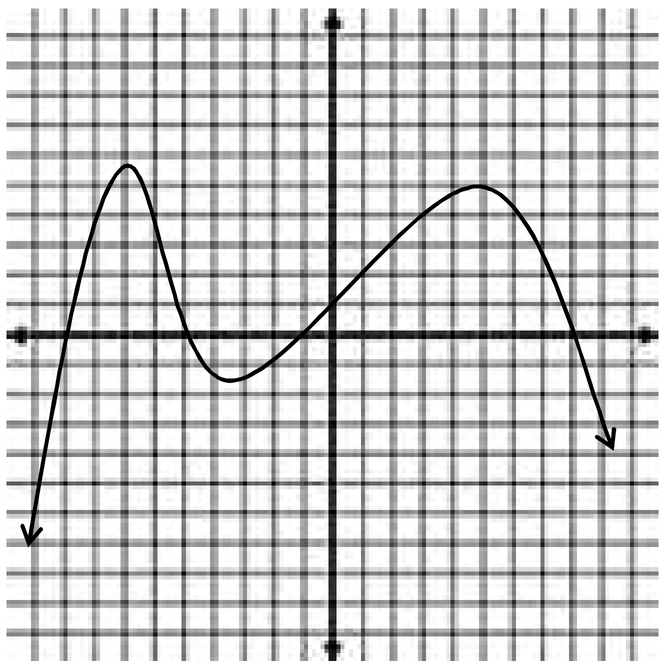
When you are done with your homework you should be able to…

- π Determine intervals on which a function is concave upward or concave downward
- π Find any points of inflection of the graph of a function
- π Apply the Second Derivative Test to find relative extrema of a function

Warm-up: Consider the graph of f' shown below.



a. Identify the interval(s) on which f is

i. increasing

ii. decreasing

b. Estimate the value(s) of x at which f has a relative

i. minimum

ii. maximum

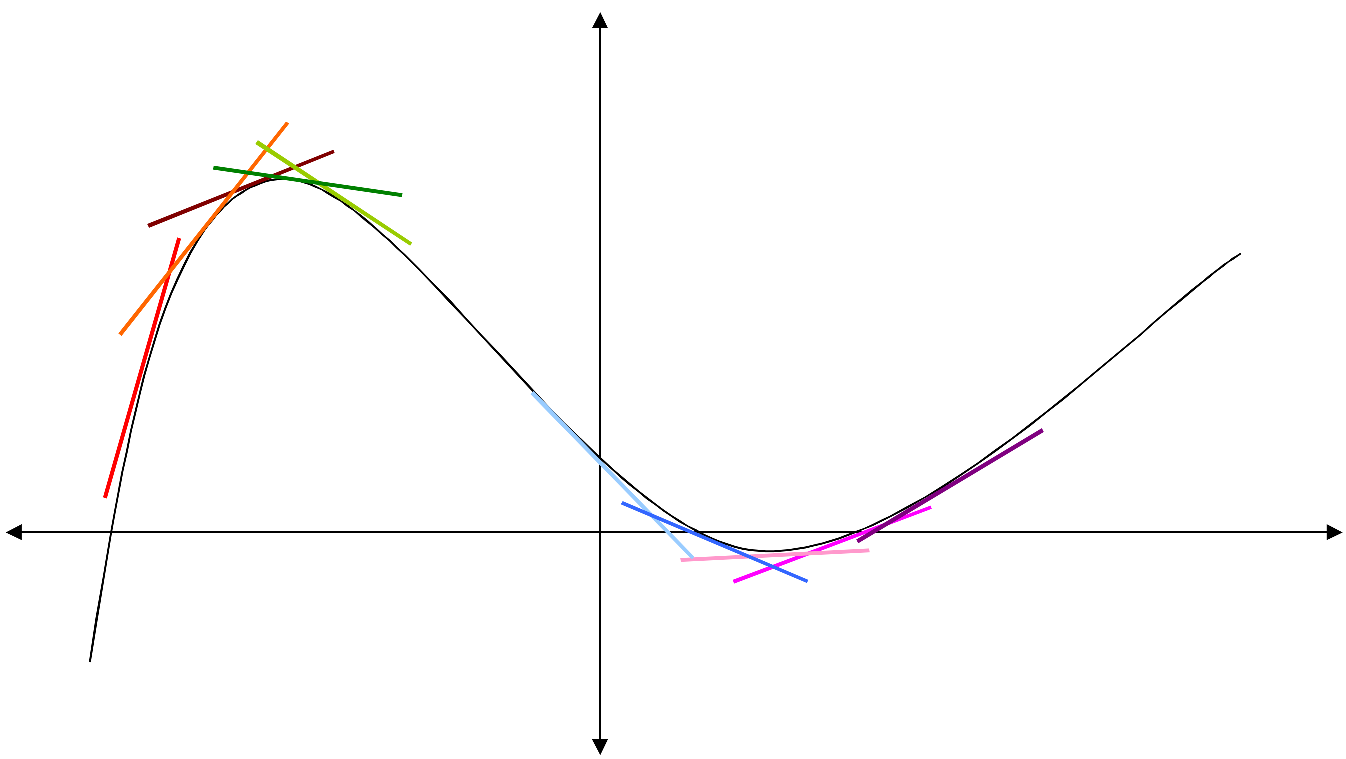
DEFINITION OF CONCAVITY

Let f be differentiable on an open interval I. The graph of f is **concave upward** on I if f' is increasing on the interval and **concave downward** on I if f' is decreasing on the interval.

THEOREM: TEST FOR CONCAVITY

Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x) > 0$ for all x in I, then f is **concave upward** on I.
2. If $f''(x) < 0$ for all x in I, then f is **concave downward** on I.



Example 1: Identify the open intervals on which the function is concave upward or concave downward.

a. $y = -x^3 + 3x^2 - 2$

b. $f(x) = x + \dfrac{2}{\sin x}, \; [-\pi, \pi]$

DEFINITION OF POINT OF INFLECTION

Let f be a function that is continuous on an open interval and let c be an element in the interval. If the graph of f has a tangent line at the point $\left(c, f\left(c\right)\right)$, then this point is a **point of inflection** of the graph of f if the concavity of f changes from upward to downward or from downward to upward at the point.

THEOREM: POINTS OF INFLECTION

If $\left(c, f\left(c\right)\right)$ is a point of inflection of the graph of f, then either $f''\left(c\right)=0$ or f'' does not exist at $x=c$.

Example 2: Consider the function $g\left(x\right)=2x^4-8x+3$.

a. Discuss the concavity of the graph of g.

b. Find all points of inflection.

THEOREM: SECOND DERIVATIVE TEST

Let f be a function such that $f'(c)=0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(x)>0$, then f has a **relative minimum** at $(c, f(c))$.
2. If $f''(x)<0$, then f has a **relative maximum** at $(c, f(c))$.
3. If $f''(x)=0$, the test FAILS and you need to run the FIRST DERIVATIVE TEST.

Example 3: Find all relative extrema. Use the Second Derivative Test where applicable.

a. $f(x)=x^3-5x^2+7x$

b. $f(x)=\dfrac{x}{x-1}$

Example 4: Sketch the graph of a function f having the given characteristics.

$f(0)=f(2)=0$

$f'(x)>0$ if $x<1$

$f'(1)=0$

$f'(x)<0$ if $x>1$

$f''(x)<0$