When you are done with your homework you should be able to…

- π Find the derivative of a function using the product rule
- π Find the derivative of a function using the quotient rule
- π Find the derivative of a trigonometric function
- π Find a higher-order derivative of a function

Warm-up: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

1. $f(x)=\dfrac{3x^2-x+2}{\sqrt{x}}$

2. $g(x)=(5x-3)^2$

3. $f(x)=\cos\left(x-\dfrac{\pi}{4}\right)$

THEOREM: THE PRODUCT RULE

The product of two differentiable functions f and g is itself differentiable. Moreover, the derivative of fg is the derivative of the first function times the second function, plus the first function times the derivative of the second function.

$$\frac{d}{dx}\left[f(x)g(x)\right]=f'(x)g(x)+f(x)g'(x)$$

This rule extends to cover products of more than two factors. For example the derivative of the product of functions $fghk$ is

$$\frac{d}{dx}[fghk]=f'(x)g(x)h(x)k(x)+f(x)g'(x)h(x)k(x)+f(x)g(x)h'(x)k(x)+f(x)g(x)h(x)k'(x)$$

Example 1: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

a. $g(x)=x\cos x$

b. $h(t)=\left(3-\sqrt{t}\right)^2$

c. $f(x)=\left(x^3-x\right)\left(x^2+2\right)\left(x^2+x-1\right)$

THEOREM: THE QUOTIENT RULE

The quotient of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$. Moreover, the derivative of f/g is the derivative of the numerator times the denominator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Example 2: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

a. $g(x) = x^4\left(1 - \frac{2}{x+1}\right)$

b. $h(s) = \frac{s}{\sqrt{s} - 1}$

c. $f(x) = \tan x$

THEOREM: DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Example 3: Find the derivative of the trigonometric functions.

a. $g(x) = -2\csc x$

b. $h(t) = \cot^2 t$

c. $r(s) = \frac{\sec s}{s}$

HIGHER ORDER DERIVATIVES

Recall that you can obtain _______________ by differentiating a position function.
You can obtain an _______________ function by differentiating a velocity function. Or, you could also think about the acceleration function as the _______________ derivative of the _______________ function.

Example 4: An automobile's velocity starting from rest is $v(t)=\frac{100t}{2t+15}$ where v is measured in feet per second. Find the acceleration at

a. 5 seconds

b. 10 seconds

c. 20 seconds

NOTATION FOR HIGHER-ORDER DERIVATIVES

First derivative					
Second derivative					
Third derivative					
Fourth derivative					
***n*th derivative**					

Example 5: Find the given higher-order derivative.

a. $f''(x)=2-\dfrac{2}{x}, \quad f'''(x)$

b. $f^{(4)}(x)=2x+1, \quad f^{(6)}(x)$