When you are done with your homework you should be able to…

- π Determine intervals on which a function is increasing or decreasing
- π Apply the First Derivative Test to find relative extrema of a function

Warm-up: Find the equation of the line tangent to the function $f(x) = \tan x$ at $x = \frac{3\pi}{4}$.

INCREASING AND DECREASING FUNCTIONS

A function is ______________________ if, as *x* moves to the right, its graph moves up, and is decreasing if its graph moves ___________________. A positive derivative implies that the function is ___________________ and a ______________________ derivative implies that the function is decreasing.

DEFINITION OF INCREASING AND DECREASING FUNCTIONS

A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

THEOREM: TEST FOR INCREASING AND DECREASING FUNCTIONS

Let f be a function that is continuous on the closed interval $[a,b]$, and differentiable on the open interval (a,b).

1. If $f'(x)>0$ for all x in (a,b), then f is **increasing** on (a,b).
2. If $f'(x)<0$ for all x in (a,b), then f is **decreasing** on (a,b).
3. If $f'(x)=0$ for all x in (a,b), then f is **constant** on (a,b).

GUIDELINES FOR FINDING INTERVALS ON WHICH A FUNCTION IS INCREASING OR DECREASING

Let f be continuous on the (a,b). To find the open intervals on which f is increasing or decreasing, use the following steps.

1. Locate the ______________________ numbers of f in (a,b), and use these numbers to determine test intervals.
2. Determine the sign of ____________ at one test value in each of the intervals.
3. Use the test for increasing and decreasing functions to determine whether f is increasing or decreasing on each ___________________.

These guidelines are also valid if the interval (a,b) is replaced by an interval of the form $(-\infty,b)$, (a,∞), or $(-\infty,\infty)$.

Example 1: Identify the open intervals on which the function is increasing or decreasing.

$f(x) = 27x - x^3$

a. Find the critical numbers of f.

b. Run the test for increasing and decreasing intervals.

i. Find the open interval(s) on which the function is decreasing.

ii. Find the open interval(s) on which the function is increasing.

Example 2: Identify the open intervals on which the function is increasing or decreasing.

$$f(x) = \cos^2 x - \cos x,\ \ 0 < x < 2\pi$$

a. Find the critical numbers of f.

b. Run the test for increasing and decreasing intervals.

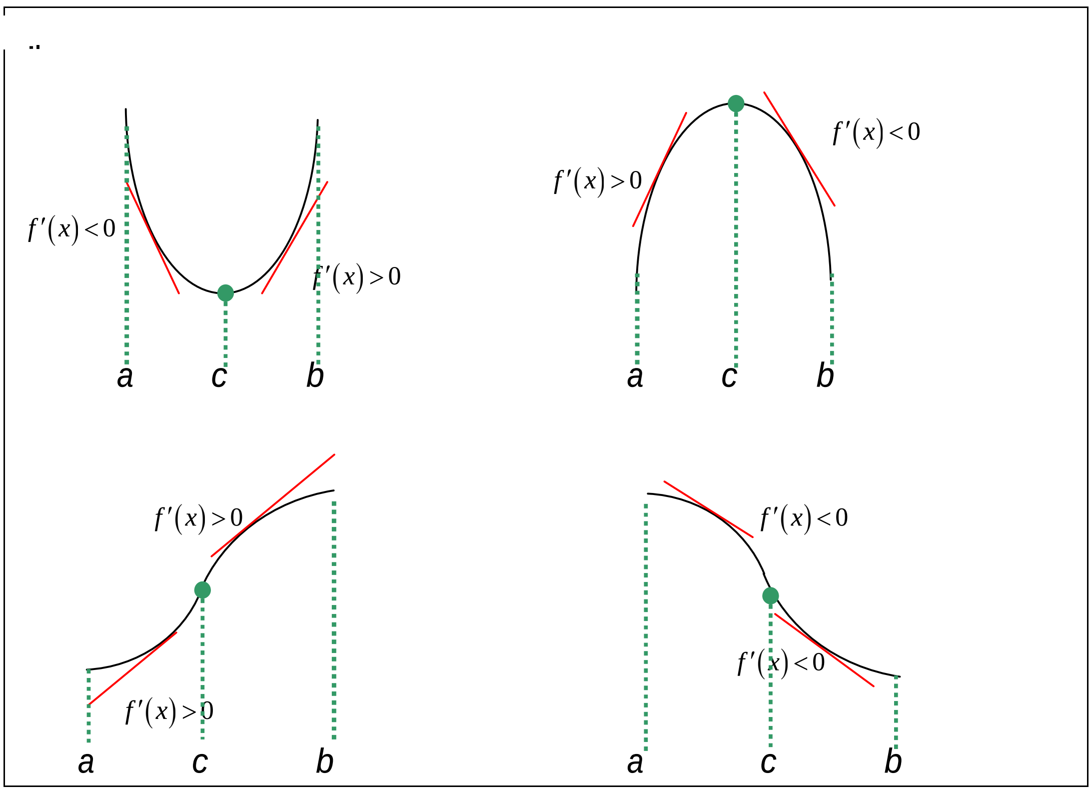
i. Find the open interval(s) on which the function is decreasing.

ii. Find the open interval(s) on which the function is increasing.

THEOREM: THE FIRST DERIVATIVE TEST

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows:

1. If $f'(x)$ changes from negative to positive at c, then f has a **relative minimum** at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a **relative maximum** at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum or relative maximum.



Example 3: Consider the function $g(x)=x^{2/3}-4$.

a. Find the critical numbers of g.

b. Run the test for increasing and decreasing intervals.

i. Find the open interval(s) on which the function is decreasing.

ii. Find the open interval(s) on which the function is increasing.

c. Apply the First Derivative Test.

i. Identify all relative minima.

ii. Identify all relative maxima.

Example 4: The graph of a function f is given. Sketch a graph of the derivative of f.