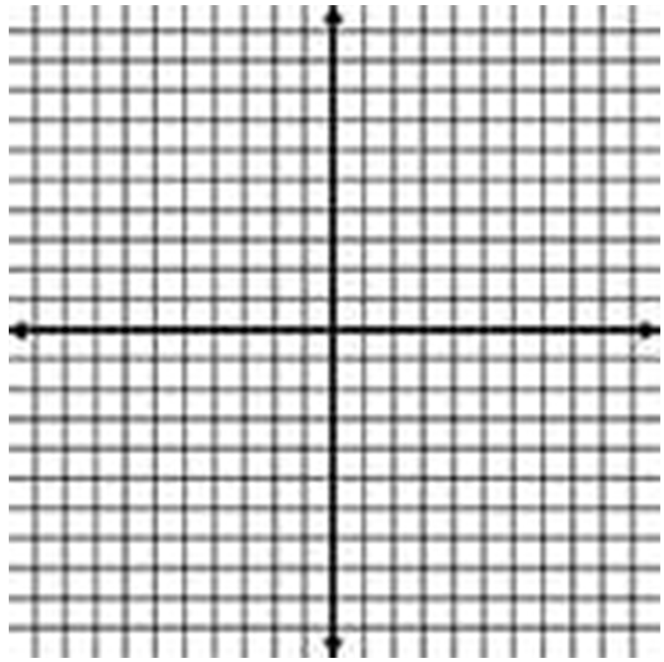
d. $\lim_{x \to \infty} \cos \frac{1}{x}$

e. $\lim_{x \to -\infty} \frac{-3x+1}{\sqrt{x^2+x}}$

f. $\lim_{x \to \infty} \frac{x}{x^2-1}$

Example 2: Sketch the graph of the equation using extrema, intercepts, symmetry, and asymptotes. Then use a graphing utility to verify your result.

a. $f(x)=\frac{1}{x^2-x-2}$



b. $h(x)=\dfrac{2x}{\sqrt{3x^2+1}}$