When you are done with your homework you should be able to…

- π Find the derivative of a function using the constant rule
- π Find the derivative of a function using the power rule
- π Find the derivative of a function using the constant multiple rule
- π Find the derivative of a function using the sum and difference rules
- π Find the derivative of the sine function and of the cosine function
- π Use derivatives to find rates of change

Warm-up: Find the following derivatives using the limit definition of the derivative.

1. $f(x) = 2$

2. $f(x) = x^2$

3. $f(x) = \cos x$

THEOREM: THE CONSTANT RULE

The derivative of a constant function is zero. That is, if c is a real number, then

$$\frac{d}{dx}[c]=0$$

Hmmm…isn't this theorem the equivalent of saying that the _________ of a ___________________ line is zero?

Example 1: Find the derivative of the function $g(x)=6$.

THEOREM: THE POWER RULE

If n is a rational number, then the function $f(x)=x^n$ is differentiable and

$$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$$

For f to be differentiable at $x=0$, n must be a number such that x^{n-1} is defined on an interval containing zero.

Example 2: Find the following derivatives.

a. $f(x)=x^5$

b. $f(x)=x^{1/2}$

c. $f(x)=x^{-5/3}$

THEOREM: THE CONSTANT MULTIPLE RULE

If f is a differentiable function and c is a real number, then cf is also differentiable and

$$\frac{d}{dx}\left[cf(x)\right] = cf'(x)$$

Example 3: Find the slope of the graph of $f(x) = 2x^3$ at

a. $x = 2$

b. $x = -6$

c. $x = 0$

THEOREM: THE SUM AND DIFFERENCE RULES

The sum (or difference) of two differentiable functions f and g is itself differentiable. Moreover, the derivative of $f+g$ (or $f-g$) is the sum (or difference) of the derivatives of f and g.

$$\frac{d}{dx}\left[f(x)+g(x)\right] = f'(x)+g'(x)$$

$$\frac{d}{dx}\left[f(x)-g(x)\right] = f'(x)-g'(x)$$

Example 4: Find the equation of the line tangent to the graph of $f(x) = x - \sqrt{x}$ at $x = 4$.

THEOREM: DERIVATIVES OF THE SINE AND COSINE FUNCTIONS

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

Example 5: Find the derivative of the following functions:

a. $f(x) = \frac{\sin x}{6}$

b. $r(\theta) = 5\theta - 3\cos\theta$

RATES OF CHANGE

We have seen how the derivative is used to determine ____________. The derivative may also be used to determine the _________ of ____________ of one _____________ with respect to another.

A common use for rate of change is to describe the motion of an object moving in a straight line. In such problems, it is customary to use either a horizontal or a vertical line with a designated origin to represent the line of motion. On such lines, movement to the __________ or _______________ is considered to be in the positive direction, and movement to the left or downwards is considered to be in the _______________ direction.

THE POSITION FUNCTION is denoted by s and gives the position (relative to the origin) of an object as a function of time. If, over a period of time Δt, the object changes its position by $\Delta s = s(t+\Delta t) - s(t)$, then, by the familiar formula

$$\text{rate} = \frac{\text{distance}}{\text{time}}$$

the **average velocity** is

$$\frac{\text{change in distance}}{\text{chang in time}} = \frac{\Delta s}{\Delta t}$$

Example 6: A ball is thrown straight down from the top of a 220-foot building with an initial velocity of -22 feet per second. The position function for free-falling objects measured in feet is $s(t) = -16t^2 + v_0 t + s_0$.

What is its velocity after 3 seconds?

What is its velocity after falling 108 feet?